Analyticity constraints for hadron spectroscopy

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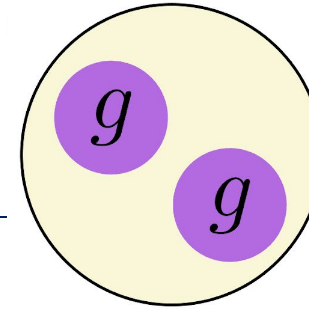
Exotic spectroscopy

Quark models are useful for **insight**



Mesons: $q\bar{q}$

$P = (-1)^{L+1}$

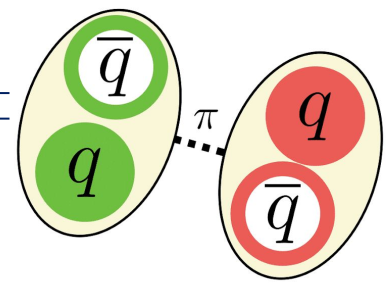
$C = (-1)^{L+S}$

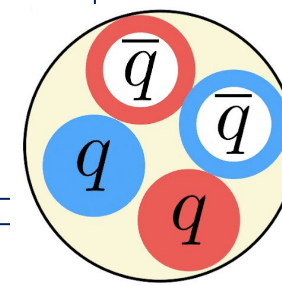
J^{PC}= $0^{-+}, 0^{++}, 1^{--}, 1^{+-}, 1^{++}, 2^{--}, 2^{-+}, 2^{++}, \ldots$

Exotic

$0^{--}, 0^{+-}, 1^{-+}, 2^{+-}, \ldots$

glueballs or hybrid mesons or multi-quark states or molecules



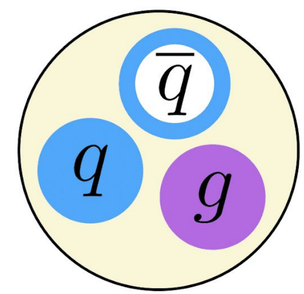


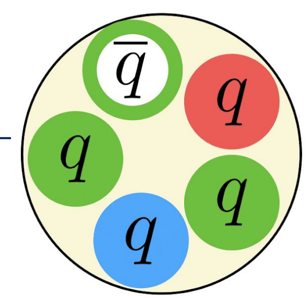
Baryons: qqq

Observation is difficult:

- ‘exotics’ hide in plain sight since they have the same quantum numbers
- Large masses

Only structure to distinguish them





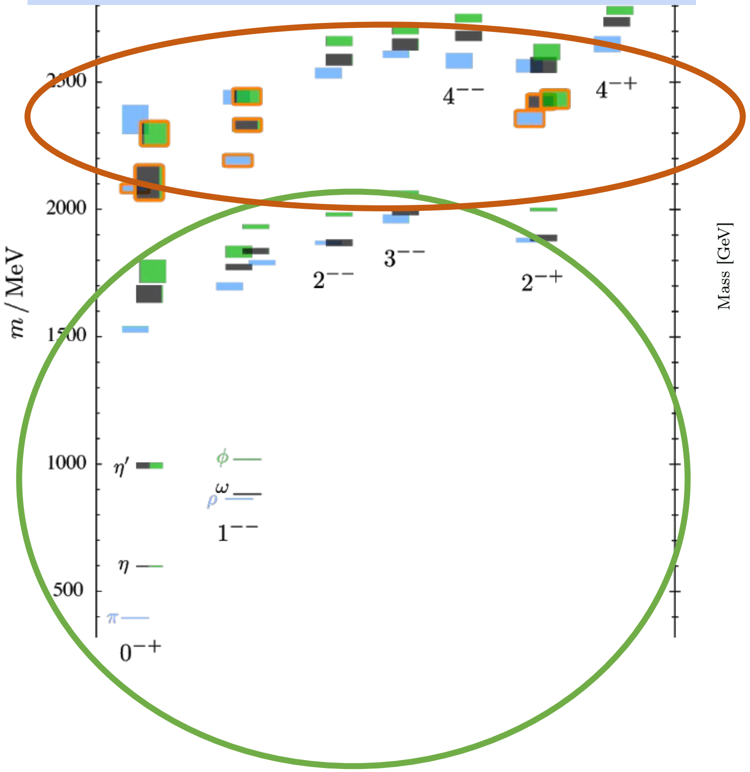
Unanswered questions

- Role of glue?
- Why did the quark model work so well up till now?
- Why does it fail in the charmonium sector (XYZ)?
- Can we extract the hadron spectrum directly from QCD?
- …

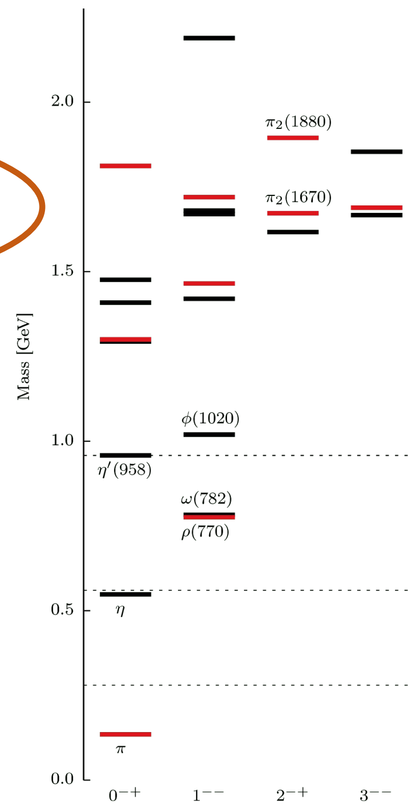
Which rules govern hadron construction?

Hadron spectrum (from QCD)

Dominant overlap with operators featuring a **chromomagnetic** construction



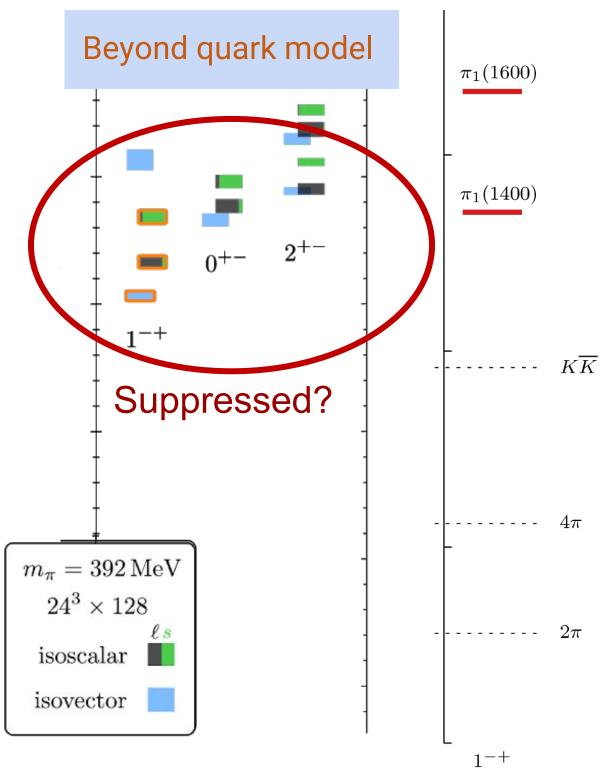
Static spectrum from Lattice QCD
[Phys.Rev. D88 (2013) no.9, 094505]



PDG, experiment

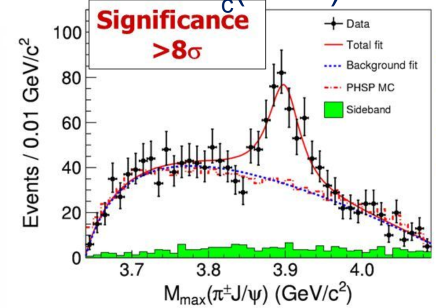
Exotic

Beyond quark model



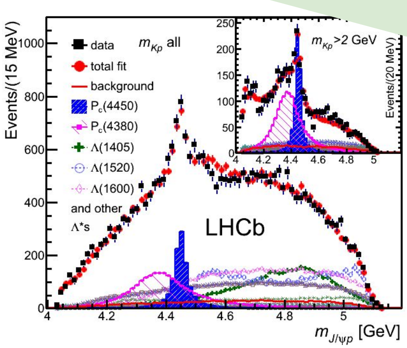
Spectroscopy programs

$Z_c(3900)$

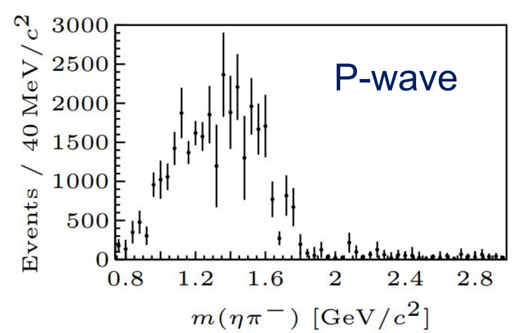


APS Highlight of the Year 2013: Four-Quark Matter

$P_c(4450)$

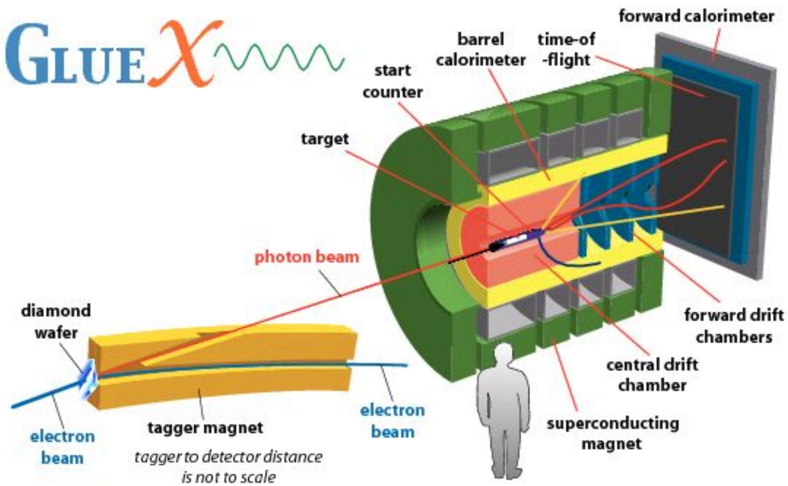


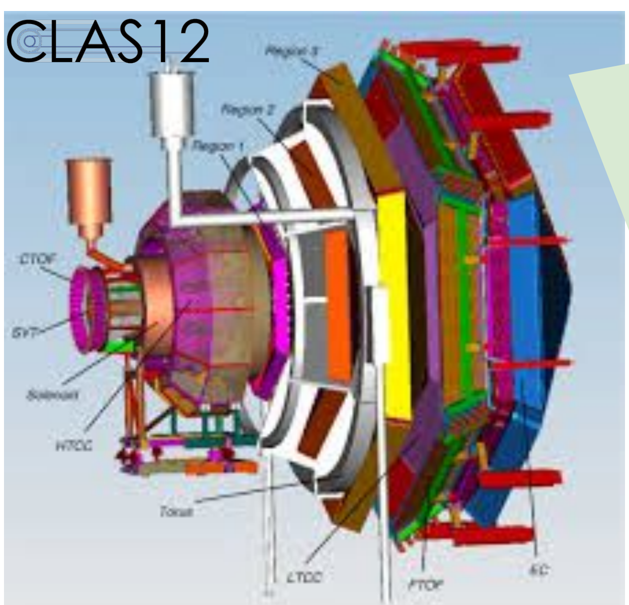
π_1 (?)

