

SPECIAL MODELS AND PREDICTIONS FOR PION
PHOTOPRODUCTION (LOW ENERGIES)*)

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I. INTRODUCTION

The theoretical investigations on pion photoproduction can be classified into two groups: the phenomenological analysis and the attempts to treat the dynamics of the $\pi N \gamma$ system.

In the phenomenological analysis¹⁾ only the general theoretical principles are used, namely

- a) Lorentz and gauge invariance (including space and time reflection);
- b) the principle of minimal electromagnetic interaction, which states that the electromagnetic interaction has to be introduced by $p_\mu \rightarrow p_\mu - eA_\mu$. It leads to a relation between the photoproduction processes in different charge states;
- c) the unitarity condition. Together with the time reversal invariance it allows one to deduce the 'final state theorem', according to which the phase of a multipole amplitude is equal to the scattering phase shift in the final state, if the energy is below the inelastic threshold

$$M_{\ell\pm} = \pm |M_{\ell\pm}| e^{i\delta_{\ell\pm}} . \quad (1)$$

In (b) and (c) the electromagnetic field is treated only to lowest order.

The aim of the phenomenological analysis (or multipole analysis) is analogous to that of the phase-shift analysis in πN scattering. One tries to determine the multipole amplitudes from the experimental data, since these amplitudes are much better suited for a comparison with the predictions of a dynamical theory than the cross-sections.

A dynamical theory which allows one to calculate the photoproduction amplitudes from first principles does not yet exist. In recent years all attempts to predict the amplitude were based on the dispersion relation approach²⁻⁴⁾, which has a more modest aim. One tries to calculate the photoproduction amplitudes from some consequences of the axioms of field theory⁵⁾ together with the experimental information from other reactions, as for instance πN and ep scattering.

Editor's Note: Professor Höhler presented this paper both at the CERN School at Bad Kreuznach and at the International Symposium on Electron and Photon Interactions held at Hamburg in June 1965. When the words "this Conference" occur in the text of the paper it is the latter Meeting to which reference is made. Thanks are due to the Springer-Verlag for permission to reprint this article.

At present the dispersion relation approach is not a systematic theory. In order to obtain a prediction one has to make drastic approximations, which were found using the static model of Chew and Low as a guide²⁾. There is no reliable way to estimate the errors.

Omitting all indices the dispersion relation at fixed t for the production amplitude reads

$$\operatorname{Re} A(s, t) = A_{\text{pole}}(s, t) + \frac{1}{\pi} P \int_{-\infty}^{\infty} ds' \operatorname{Im} A(s', t) \left\{ \frac{1}{s' - s} \pm \frac{1}{s' - \bar{s}} \right\} \quad (2)$$

$$(M+1)^2$$

where $s = W^2$, $W =$ total energy in the c.m. system, $t =$ invariant momentum transfer squared, $\bar{s} = -s - t + 2M^2 + 1$, $m_{\pi} = 1$, $M =$ mass of the nucleon.

The pole term follows from the one-nucleon intermediate states. It is also called the 'Born term', since it happens to agree with the amplitude calculated from the Feynman graphs of Fig. 1, if the pseudoscalar πN coupling is used ($f^2 = g^2/4M^2 = 0.081$) and the electromagnetic coupling of the anomalous moment μ' of the nucleon is taken into account explicitly.

The most important contribution to $\operatorname{Im} A(s', t)$ in the dispersion integral is expected to belong to the isobar intermediate state

$$\pi + N \rightarrow \Delta \rightarrow \pi + N \quad ,$$

where Δ denotes the $\mathfrak{33}$ -resonance at the total c.m. energy $W = 1236$ MeV. This transition can proceed via the magnetic dipole amplitude M_{33} and the electric quadrupole amplitude E_{33} only. If all the other contributions to $\operatorname{Im} A$ are neglected and $\operatorname{Re} A$ is calculated from Eq. (2), the result will be called in the following the 'isobar approximation'.

II. THE ISOBAR APPROXIMATION

1. The resonant multipoles

The first problem in all investigations using the isobar approximation is to find an expression for the energy dependence of the resonant multipoles M_{33} and E_{33} .

CGLN²⁾ derived approximate 'dispersion relations' for the resonant partial waves by a projection of the fixed- t dispersion relations, neglecting the non- $\mathfrak{33}$ contributions to the dispersion integral and considering the static limit and some recoil corrections. They found that the 'dispersion relations' for the resonant πN partial wave f_{33} and the resonant multipole M_{33} agree, if the formula

$$M_{33} = \frac{u}{f} \frac{k}{q} f_{33} ; \quad f_{33} = \frac{e^{i\alpha_{33}} \sin \alpha_{33}}{q}$$

is assumed to hold and only a certain part of the Born term of M_{33} is taken into account,

namely the static limit of the anomalous magnetic part plus the recoil correction of order $1/M$ to the electric part. In Eq. (3) $\mu_v = (\mu_p - \mu_n)/2$ denotes the isovector part of the total magnetic moment of the nucleon, k the photon momentum, and q the pion momentum in the c.m. system^{*) 6)}. CGLN also gave an estimation of the remaining part of M_{33} and of E_{33} , assuming that these multipole amplitudes have a similar behaviour as in the static model. The contribution of these terms is small in comparison with Eq. (3).

Several authors have tried to improve the result of CGLN. Recently Finkler⁷⁾ used the Omnès method, assuming that f_{33} , M_{33} , and E_{33} have the same real phase at all energies. His further assumption that all non- 33 contributions to the dispersion integrals can be neglected has to be discussed critically, since Donnachie and Hamilton⁸⁾ had found an appreciable $T = 0$ $\pi\pi$ contribution in their investigation of the resonant f_{33} amplitude. Finkler corrected Eq. (3) by a factor on the right-hand side, which is about 0.9 at the resonance and decreases at higher energies, similar to McKinley's result⁹⁾. Furthermore, he obtained a large correction to the CGLN estimate of the E_{33}/M_{33} ratio, which is of special importance for the π^0 production with polarized γ rays¹⁰⁾. A detailed discussion has not yet been made, but it seems that the discrepancy found in earlier discussions will be reduced considerably by Finkler's treatment.

2. The work of Ball and Schmidt

Ball¹¹⁾ approximated $\text{Im } A$ by the contribution of $\text{Im } M_{33}$ alone, taking M_{33} from Eq. (3). Then he evaluated the dispersion integrals and calculated the cross-sections without further approximations. Unfortunately his numerical results are not reliable, because there is an error in his Eq. (8.29), and also in the D coefficient for π^0 production.

Ball's work was continued by Schmidt¹²⁻¹⁴⁾, who calculated predictions for all measured quantities up to 500 MeV, except for the polarization of the recoil proton in π^0 production, which was treated by Müllensiefen¹⁵⁾.