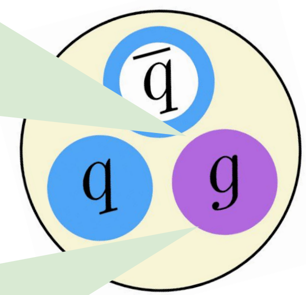


APS Highlight of the Year 2015: Particle High Five

Light-quark exotics: experiment

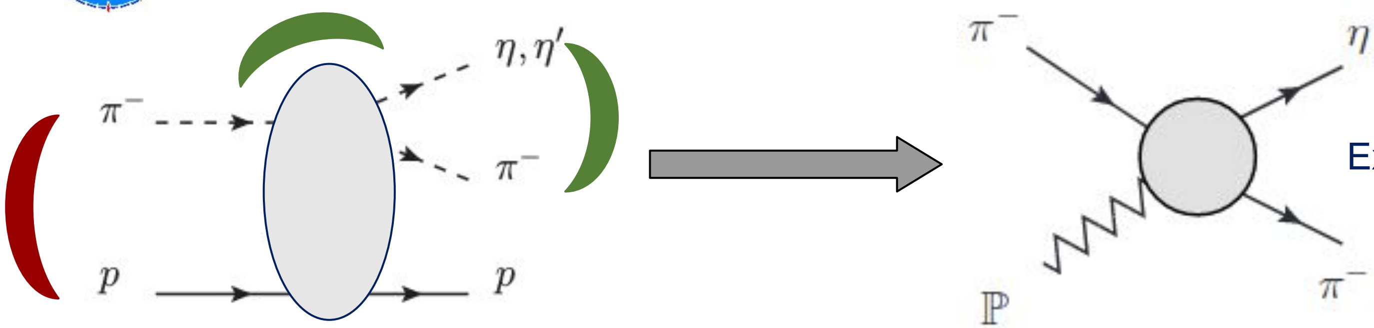






Meson production

Example: π_1 production

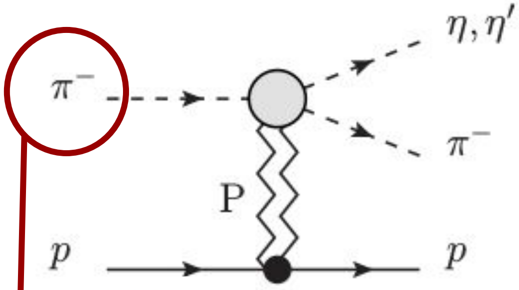


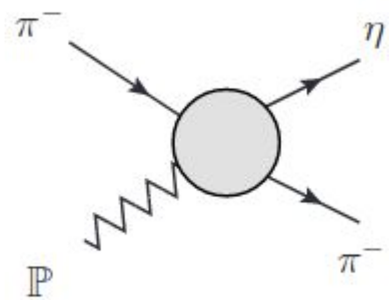
Exotic P-wave

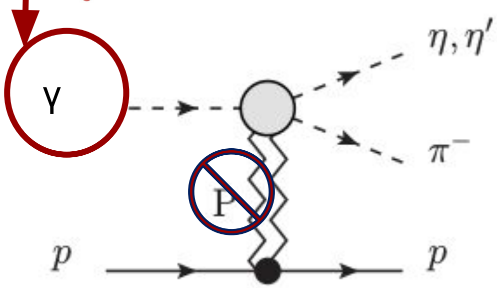
Energy **scale separation**: factorization possible

Knowledge of the production process required to carry out PWA
Multiple production processes required for confirmation

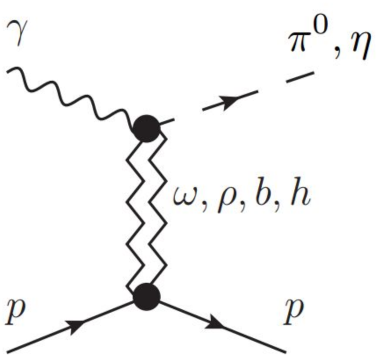
Production process



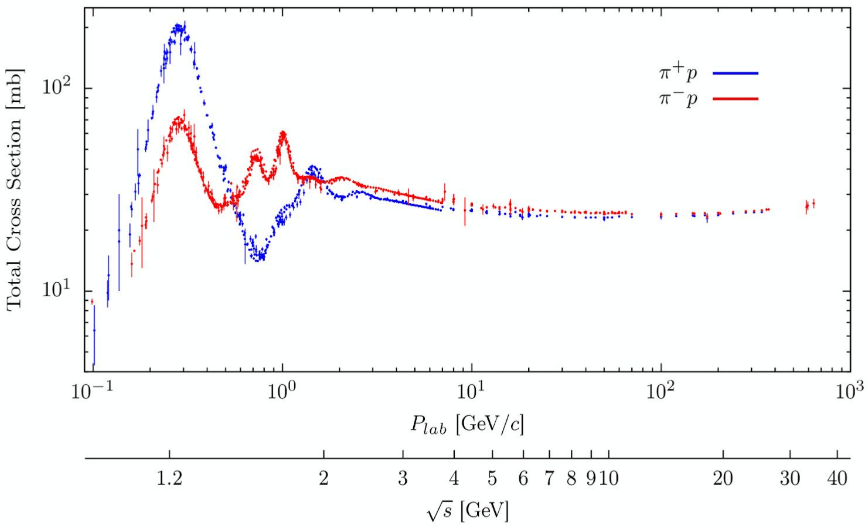




$\pi^0, \eta(0^{-+})$ **have same production as** $\pi_1^0, \eta_1(1^{-+})$



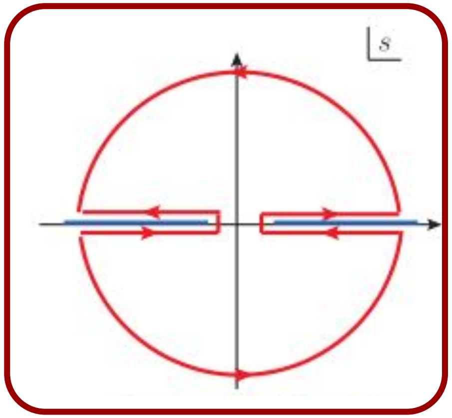
S-matrix theory



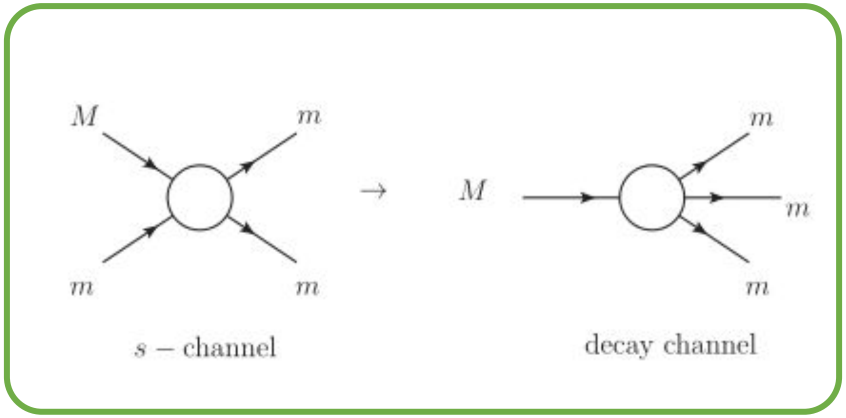
S-matrix theory

Build models: general principles

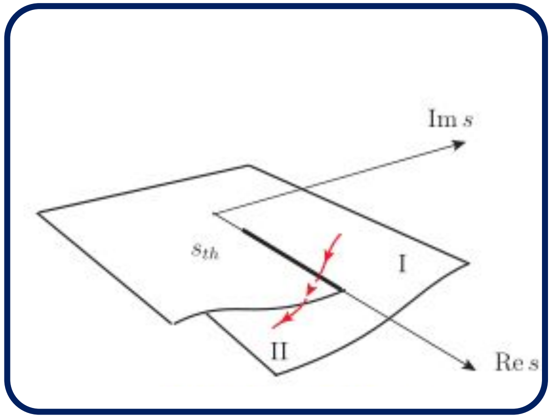
- Analyticity
- Crossing symmetry
- Unitarity
- Lorentz symmetries
- Global symmetries of QCD



ANALYTICITY



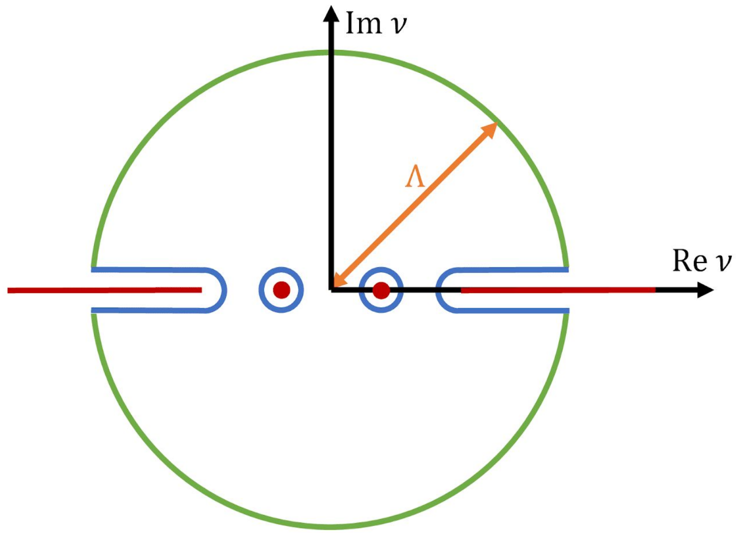
CROSSING SYMMETRY



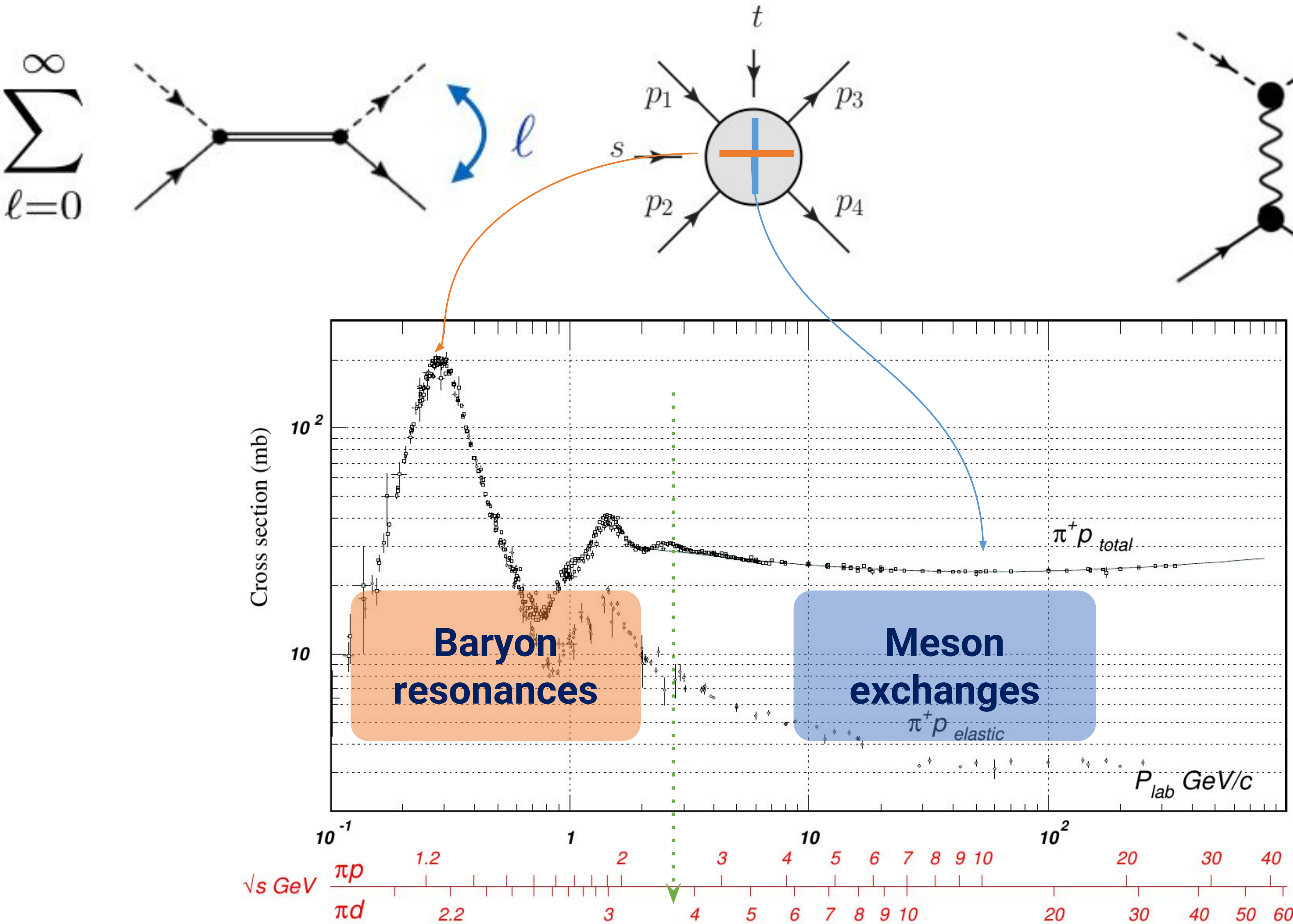
UNITARITY

Sum rules

$$\oint_C \mathcal{M}(s,t)\mathrm{d}s = 0$$



Sum rules



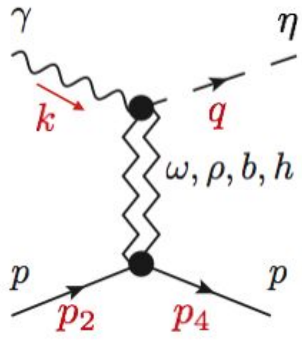
Connect low- and high-energy dynamics.

Choice of amplitudes

$$A_{\lambda';\lambda\,\lambda_\gamma}(s,t) = \overline{u}_{\lambda'}(p')\left(\sum_{k=1}^{4} A_k(s,t) M_k\right) u_\lambda(p)$$

- No kinematic singularities
- No kinematic zeros
- Discontinuities:
  - Unitarity cut
  - Nucleon pole

$$M_1 = \frac{1}{2}\gamma_5\gamma_\mu\gamma_\nu F^{\mu\nu},$$
$$M_2 = 2\gamma_5 q_\mu P_\nu F^{\mu\nu},$$
$$M_3 = \gamma_5\gamma_\mu q_\nu F^{\mu\nu},$$
$$M_4 = \frac{i}{2}\epsilon_{\alpha\beta\mu\nu}\gamma^\alpha q^\beta F^{\mu\nu}.$$



A_i	I^G	J^{PC}	η	Leading exchanges
A_1	$0^-, 1^+$	$(1,3,5,...)^{--}$	$+1$	$\rho(770), \omega(782)$
A_2'	$0^-, 1^+$	$(1,3,5,...)^{+-}$	-1	$h_1(1170), b_1(1235)$
A_3	$0^-, 1^+$	$(2,4,...)^{--}$	-1	$\rho_2(??), \omega_2(??)$
A_4	$0^-, 1^+$	$(1,3,5,...)^{--}$	$+1$	$\rho(770), \omega(782)$

$$\gamma p \to \eta p\,, \qquad A = (\omega + h + \omega_2) + (\rho + b + \rho_2)$$
$$\gamma n \to \eta n\,, \qquad A = (\omega + h + \omega_2) - (\rho + b + \rho_2)$$

s-channel: *truncated* partial-wave analysis

$$\sum_{l=0}^{\not\infty} A_l(s) P_l(z_s)$$

- Various models available for extracting baryon resonances (**W < 2 GeV**)
  - SAID
  - MAID
  - Bonn-Gatchina
  - Juelich-Bonn
  - …

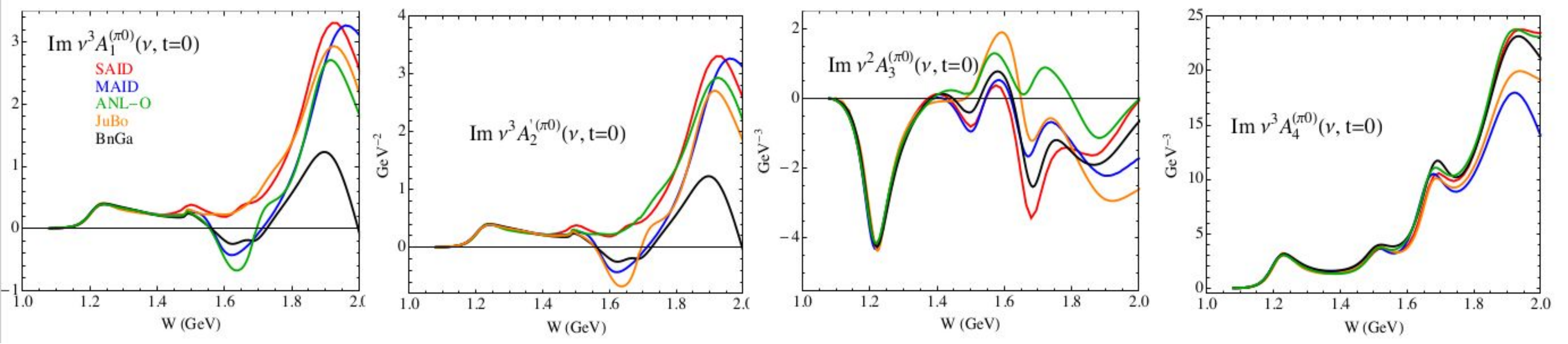
Low energies

$$\int_{\nu_\pi}^{\Lambda} \mathrm{Im}\ A_i^{\sigma}(\nu', t) \left(\frac{\nu'}{\Lambda}\right)^k d\nu'$$

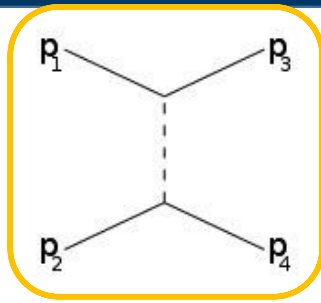
Low energy models

- BnGa, Juelich-Bonn, ANL-Osaka, SAID, MAID,...