Since f^2 and α_{33} were taken from the scattering data, the prediction is an absolute one, it contains no adjustable parameters.

A comparison with the experimental data shows that the Ball-Schmidt calculation leads to a reasonable zero order approximation for the photoproduction cross-sections, including the measurements with polarized γ rays and the π^-/π^+ ratios. However, one should notice that the agreement is mainly due to the fact that the cross-sections are dominated by the pole terms and the resonance effects. From the experience with the isobar approximation in πN scattering^{**) 16)} one would expect that the Ball-Schmidt result could be quite wrong for some of the small multipoles, especially for those leading to $T = 1/2$ final states.

Figure 2 shows a contour diagram of the difference between Schmidt's prediction for π^+ production and the experimental data, interpolated by a Moravcsik fit. The deviation amounts to 15% at its maximum near 280 MeV. In the region of small angles the extrapolation of the experimental data is only a crude estimate, and the magnitudes of the deviations will be much better known after the completion of the new measurements at Orsay and Bonn.

*) Other derivations of Eq. (3) are discussed in part 9.3 of Ref. 1, and part 2.2.3 of Ref. 6.

**) See Fig. 6 of Ref. 16.

Baldin¹⁷⁾ has pointed out that it is interesting to consider the cross-section at a fixed momentum transfer t which is equal to its value at threshold, since in this case the integrand of the dispersion integral (2) is used in the physical region of (s',t) only. In all other cases there is an additional uncertainty following from the extrapolation of $\text{Im } A(s',t)$ into the unphysical region. According to Fig. 3 the agreement between the Ball-Schmidt prediction and the experimental data is good up to 260 MeV. It is interesting to notice that in this region the data can be as well described by the cross-section calculated from the pole term alone (= second order Born approximation). At higher energies there is an increasing and rather large deviation (cf. the curve $t = -0.87$ in Fig. 2) which must be due to an error of $\text{Im } A$ in the physical region.

The error caused by the extrapolation into the unphysical region should be especially large for the excitation curve at 180° ; however, Fig. 2 shows that in this case the Ball-Schmidt prediction agrees very well with the experiments.

Although the recent π^+ production data at 90° near threshold are very accurate, they do not allow one to test the dispersion integral contribution, since the Born term is so much larger (Fig. 4). Unfortunately, one cannot use these data for an accurate determination of the coupling constant f^2 , since one expects several other slowly energy-dependent contributions from the dispersion integral, which cannot be estimated in a reliable way. The same difficulty is present, if one wants to test the well-known relation between the Panofsky ratio, the difference of the πN S-wave scattering lengths, and the photoproduction data¹⁸⁾.

The experimental data on π^+ production with polarized γ rays are compared with the prediction in Fig. 5.

The experimental information on π^0 production is not as good as for π^+ production. Figures 6, 7, and 8 show that the Ball-Schmidt prediction again is a zero order approximation. However, it is easier to find large deviations, since the Born term is smaller than for π^+ production and partly compensated by an indirect effect of the resonance. Therefore, the cross-sections are more sensitive to the 'small' multipoles, for which the isobar approximation is not reliable. For instance, there is a large discrepancy¹³⁾ in the energy dependence of B and C ($\sigma = A + B \cos \Theta + C \cos^2 \Theta$) below 300 MeV and in the ratio A/C , which follows from the experiments with polarized γ rays¹⁰⁾.

It is astonishing that the simple Ball-Schmidt calculation describes so many features of pion photoproduction, although it does not contain adjustable parameters. If one discusses the comparison with the experimental data, one should keep in mind that appreciable corrections are expected from several other contributions to the amplitude, but at present they cannot be calculated in a reliable way. The most interesting experiments are those which show deviations from the prediction far outside the errors, since they help to identify those parts of the theory which are in need of improvement.

The work of Schmidt is only the simplest version of the isobar approximation. It should be improved by taking into account the unitarity condition for the 33 -multipoles in a better way. First steps in this direction have already been made by CGLN²⁾, Ball¹¹⁾, and by McKinley⁹⁾. It will be interesting to see to what extent Finkler's careful treatment⁷⁾ of the unitarity condition for the resonant multipoles diminishes the discrepancies found by Schmidt.

3. Feynman graphs

Amati and Fubini¹⁹⁾ have pointed out that in the limit of a narrow resonance the isobar approximation of the dispersion integral leads to the same result as the evaluation of the Feynman graphs of Fig. 9. The first term in the integrand of Eq. (2) corresponds to the graph I, and the second term to II.

It is interesting to notice that the $1/(s'-s)$ term in the dispersion integral gives a large contribution to $\text{Re } E_{0+}^{(+)}$ and thereby to a $J = 1/2$ final state¹⁸⁾. For the corresponding graph I in Fig. 9 this is somewhat unexpected, since the intermediate isobar state has the spin $3/2$. However, if one starts from the usual interaction Lagrangian one finds an additional term in the interaction Hamiltonian which leads to the $J = 1/2$ final state^{*) 20)}.

Gourdin and Salin²¹⁾, and Rashid and Moravcsik²²⁾ treated the isobar intermediate states in such a way that the graph I does not contribute to $J = 1/2$ final states. It is not clear to me how these calculations can be justified from the general theoretical principles.

The comparison between Schmidt's calculation and the Feynman graph formulae gives the values of the γNN^* coupling constant, the πNN^* coupling constant following from a similar treatment of πN scattering¹⁸⁾. Furthermore, it shows that the description of the finite width by a constant imaginary part of the isobar mass ($M^* + i\Gamma$) is not sufficient for quantitative purposes. If one wants to use a Breit-Wigner type formula one has to assume a strongly energy-dependent width^{**) 23)}.

III. CORRECTIONS TO THE ISOBAR APPROXIMATION

1. Final-state corrections to the non- 33 multipoles

In the isobar approximation the unitarity condition is not fulfilled for the non- 33 multipoles. An estimation of the final-state corrections was given by CGLN²⁾. It was improved by taking into account relativistic kinematics in the paper of McKinley⁹⁾, but the theoretical derivation is still more or less doubtful. Also the addition of these corrections to the isobar approximation amplitude does not lead to a better agreement with the experiments.

In my opinion one does not gain much information if the experiments are compared with a prediction which contains many uncertainties. It is better to determine the multipoles by a phenomenological analysis (part 4) and to compare each multipole with the theoretical expression as given, for instance, in the paper of McKinley.

*) I am much indebted to Professor H. Umezawa, Professor A. Visconti, and Dr. G. von Gehlen for discussions on this question. Also, I would like to thank Dr. F. Hadjiannou for sending me a preprint which treats a closely related aspect of this problem²⁰⁾.