$\gamma N \to \pi^0 N$



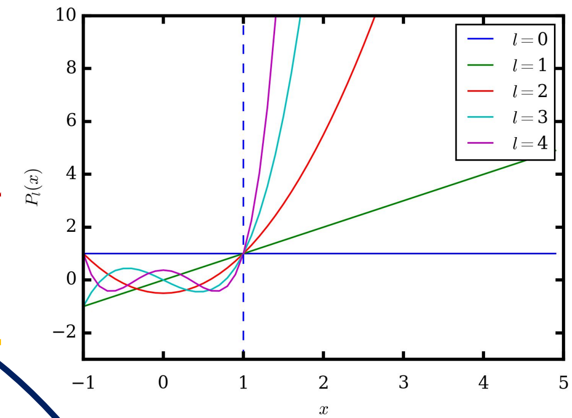
Using the right degrees of freedom



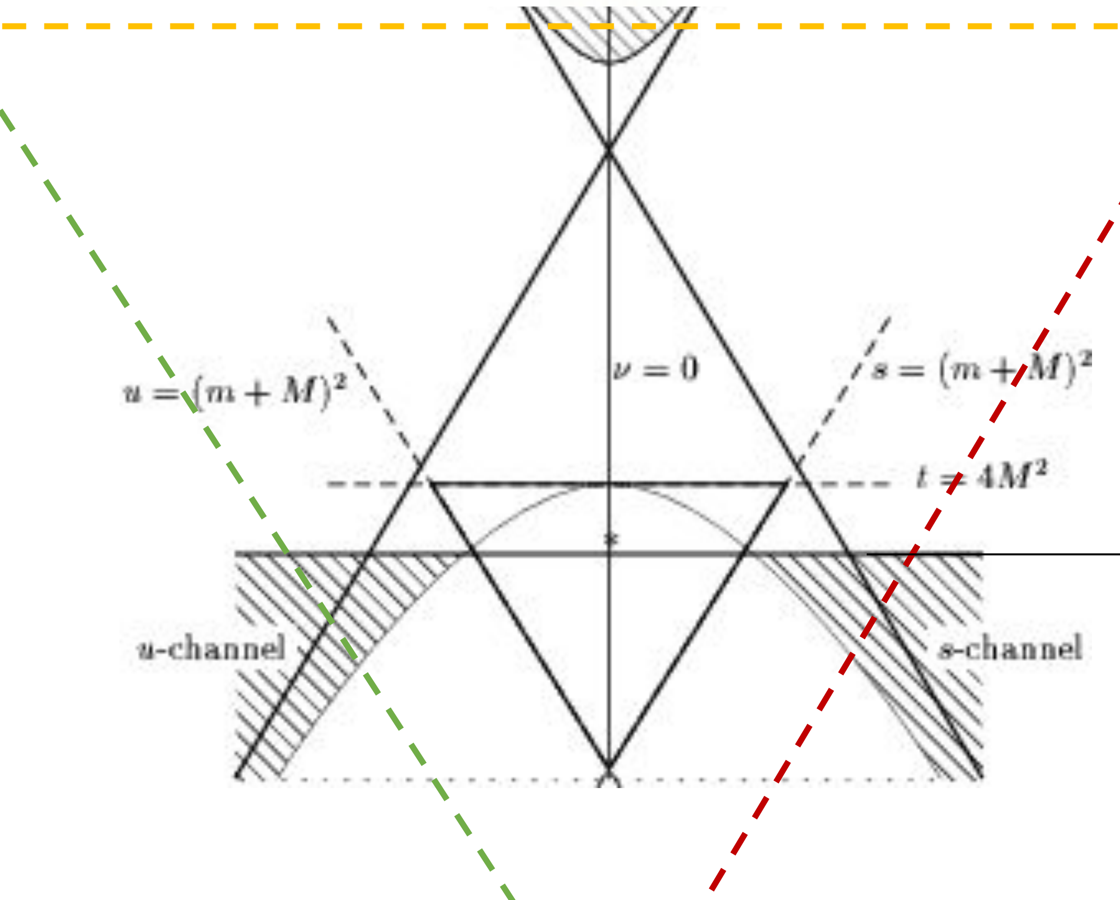
t-channel

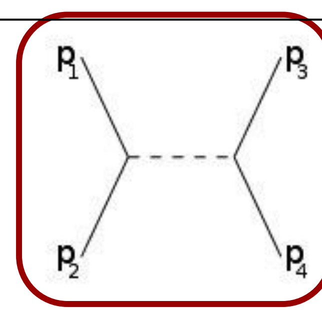
$$\sum_{l=0}^{\infty} A_l(t) P_l(z_t)$$

Convergent here



Not convergent here





s-channel

$$\sum_{l=0}^{\infty} A_l(s) P_l(z_s)$$

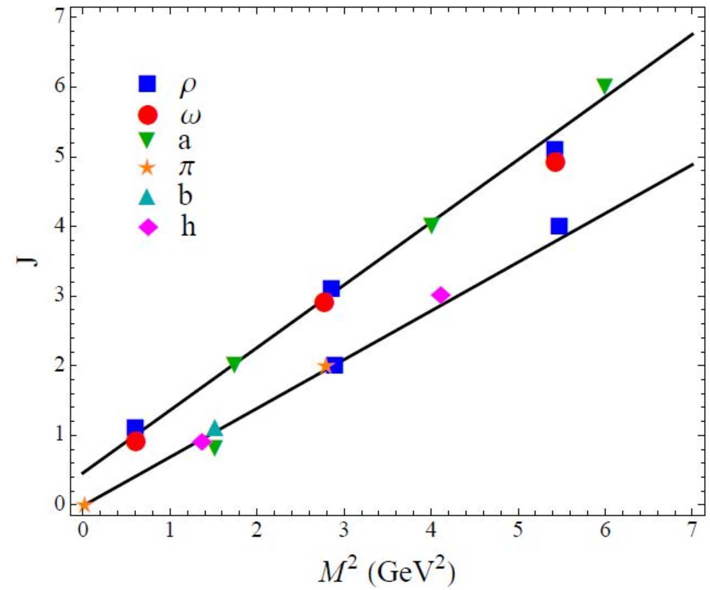
High-energy model

Regge pole model

$J(m^2)$

$$A_i(s,t) = -\beta_i(t)\frac{(-1)^J + e^{-i\pi\alpha(t)}}{\sin\pi\alpha(t)} s^{\alpha(t)-1}$$

Dominant: vector exchanges



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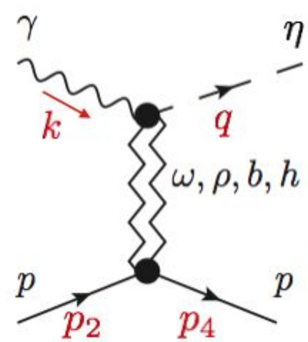


$$\gamma p \to \eta p\,, \qquad A = (\omega + h + \omega_2) + (\rho + b + \rho_2)$$

$$\gamma n \to \eta n\,, \qquad A = (\omega + h + \omega_2) - (\rho + b + \rho_2)$$

Dispersion relations - FESR

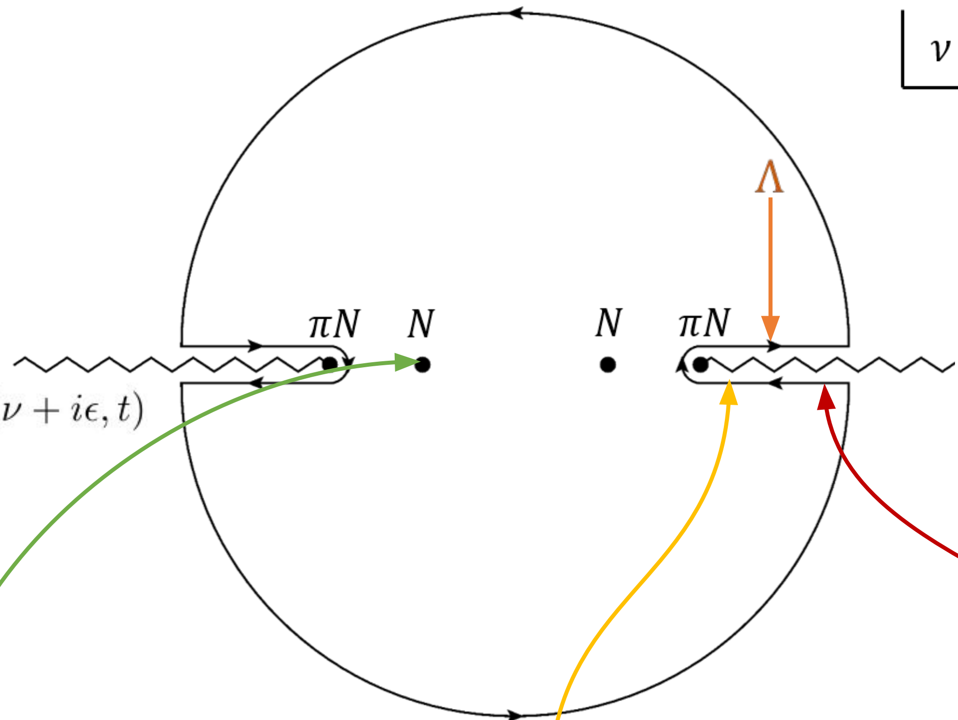
$A_i(\nu, t)$

Fixed t

Crossing variable: $\nu = \frac{s-u}{4\,M}$

s-channel: $+\,\nu$

u-channel: $-\,\nu$



$$A_i^\sigma(-\nu - i\epsilon, t) = \xi_i A_i^\sigma(\nu + i\epsilon, t)$$

$$-\beta_i(t)\frac{\tau + e^{-i\pi\alpha(t)}}{\sin \pi\alpha(t)}(r_i\nu)^{\alpha(t)-1}$$

$$\pi B_i^\sigma(t)\left(\frac{\nu_M}{\Lambda}\right)^k + \int_{\nu_\pi}^{\Lambda} \text{Im } A_i^\sigma(\nu', t)\left(\frac{\nu'}{\Lambda}\right)^k d\nu' = \beta_i^\sigma(t)\frac{(\Lambda/\nu_0)^{\alpha(t)}}{\alpha(t)+k}\nu_0$$

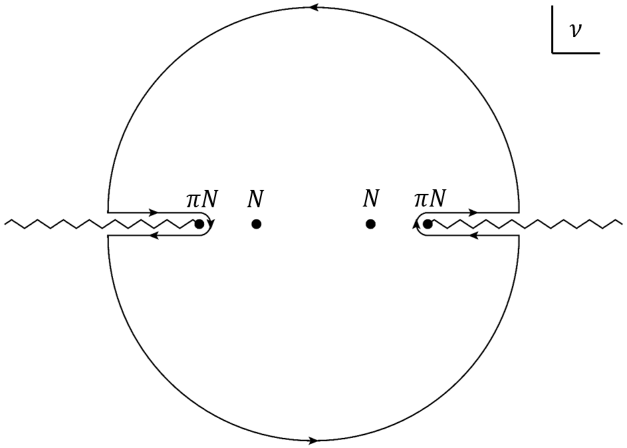
Nucleon pole

Low-energy model

Regge pole

Analyticity results in Finite-Energy Sum Rules.

Finite-Energy Sum Rules

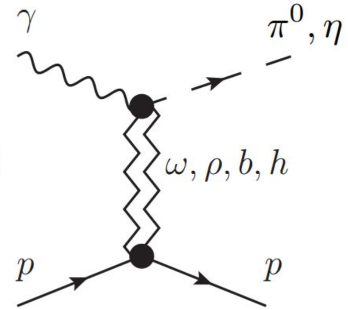


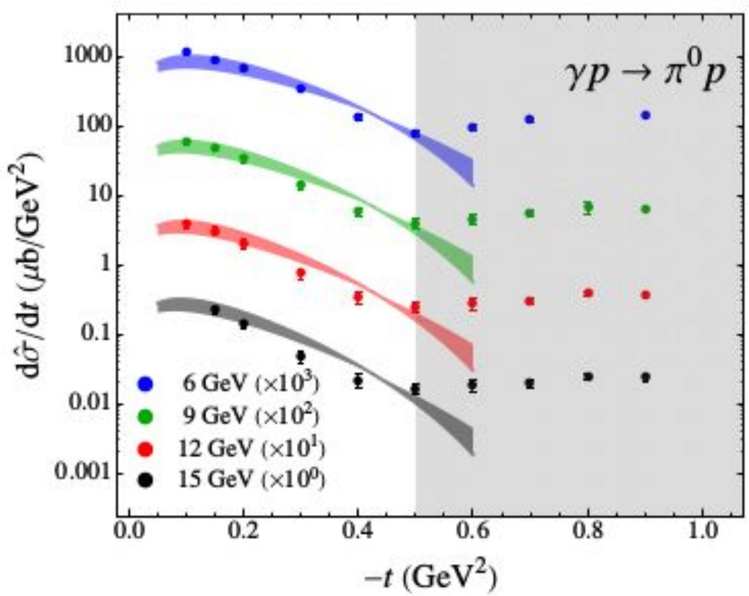
$$\int_0^{\Lambda} \text{Im } A_i(\nu, t)\nu^k d\nu = \beta_i(t) \frac{\Lambda^{\alpha(t)+k}}{\alpha(t) + k}$$