**) Compare the interesting discussion of the isobaric model in the review article of Gell-Mann and Watson²³⁾.

2. The ρ -exchange contribution

In the isobar approximation the dispersion integral does not contribute to the isoscalar part A_1^0 of the amplitude. But if one considers the fixed- s dispersion relation¹¹⁾ or the Cini-Fubini approximation to the Mandelstam representation²⁴⁾, one is led to expect a contribution from the ρ -meson exchange in the t channel (Fig. 10a). Ball¹¹⁾ succeeded in expressing the result by the isovector part of the electromagnetic nucleon form factor which is assumed to be dominated by the ρ -exchange effect (Fig. 10b).

If an empirical fit to the experimental form factors and the new data for the ρ resonance are inserted into Ball's result, one finds for the ρ -exchange contribution to the amplitude¹⁸⁾ (we give A_1^0 only)

$$A_{1\rho}^0(s,t) = 4.1 \frac{\Lambda}{e} \mu_V' \left[0.50 + \frac{0.60t}{18-t} \right]. \quad (4)$$

For energies in the region of the first resonance or below, t is so small that the first term in the bracket is dominating and a multipole decomposition shows that the main contributions belong to E_{0+} and M_{1-} . It will be very difficult to distinguish the ρ -exchange parts from other corrections to these multipoles (for instance, from final-state interactions or high-energy contributions to the dispersion integrals) as long as one considers π^+ or π^0 production directly. The situation is more favourable if the data are combined in such a way that the isoscalar part is isolated or enhanced (π^-/π^+ ratio). At present there is no convincing evidence for the ρ -exchange effect¹⁸⁾. Ball's coupling constant Λ is smaller than 0.5 e, unless the ρ exchange is masked by another correction to the isobar approximation.

Gourdin et al.²⁴⁾ have also noticed the relation to the nucleon form factor, but instead of evaluating the first term in the bracket of Eq. (4) they suggested treating it as an adjustable parameter. The same suggestion was made by de Tollis et al.²⁵⁾. Since McKinley⁹⁾ neglected this term which is the dominating one in Ball's result, without giving a reason, his treatment of the ρ -exchange effect is questionable.

The values of Λ given in the experimental papers should not be compared with each other without a critical examination of the underlying theoretical analysis. In several cases the results for Λ differ not only because of the experimental data but also because of different assumptions and definitions.

IV. PHENOMENOLOGICAL ANALYSIS

1. Summary of the results of the phase shift analysis

Because of the close connection between pion photoproduction and πN scattering it is useful to summarize first our knowledge of the πN system as obtained from the scattering data, which have considerably improved during the last year^{*)}. The quantum numbers of the 2nd, 3rd, and 4th resonances are now well established and it is clear that other strong effects occur mainly in the states P_{11} , S_{11} , S_{13} (indices: $2T, 2J$)^{**)}. The following table gives some of the properties which are relevant for photoproduction. All energies are γ -laboratory energies, $E_\gamma = T_\pi + 150$ MeV.

	T	J ^P	multipoles	$\delta = 90^\circ$ E_γ (MeV)	η_{\min}	$ Re f = \max$ at E_γ (MeV)
Δ (1236) 1st res.	$\frac{3}{2}$	$\frac{3}{2}^+ P_{33}$	M_{1+}, E_{1+}	345	1.0	290, 480
N (1525) 2nd res.	$\frac{1}{2}$	$\frac{3}{2}^- D_{13}$	M_{2-}, E_{2-}	770	0.25	680
N (1680) 3rd res.	$\frac{1}{2}$	$\frac{5}{2}^+ F_{15}$	M_{3-}, E_{3-}	1040	0.6	960, 1120
Δ (1920) 4th res.	$\frac{3}{2}$	$\frac{7}{2}^+ F_{37}$	M_{3+}, E_{3+}	1495	0.2	1290, 1680
N (1400) ?	$\frac{1}{2}$	$\frac{1}{2}^- P_{11}$	M_{1-}	≈ 750 ($\delta \approx 80^\circ$)	0.2	490

δ is the real part of the phase shift, η the absorption parameter. The last column gives the energy at which the real part of the resonant scattering amplitude has its maximum. It is an estimate of the position of a peak or a dip caused by the interference between the real part of the resonant multipole and a slowly varying real background amplitude.

In the discussion of the question whether the P_{11} phenomenon is a 'resonance' one should keep in mind that the notion of a resonance is not sharply defined. There is a continuous transition to several other phenomena and therefore to a certain extent it is a matter of convention and of convenience how to define a resonance. For instance, the

*) Compare the papers presented at the Royal Society Meeting in London (11 February 1965), to be published in the Proceedings of the Royal Society.

***) Note added in proof: recently several authors have found evidence for a $T = \frac{1}{2} \frac{5}{2}^- (D_{15})$ resonance near N(1680).

properties of a resonance are considerably changed if a threshold is nearby or if there is a large background in the same partial wave. Also, in many models a resonance occurs together with strong variations in other partial waves, and it might be unsuitable to consider it separately.

It will be very interesting to see if the P_{11} phenomenon occurs in photoproduction as inconspicuously as in πN scattering, or if it is enhanced for some reason as in the final state of pp scattering at small momentum transfer and high energies²⁶⁾.

Presumably the 'shoulder' in the total π^+p cross-sections near $T_\pi = 750$ MeV is not caused by a resonance, but it seems²⁷⁾ that an important contribution comes from a strong variation in S_{31} .

2. The work of Gourdin and Salin

In their well-known work on the 'isobaric model' Gourdin and Salin²¹⁾ have described the π^+ and π^0 production data up to $E = 800$ MeV by an ansatz which uses Breit-Wigner type formulae for the resonances, treating the coupling constants, the widths, and several background terms as adjustable parameters. This investigation was performed three years ago. In the meantime our knowledge of the πN system has considerably improved, and the present status leaves little hope that the simple ansatz of Gourdin and Salin, or Rashid and Moravcsik²²⁾ is adequate for a quantitative description of photoproduction.

Of course, the ansatz could be extended by admitting additional parameters for the background multipoles and introducing the important energy dependence²³⁾ of the resonance widths Γ (Γ_{33} for πN scattering changes by a factor of two between $\alpha_{33} = 45^\circ$ and 135°). But there remains the question of uniqueness which has led to so many difficulties in the simpler case of πN scattering, and has not yet been discussed in photoproduction.

3. The work of Schmidt, Schwidersky and Wunder

As mentioned in II.2, the general features of pion photoproduction below 500 MeV are well described by the results of Schmidt's evaluation of the isobar approximation. Therefore, it seems reasonable to consider the possibility that the exact multipole amplitudes differ only by small corrections from the multipoles of the Ball-Schmidt approximation. In order to find these corrections, Schmidt²⁸⁾ has calculated the variation of his prediction for the cross-section σ_{BS} if the real and imaginary part of one of the s, p, or d-wave multipoles is changed by a small amount. The result is plotted at fixed energies as a function of angle. It allows easy discussion of the different possibilities for corrections of σ_{BS} which lead to a better agreement with the experimental data.

This method corresponds to a multipole analysis which is limited to sets of multipoles in the neighbourhood of the Ball-Schmidt amplitudes.

The π^0 and π^+ production data near the 2nd resonance were analysed by Schmidt, Schwidersky and Wunder²⁹⁾, assuming that the cross-sections can be described by the Ball-Schmidt amplitude and additions (which are not necessarily small) to E_{2-} , M_{2-} , and a few other multipoles. The isobar approximation to the dispersion relation approach cannot be applied in this region because the polynomial expansion in $\cos \Theta$ of $\text{Im } A(s', t)$ does not converge any more. However, this is no objection against using the empirical fact that the extension of Schmidt's calculations to these energies reproduces the general features of the non-resonant background.

It turns out that a good fit can be obtained (Fig. 11) for a resonance-like behaviour of E_{2-} and M_{2-} . However, the present data admit several solutions and there could be others which were not found because of the restricted assumptions.

The discussion of the polarization of the recoil proton in π^0 production^{15, 29)} has shown that a broad peak above 500 MeV is expected from the background effects alone. It would be very interesting to look for a superimposed structure at the energy of the 2nd resonance, taking into account that according to the recent results of the phase shift analysis its width is much narrower than formerly supposed²⁷⁾.

Finally, we compare in Fig. 12 the position of the resonances defined by $\delta = 90^\circ$ with the sum of the total π^0 and π^+ cross-sections. This quantity was chosen in order to eliminate all interference terms which could cause a shift of the peaks. It is seen that in all three cases there is a shift to the low-energy side, which presumably has to be explained for the higher resonances in the same way as for the well-known first resonance.

V. CONCLUDING REMARKS

In earlier summaries on the status of photoproduction several simple models played an important role which were not treated above, for instance, the model of Peierls³⁰⁾ which has led to the correct predictions for the quantum numbers of the 2nd and 3rd resonances.

Unfortunately, there are many indications that photoproduction is more complicated than assumed in these models and cannot be described quantitatively by a small number of simple terms.

We have seen that in the low-energy region there is a remarkable success of the isobar approximation to the dispersion relation approach. Although it does not contain adjustable parameters, the predictions for the absolute values of the cross-sections are in most cases in reasonable agreement with the experimental data. So it seems that this approximation is a good starting point for further improvements.

At present there is no convincing evidence for the contributions of ω and ρ exchange processes to single-pion photoproduction. This question deserves further study since it would be very interesting to determine the coupling constants. Possibly one will encounter similar difficulties as those mentioned by Professor van Hove³¹⁾ for πN charge exchange scattering. In this case the data seem to be compatible with the assumption of the exchange of a ρ Regge pole³²⁾.

Presumably further progress in the theory of pion photoproduction will be made in a similar way as in pion-nucleon scattering during the last years²⁷⁾. The phenomenological analysis of the forthcoming data will give information on the energy dependence of the multipole amplitudes which can be compared with the results of a more detailed study of their dispersion relations. This does not necessarily mean that the theory will become more and more complicated since one could hope that someone will find a simple physical picture for the dominating parts of the production amplitude.

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Figure captions

- Fig. 1 Feynman graphs of the Born terms.
- Fig. 2 Contour diagram of the difference between the experimental differential cross-sections and Schmidt's prediction in $\mu\text{b}/\text{st}$. The curve denoted by $t = -0.87$ gives the relation between energy and angle if the four-momentum transfer t is equal to its value at threshold.
- Fig. 3 π^+ cross-section for a momentum transfer equal to its threshold value (cf. Fig. 2).
- Fig. 4 π^+ cross-section at 90° c.m. near threshold. $\sigma_p =$ Born cross-section.
- Fig. 5 π^+ excitation curve for plane-polarized γ rays.
- Fig. 6 Excitation curve for π^0 production at 90° c.m.s.
- Fig. 7 Excitation curve for π^0 production at 0° . Dashed line: estimation of the correction from $\text{Im } M_{33} \cdot \text{Im } E_{0+}$.
- Fig. 8 π^0 angular distributions at 360 and 450 MeV. Dashed line as in Fig. 7.
- Fig. 9 Feynman graphs for isobar intermediate states.
- Fig. 10 Graphs for ρ -exchange processes.
- Fig. 11 π^0 production in the region of the 2nd resonance. Experimental points: Δ gives the difference between the data and Schmidt's calculation which is assumed to describe the main background effects. Solid line: best fit obtained by additional contributions in E_{2-}, M_{2-}, E_{0+} .
- Fig. 12 Sum of total π^0 and π^+ cross-sections. $\delta =$ real part of the resonant phase shift.