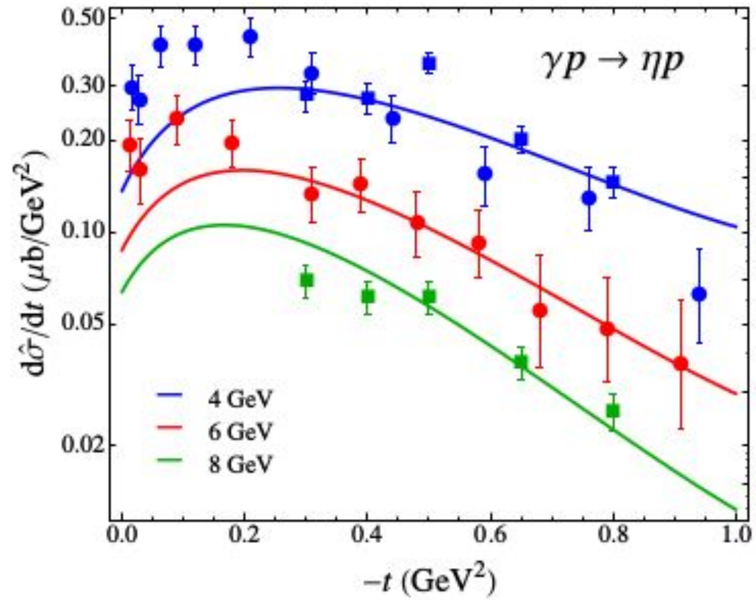
$$\beta_i(t) = \frac{\alpha(t) + k}{\Lambda^{\alpha(t)+k}} \int_0^{\Lambda} \text{Im } A_i(\nu, t)\nu^k d\nu$$

Finite-Energy Sum Rules

[V. Mathieu, J.N. *et al.* (JPAC) 1708.07779 (2017)]







Combine energy regimes

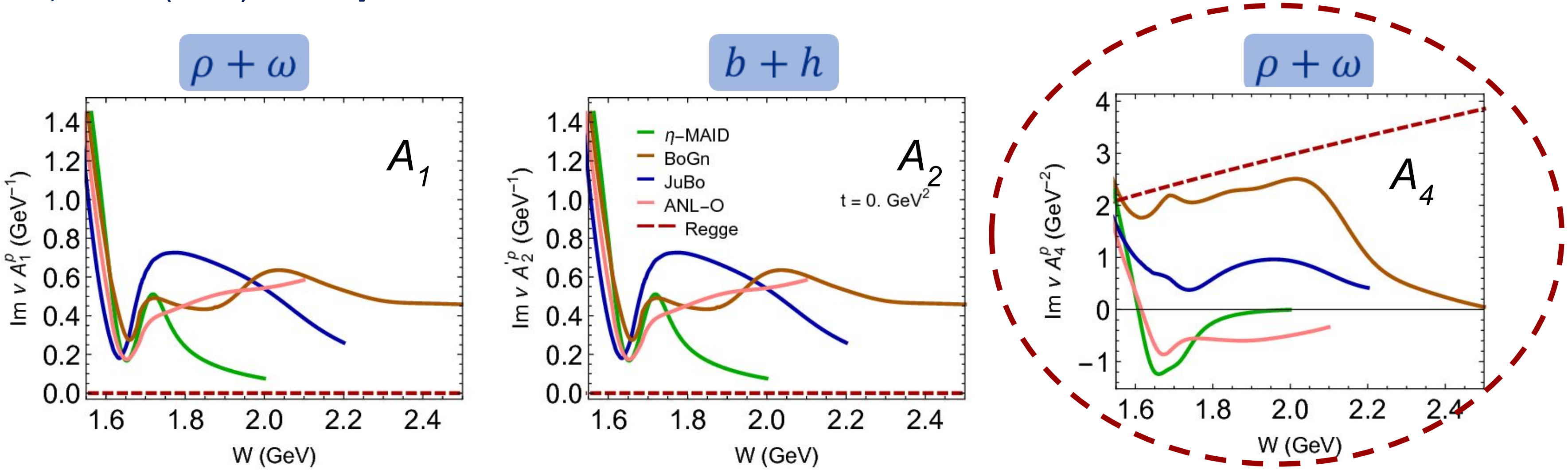
- Low-energy model
- Predict high-energy observables

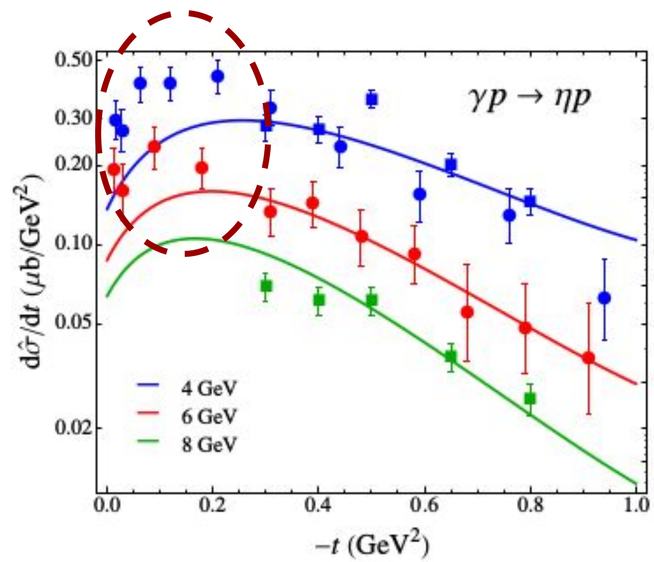
Two applications

- Understand high-energy dynamics
- Constraining low-energy models

Low-energy models (η)

[J.N. *et al.*, PRD95 (2017) 034014]





Ambiguities in the low-energy model (η-MAID)
→ Mismatch with high-energy data

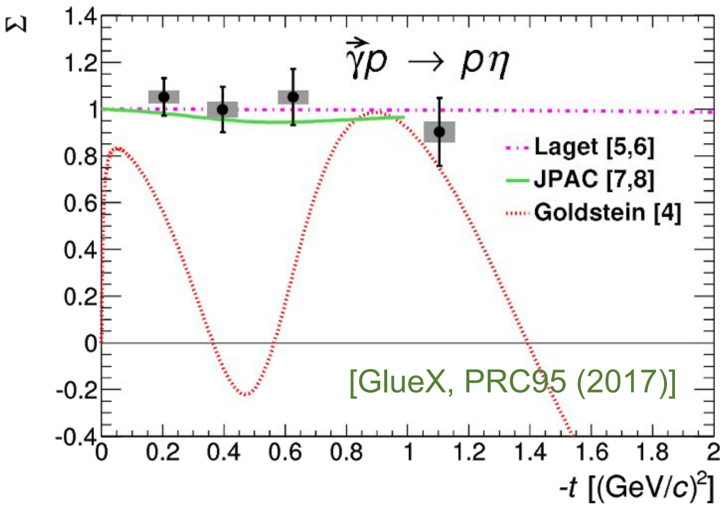
Possibilities

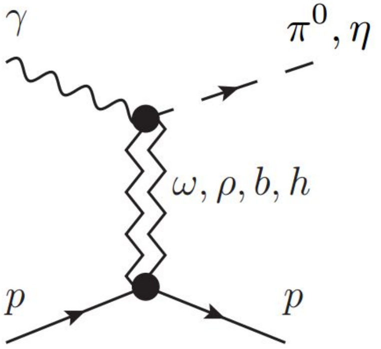
- Low-energy model inconsistent
- Cut-off not high enough
  - High mass resonances!

High-energy predictions

Natural dominant: $\Sigma = +1$

Unnatural dominant: $\Sigma = -1$





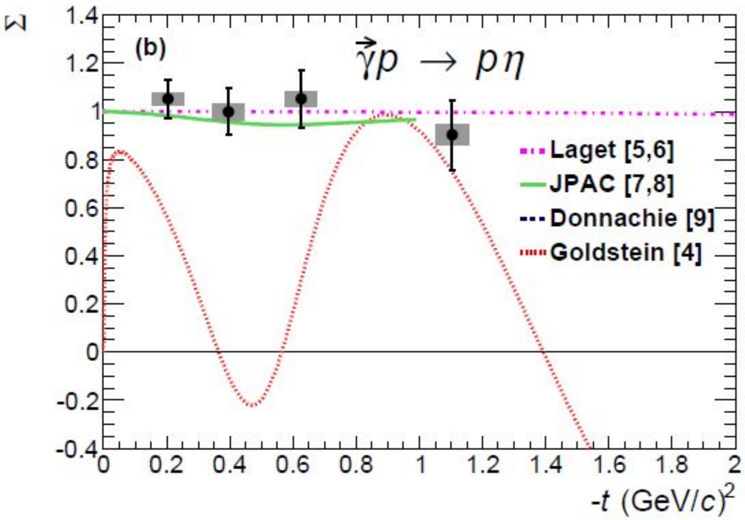
$$\Sigma = \frac{\sigma_\perp - \sigma_\parallel}{\sigma_\perp + \sigma_\parallel} = \frac{|\rho + \omega|^2 - |b + h|^2}{|\rho + \omega|^2 + |b + h|^2}$$

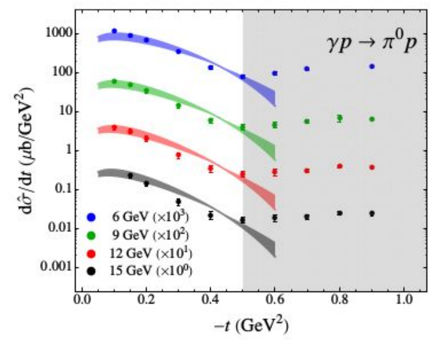
$$\Sigma = +1 \quad : \quad \rho, \omega$$

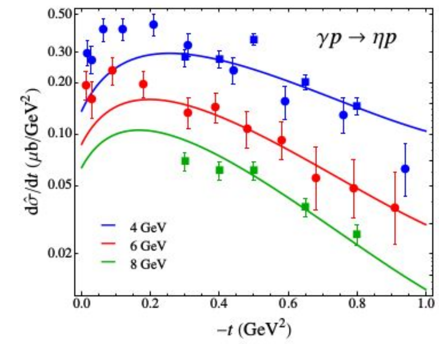
$$\Sigma = -1 \quad : \quad b, h$$

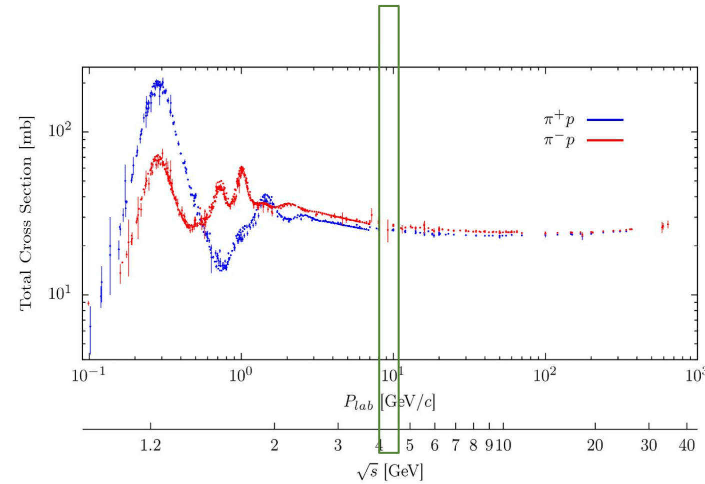
Production process: example

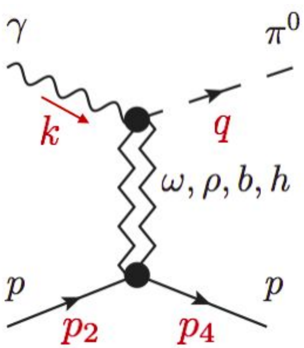
$$\Sigma = +1 \quad : \quad \rho, \omega$$
$$\Sigma = -1 \quad : \quad b, h$$

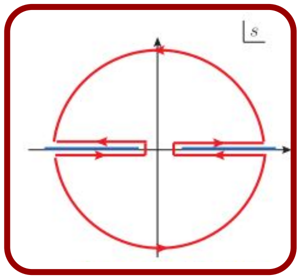












ANALYTICITY

High-energy predictions

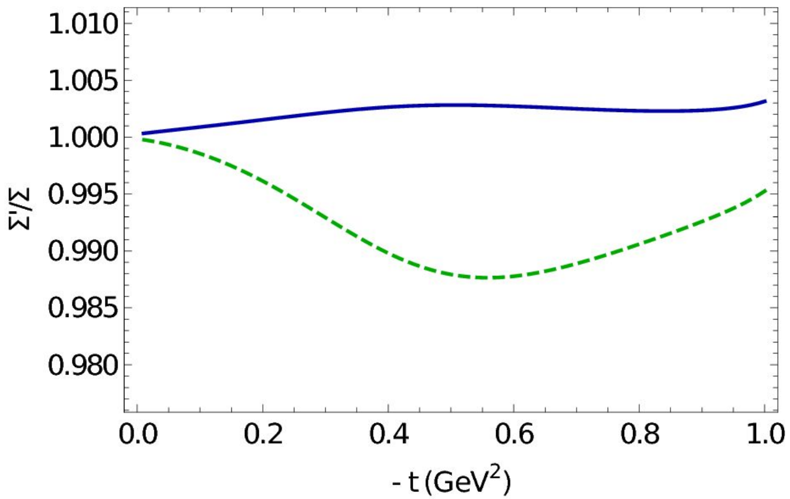
- Unnatural components have little effect
- Φ, h' components are subleading

$$\Sigma = \frac{d\sigma_\perp - d\sigma_\parallel}{d\sigma_\perp + d\sigma_\parallel} \qquad (\gamma p \to \eta p)$$

$$\Sigma = \frac{|\rho+\omega|^2 - |b+h|^2}{|\rho+\omega|^2 + |b+h|^2} = \Sigma'$$

$$\Sigma' = \frac{d\sigma'_\perp - d\sigma'_\parallel}{d\sigma'_\perp + d\sigma'_\parallel} \qquad (\gamma p \to \eta' p)$$

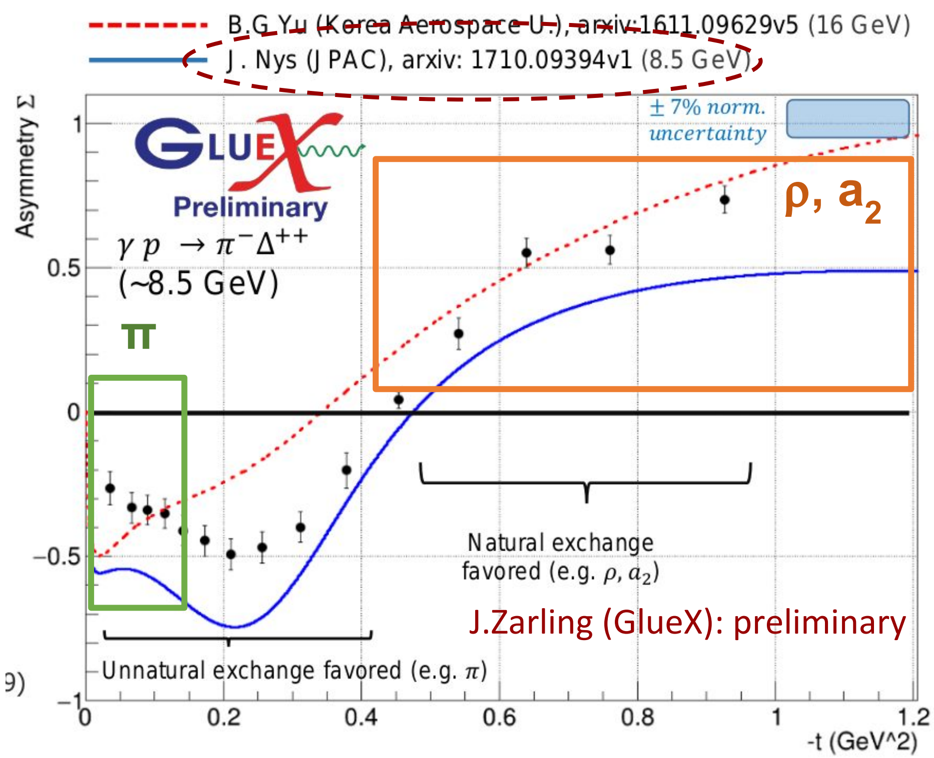
$$\Sigma = \frac{|\rho+\omega+\phi|^2 - |b+h+h'|^2}{|\rho+\omega+\phi|^2 + |b+h+h'|^2} \neq \Sigma'$$

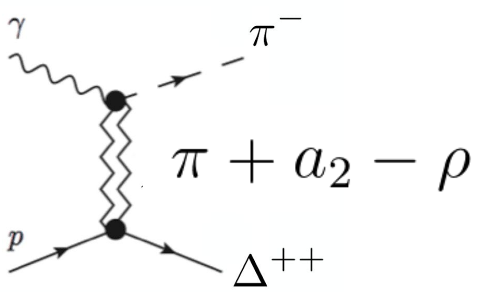


Prediction: $\Sigma = \Sigma'$

High-energy predictions

- Dominated by charged pion exchanges
- Model includes
  - Absorbed pion exchange
  - ρ, a2 exchange (cuts)



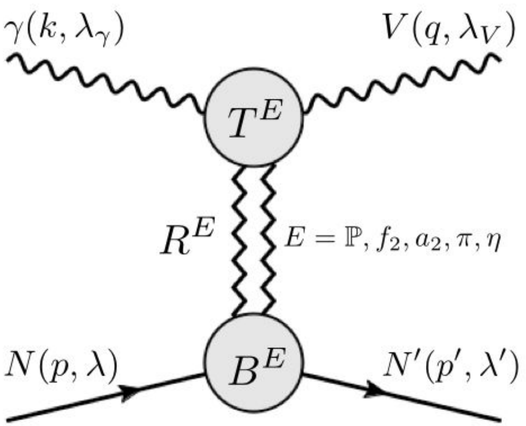


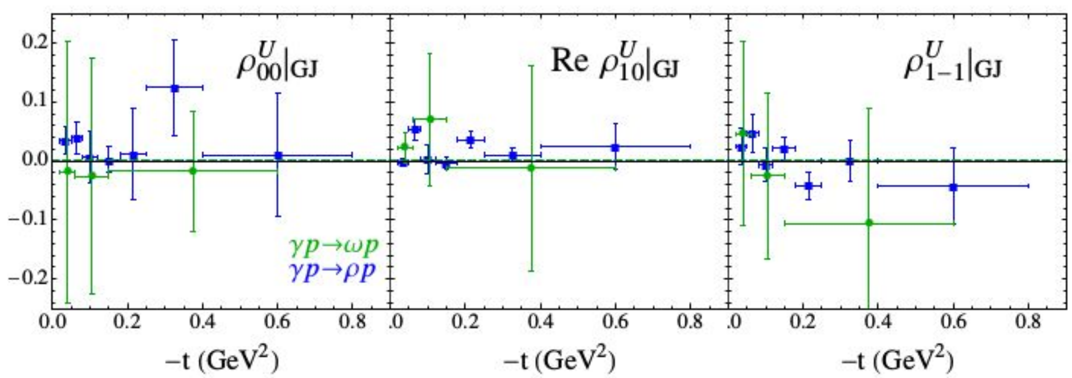
Neutral vector mesons

- Pomeron dominates at high energies
- Isoscalar exchanges dominantly helicity non-flip (λ=λ’)
- Unnatural exchanges: only helicity flip (|λ-λ’|=1)

$$\mathcal{M}_{\substack{\lambda_V,\lambda_\gamma \\ \lambda',\lambda}}(s,t) = \sum_{E=\pi,\eta,\mathbb{P},f_2,a_2} \mathcal{M}^E_{\substack{\lambda_V,\lambda_\gamma \\ \lambda',\lambda}}(s,t)$$

$$\mathcal{M}^{\substack{N \\ U}}_{\substack{-\lambda_\gamma,-\lambda_V \\ \lambda,\lambda'}} = \pm(-1)^{\lambda_\gamma-\lambda_V}\,\mathcal{M}^{\substack{N \\ U}}_{\substack{\lambda_\gamma,\lambda_V \\ \lambda,\lambda'}}$$





$$\rho^{\substack{N \\ U}}_{00} = \frac{1}{2}\left(\rho^0_{00} \mp \rho^1_{00}\right),$$

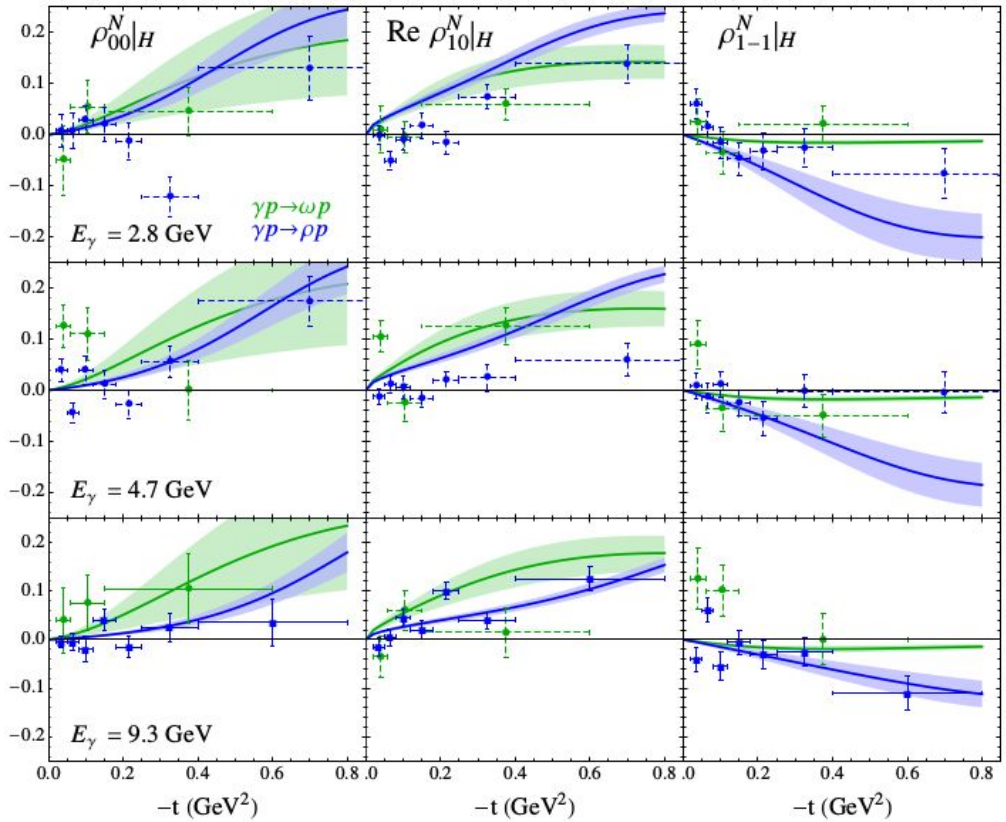
$$\mathrm{Re}\,\rho^{\substack{N \\ U}}_{10} = \frac{1}{2}\left(\mathrm{Re}\,\rho^0_{10} \mp \mathrm{Re}\,\rho^1_{10}\right),$$