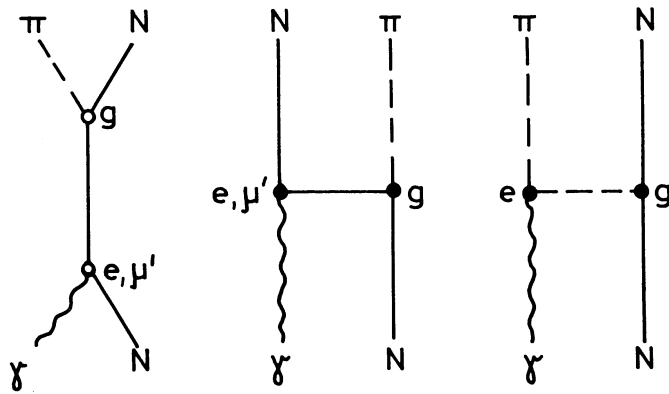


Fig. 1

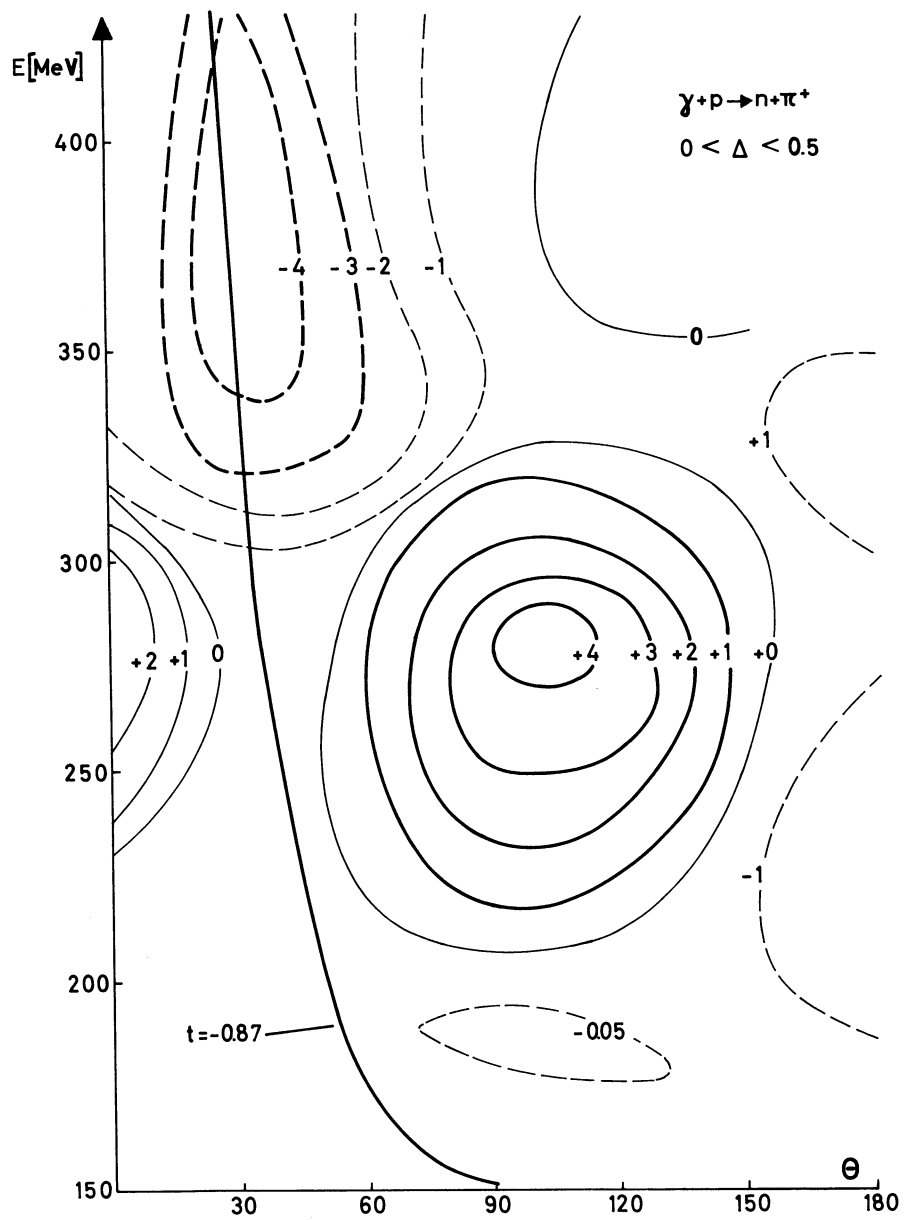


Fig. 2

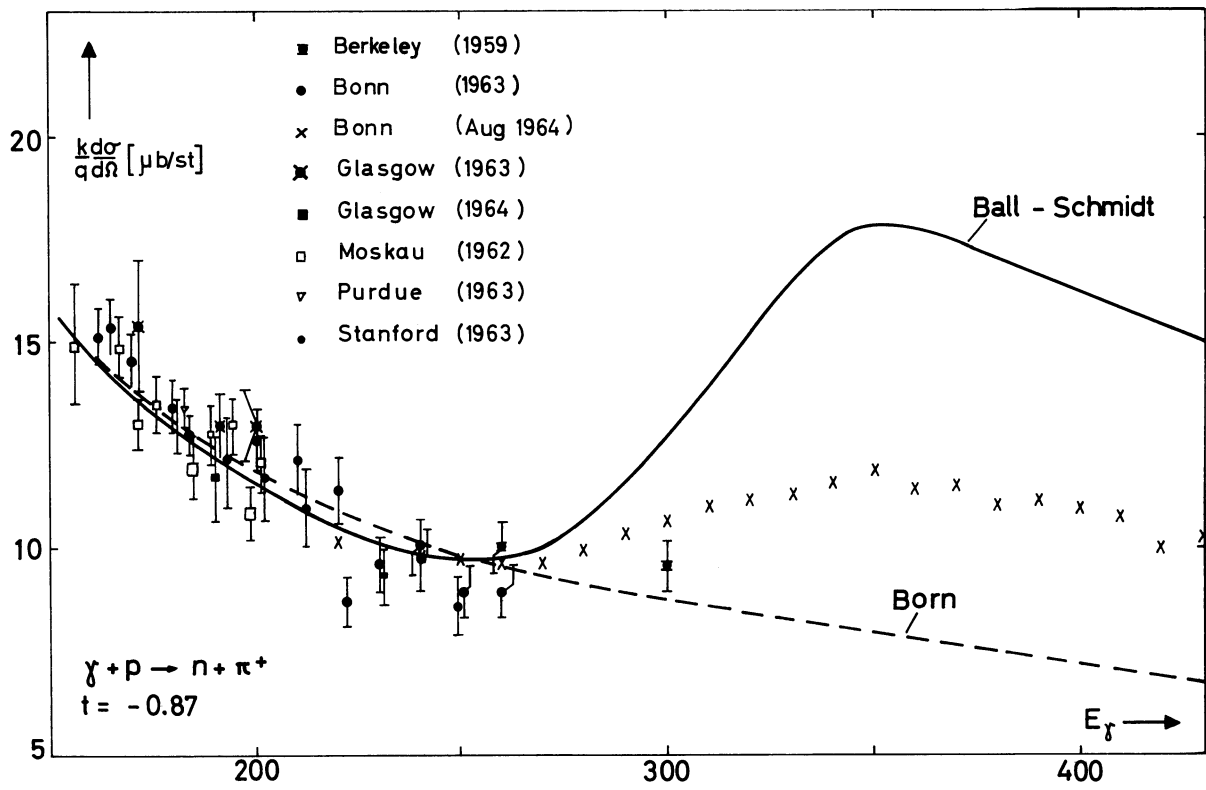


Fig. 3

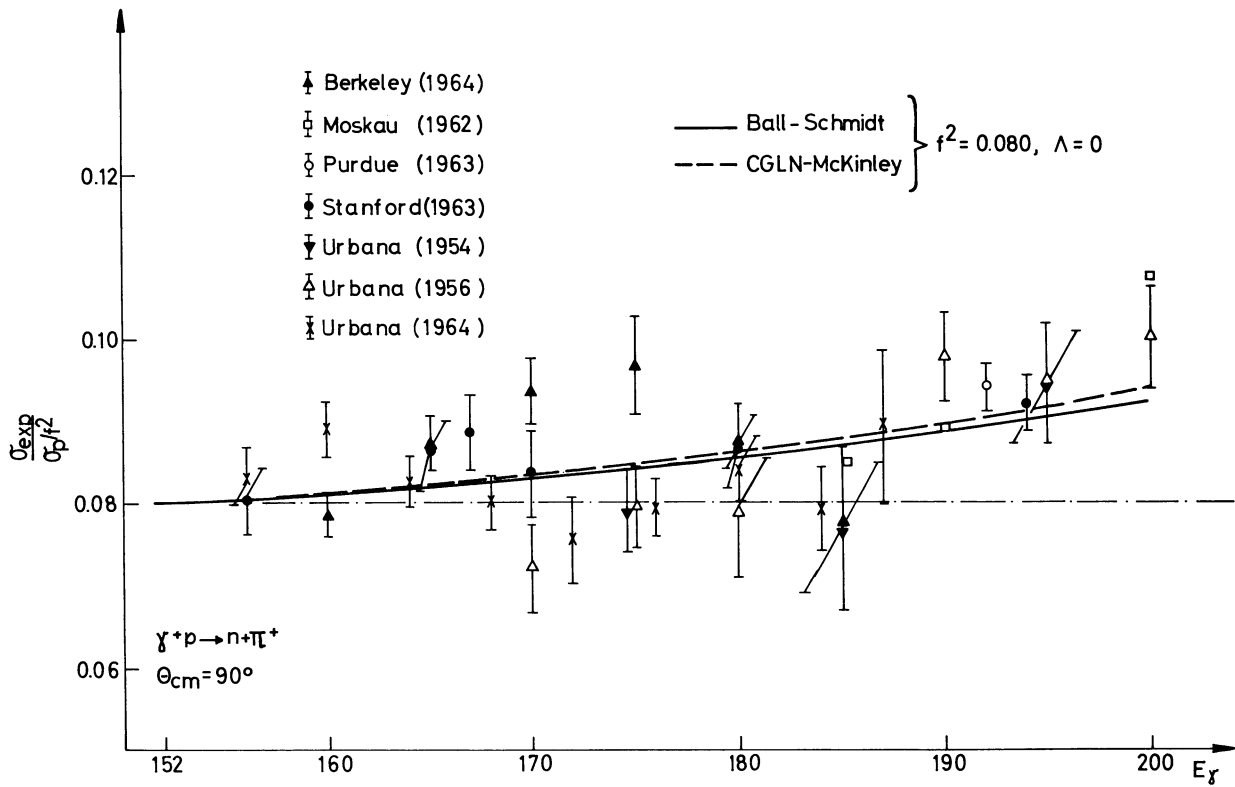


Fig. 4

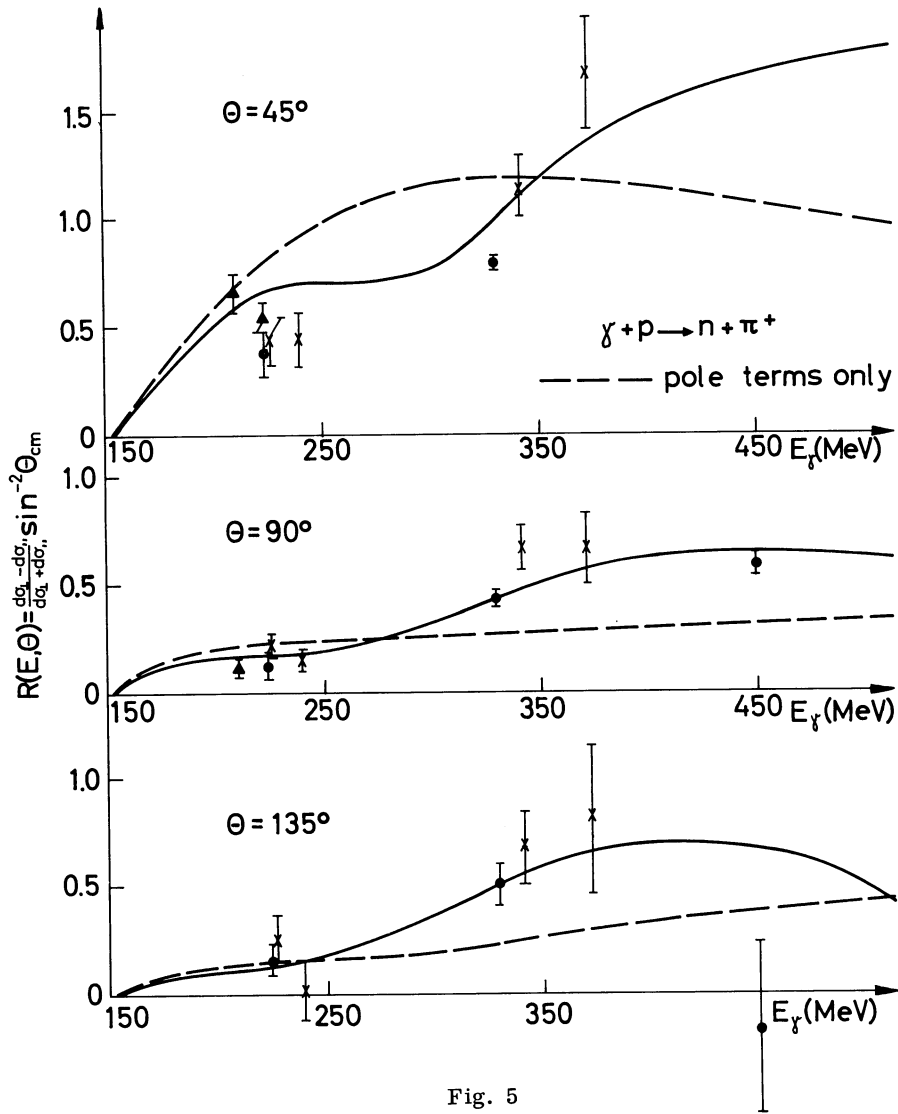


Fig. 5

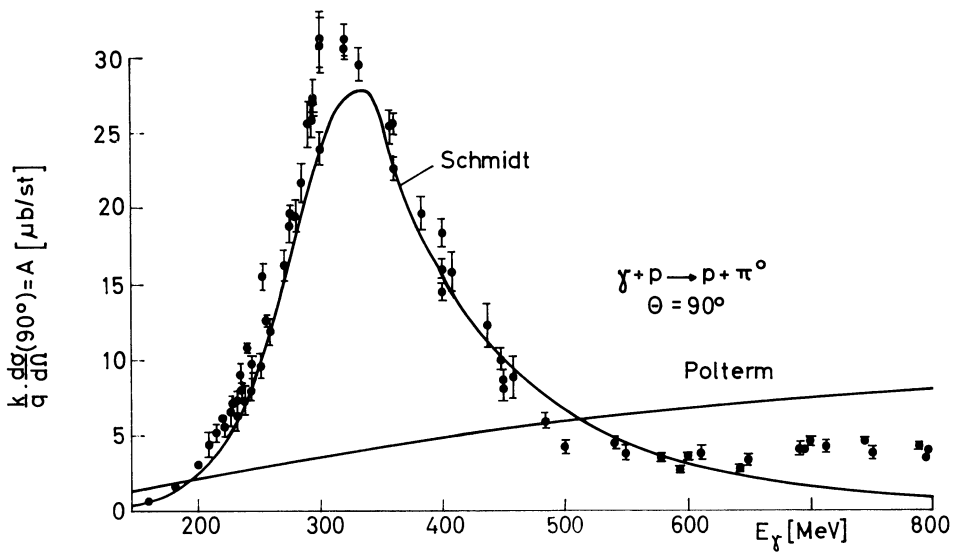


Fig. 6