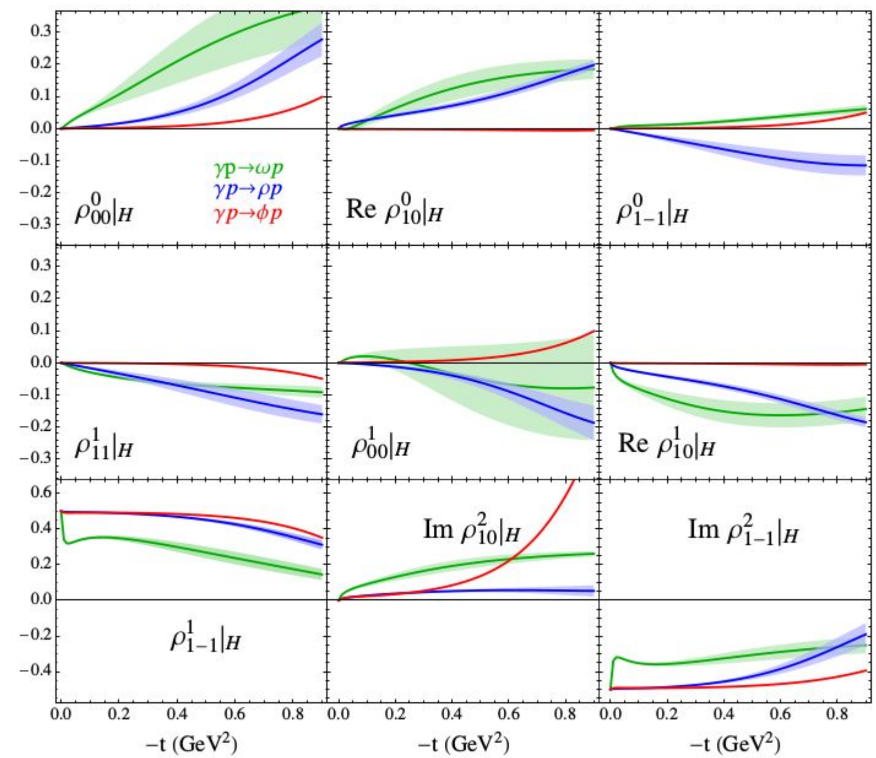
$$\rho^{\substack{N \\ U}}_{1-1} = \frac{1}{2}\left(\rho^1_{1-1} \pm \rho^1_{11}\right).$$

[V.Mathieu, J.N. *et al.*, (2018) arXiv:1802.09403]

Neutral vector mesons

$$\mathcal{M}_{\substack{\lambda_V,\lambda_\gamma \\ \lambda',\lambda}}(s,t) = \sum_{E=\pi,\eta,\mathbb{P},f_2,a_2} \mathcal{M}^{E}_{\substack{\lambda_V,\lambda_\gamma \\ \lambda',\lambda}}(s,t)$$

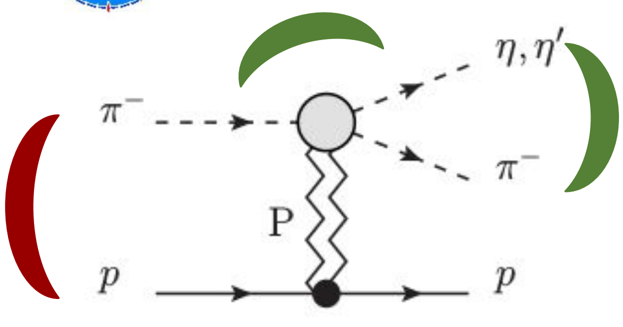


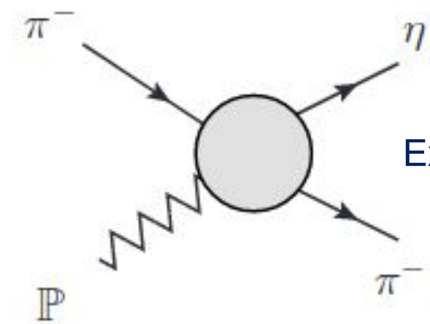


[V.Mathieu, J.N. *et al.*, (2018) arXiv:1802.09403]

Meson production

Example: π_1 production





Exotic P-wave

Energy **scale separation**: factorization possible

Knowledge of the production process required to carry out PWA
Multiple production processes required for confirmation

Kinematic singularities

Constraints from **analyticity**

- Reaction amplitude is a smooth function, i.e. analytic
- Dynamics introduces singularities (on the unphysical sheets, or the real axis)
- Spin projections introduce singularities related to their Lorentz transformations
  - Track them down & remove them
  - Create 'kinematic singularity free amplitudes', where you can plug in the **dynamics**.
- Amplitude must be **crossing symmetric**: resonance properties are the same for different kinematics

S-matrix theory: we do not use the underlying field theory, so no Feynman diagrams to help us out

Question: *"What are the **minimal kinematic factors** to include to have obtain analytic amplitude?"*

Types of singularities: $$A_\lambda(s,t,u) = \frac{1}{4\pi}\sum_{j=|\lambda|}^{\infty}(2j+1)A^j_\lambda(s)\, d^j_{\lambda 0}(z_s)$$

- Half-angle factors: kinematic singularities in t $\hat{d}^j_{\lambda\lambda'}(z_s) = \dfrac{d^j_{\lambda\lambda'}(z_s)}{\xi_{\lambda\lambda'}(z_s)}$ $\quad \xi_{\lambda\lambda'}(z_s) = \left(\sqrt{1-z_s}\right)^{|\lambda-\lambda'|}\left(\sqrt{1+z_s}\right)^{|\lambda+\lambda'|}$
- (pseudo)threshold factors $A^{j\eta}_{\lambda_p,\lambda_b\lambda_\psi}(s) \sim p^{L_1}$ $\left(A^{j\eta}_{\lambda_p,\lambda_b\lambda_\psi}(s) \sim q^{L_2}\right)$
- s=0: little group changes $|\mathcal{P}\rangle \otimes |pJM;\mu_1\mu_2\rangle$ $\quad s = \mathcal{P}^2 \to 0$ $\quad \mathcal{P}^\mu \to (0,0,0,0)$

Kinematic singularities

<u>Tools in the toolbox:</u>

- Helicity formalism
  - Jacob, Wick, Annals Phys. 7, 404 (1959)
- LS formalism
- Covariant tensor formalism
  - Chung, PRD48, 1225 (1993)
  - Chung, Friedrich, PRD78, 074027 (2008)
  - Filippini, Fontana, Rotondi, PRD51, 2247 (1995)
  - Anisovich, Sarantsev, EPJA30, 427 (2006)

Kinematic singularities

- General covariant structures: scalar functions are kinematic singularity and zero free

$$A_\lambda(s,t) = \epsilon_\mu(\lambda,p_1)\left[(p_3-p_4)^\mu - \frac{m_3^2-m_4^2}{s}(p_3+p_4)^\mu\right]C(s,t) + \epsilon_\mu(\lambda,p_1)(p_3+p_4)^\mu \underline{B(s,t)}$$

- Helicity partial wave decomposition **+ matching with covariant basis**

$$\mathcal{A}_\lambda(s,t,u) = \frac{1}{4\pi}\sum_{j=|\lambda|}^{\infty}(2j+1)A^j_\lambda(s)\, \underline{d^j_{\lambda 0}(z_s)}$$

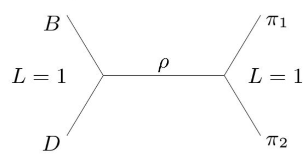
singularities in t

Cohen-Tannoudji, *et al.* Annals Phys. (1968)
Collins' book
Martin & Spearman book

- LS decomposition

$$A^j_\lambda(s) = p^{j-1}q^j\left(\sqrt{\frac{2j-1}{2j+1}}\langle j-1,0;1,\lambda|j,\lambda\rangle \hat{G}^j_{j-1}(s) + \sqrt{\frac{2j+3}{2j+1}}\langle j+1,0;1,\lambda|j,\lambda\rangle p^2 \hat{G}^j_{j+1}(s)\right)$$

- Covariant projection method: **scattering**

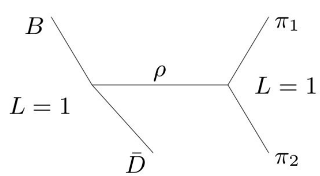


$$X_\nu(q,P) = q^\perp_\nu = q_\nu - P_\nu P\cdot q/s$$

$$A_\lambda(s,t) = \epsilon_\mu(\lambda,p_1)\left(-g^{\mu\nu} + \frac{P^\mu P^\nu}{s}\right)X_\nu(q,P)g_S(s) + \epsilon^\rho(\lambda,p_1)X_{\rho\mu}(p,P)\left(-g^{\mu\nu} + \frac{P^\mu P^\nu}{s}\right)X_\nu(q,P)g_D(s)$$

Spin 1 isobar — P-wave (final state) — D-wave (initial state)

- Covariant projection method: **decay**



$$A_\lambda(s,t) = \epsilon^*_\mu(\lambda,\bar{p}_1)\left(-g^{\mu\nu} + \frac{P^\mu P^\nu}{s}\right)X_\nu(q,P)g_S(s) + \epsilon^{\rho *}(\lambda,\bar{p}_1)X_{\rho\mu}(\hat{p},p_2)\left(-g^{\mu\nu} + \frac{P^\mu P^\nu}{s}\right)X_\nu(q,P)g_D(s)$$

Orthogonal to B

Covariant projection method

Based on the construction of explicitly covariant expressions.