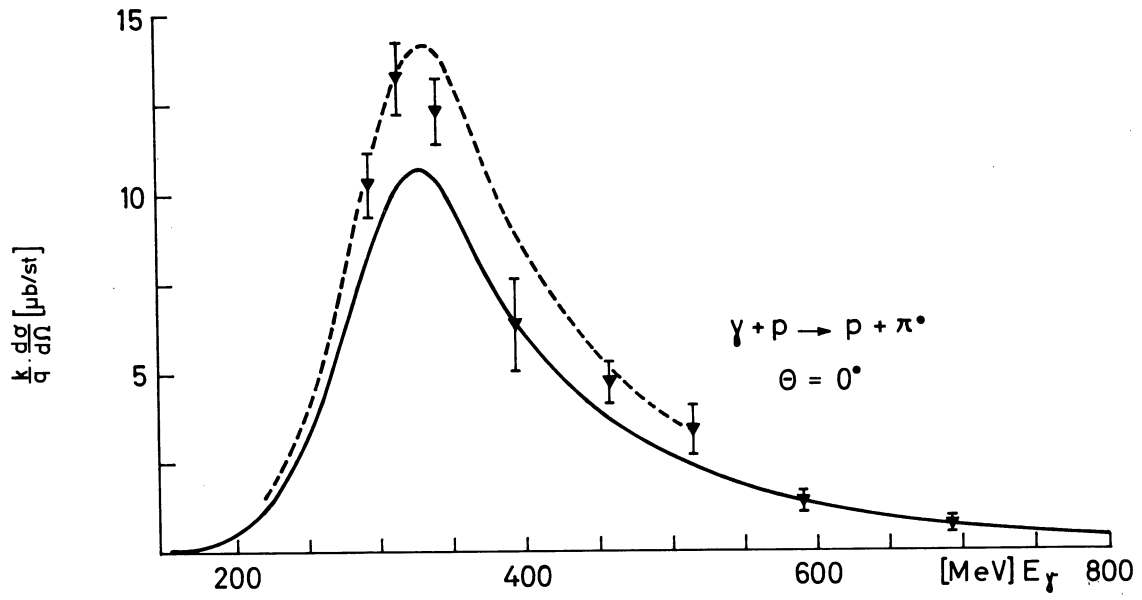


Fig. 7

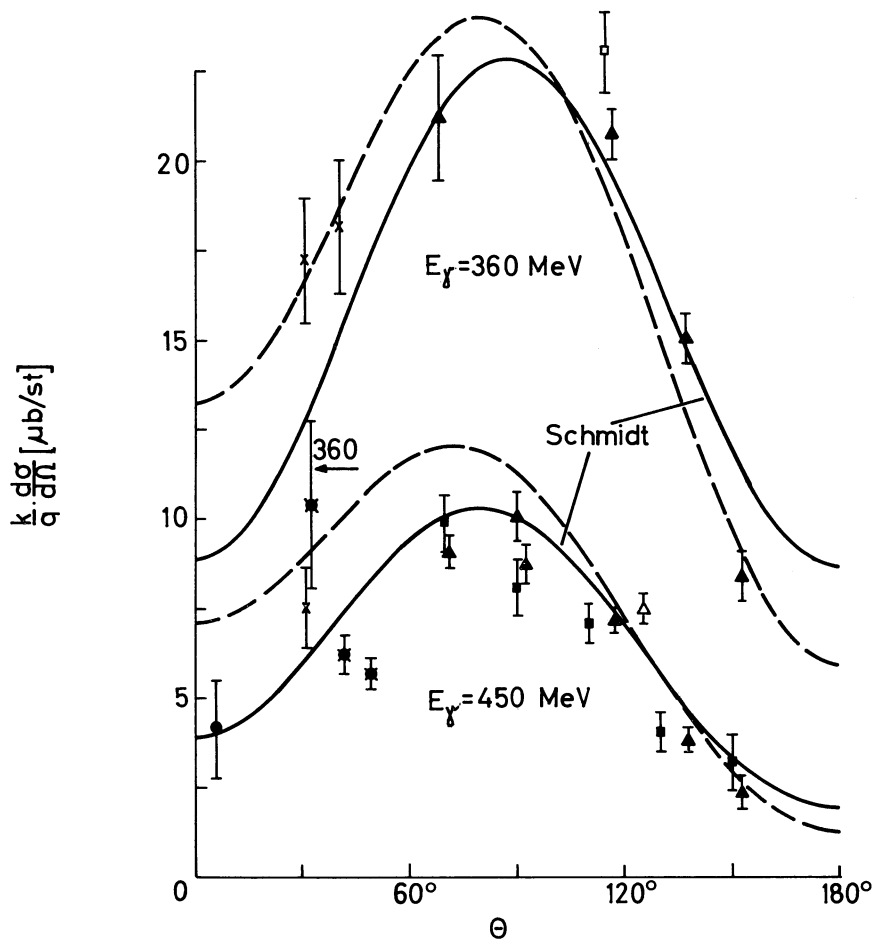


Fig. 8

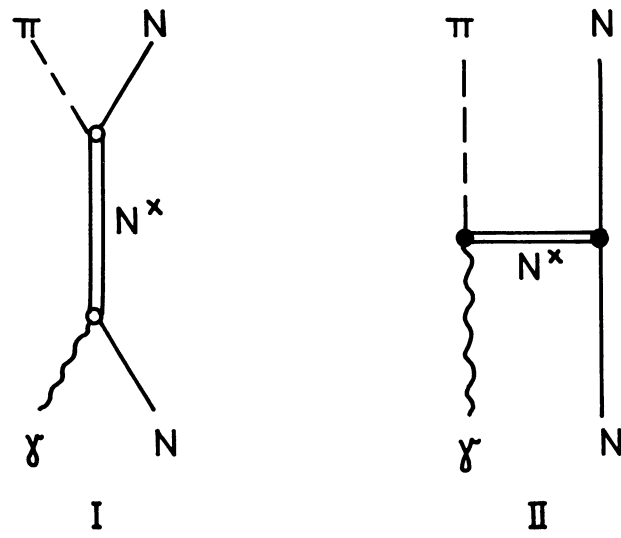


Fig. 9

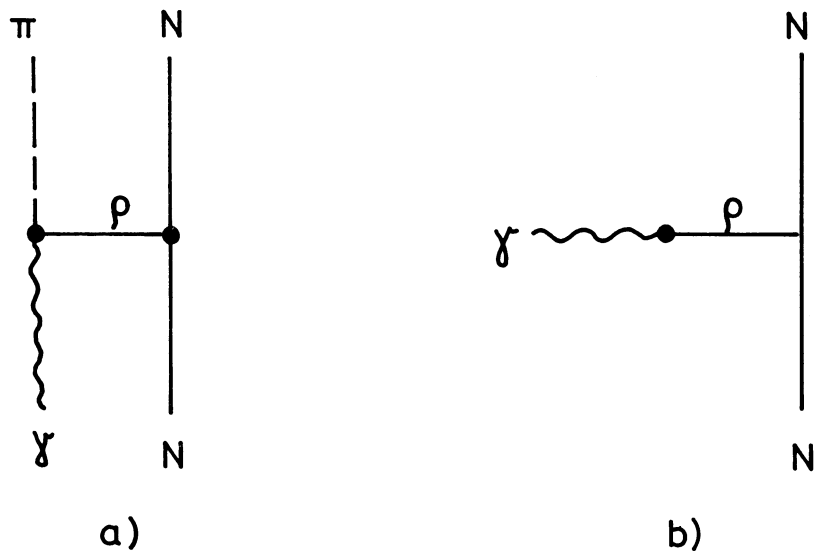


Fig. 10

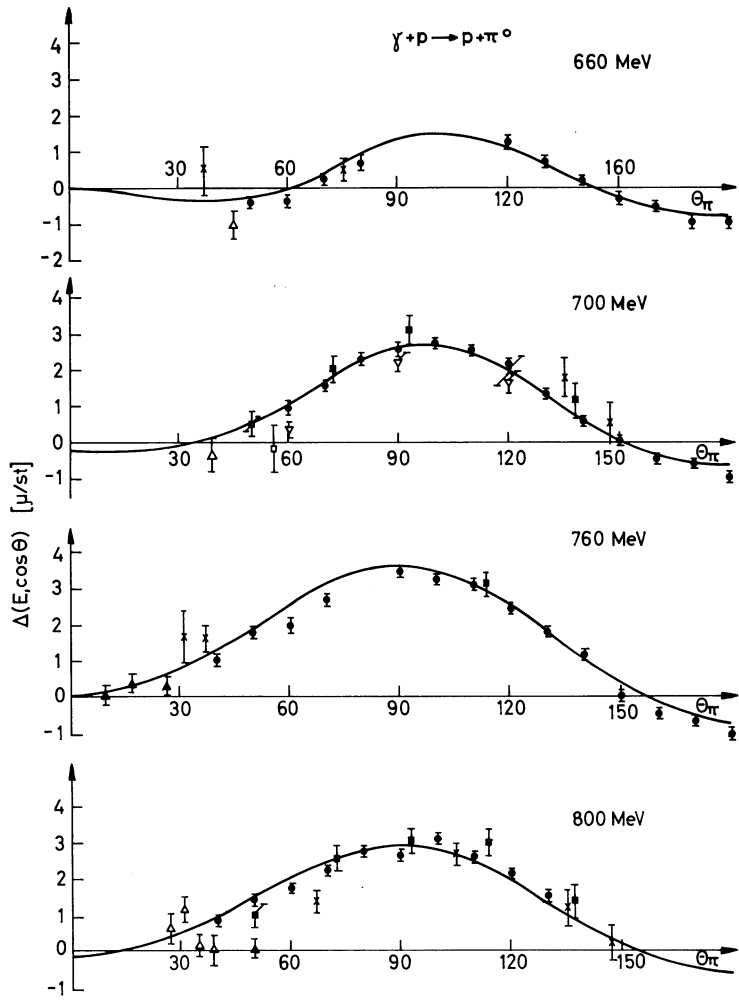


Fig. 11

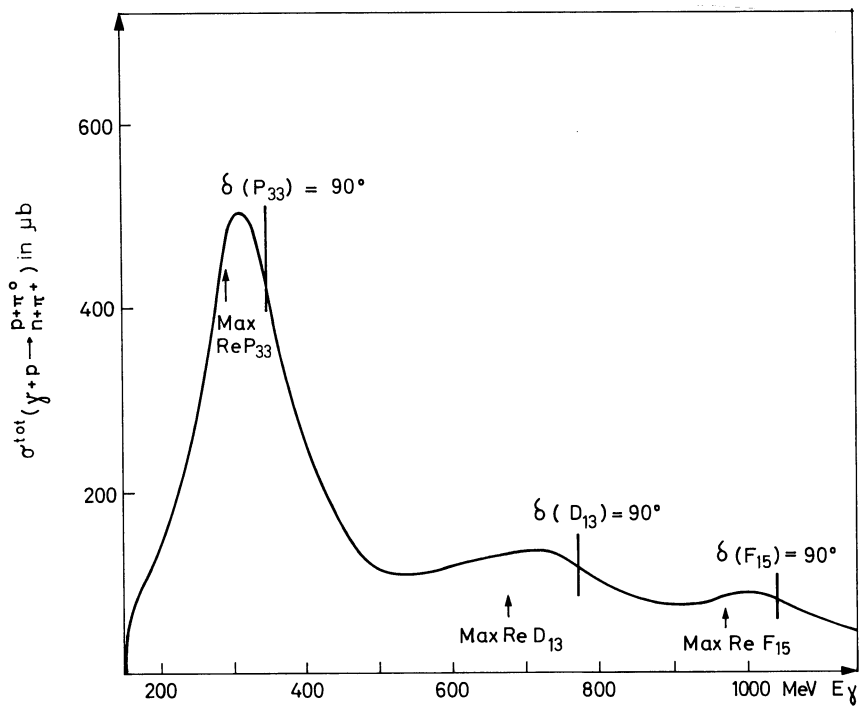


Fig. 12