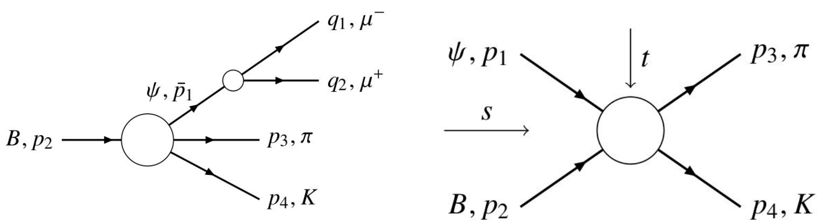
Routine:

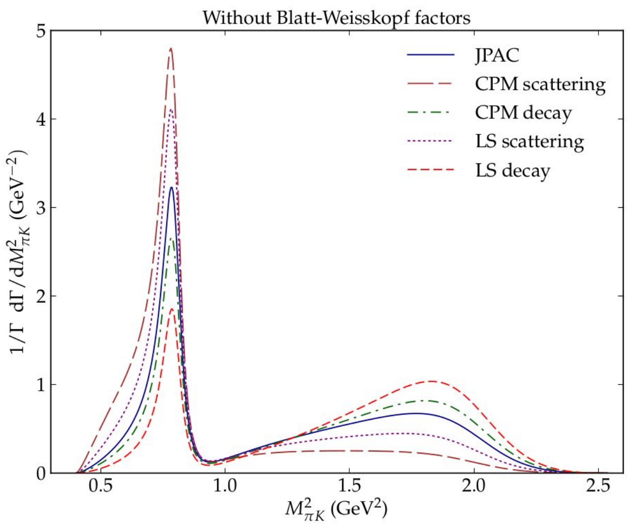
- To describe the decay a → bc, we first consider the polarization tensor of each particle, $\varepsilon^{i}_{\mu_1 \ldots \mu_{j_i}}(p_i)$
- We combine the polarizations of b and c into a "total spin" tensor, $S_{\mu_1 \ldots \mu_S}(\varepsilon_b, \varepsilon_c)$
- Using the decay momentum, we build a tensor $L_{\mu_1 \ldots \mu_L}(p_{bc})$ to represent the orbital angular momentum of the bc system, orthogonal to the total momentum of p_a
- We contract S and L with the polarization of a

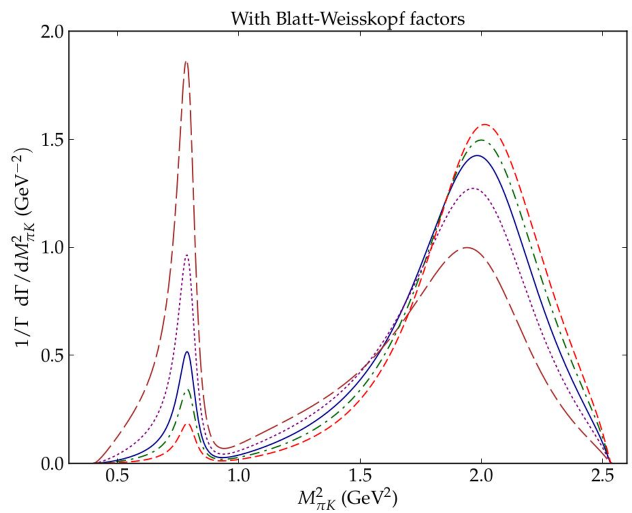
Advantages

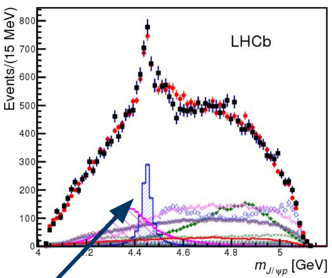
- The procedure is recursive, and relatively simple for low spins.
- The tensor multiply the dynamic functions which contain resonances and form factors

Kinematic singularities







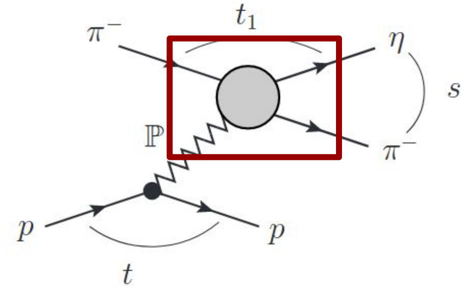


Conclusions:

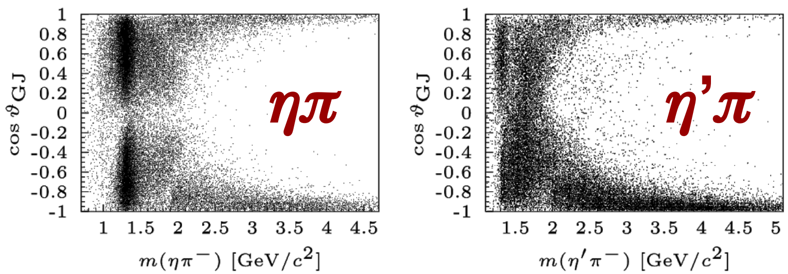
- LS only gives correct threshold behavior (not pseudothreshold and s=0)
- LS is relativistic
- CPM is not crossing symmetric
- CPM differs from LS
- CPM yields redundant kinematic factors, which are not required by analyticity

[M. Mikhasenko, A. Pilloni, JN *et al.* EPJC78 (2018)] & [A. Pilloni, JN, M. Mikhasenko *et al.* arxiv today (?) (2018)]

Pole extraction

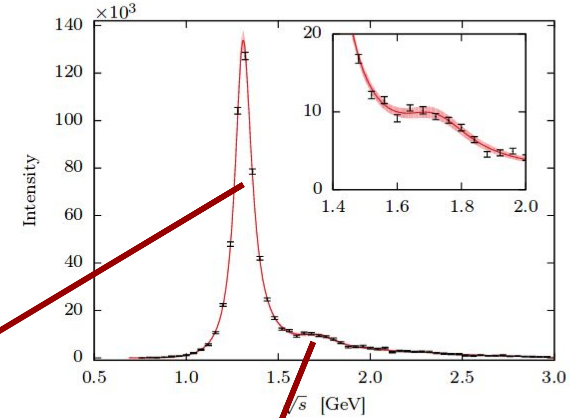


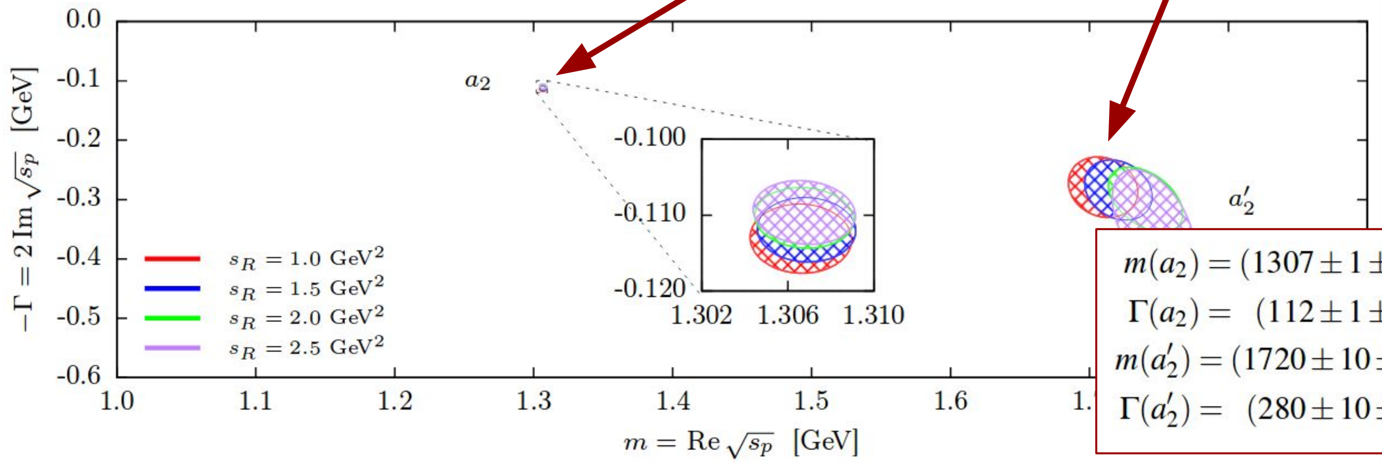
[DATA: COMPASS, PLB (2015) 303]



(a) $m(\eta\pi^-)$ vs. $\cos\vartheta_{GJ}$

(b) $m(\eta'\pi^-)$ vs. $\cos\vartheta_{GJ}$

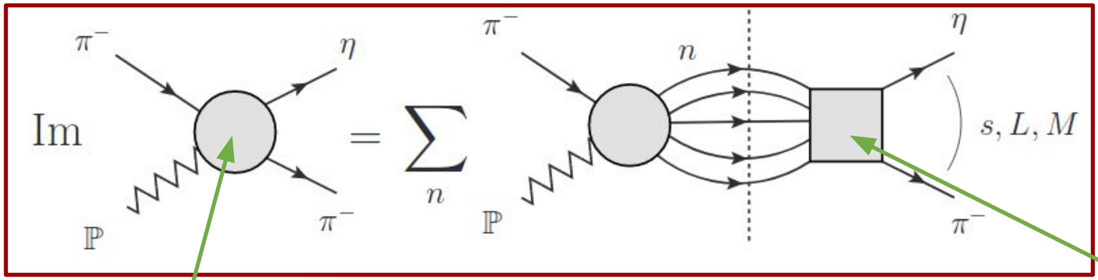




$$m(a_2) = (1307 \pm 1 \pm 6)\ \text{MeV}$$
$$\Gamma(a_2) = (112 \pm 1 \pm 8)\ \text{MeV}$$
$$m(a_2') = (1720 \pm 10 \pm 60)\ \text{MeV}$$
$$\Gamma(a_2') = (280 \pm 10 \pm 70)\ \text{MeV}$$

Partial-wave analysis

- Unitarity, analytic N/D model
- N contains left-hand cuts (exchange forces)
- D contains right-hand cuts (resonance content)



$$\mathrm{Im}\, D(s) = -\rho(s) N(s)$$

$$\hat{a}(s) = \frac{n(s)}{D(s)}$$

$$\hat{f}(s) = N(s)/D(s)$$

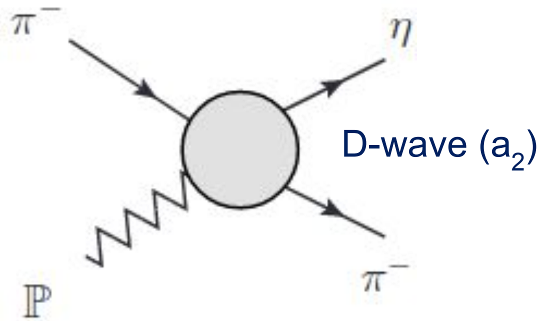
$$\rho(s) N(s) = g \frac{\lambda^{5/2}(s, m_\eta^2, m_\pi^2)}{(s + s_R)^n}$$

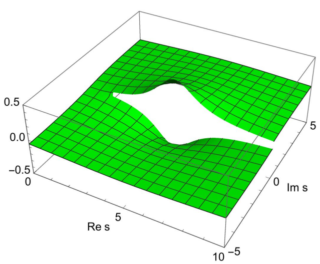
$$D(s) = D_0(s) - \frac{s}{\pi} \int_{s_{th}}^{\infty} ds' \frac{\rho(s') N(s')}{s'(s' - s)}$$

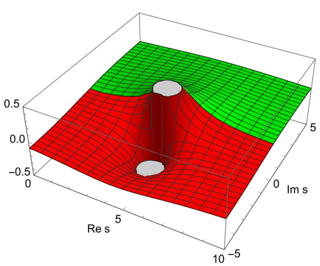
$$D_0(s) = c_0 - c_1 s - \frac{c_2}{c_3 - s}$$

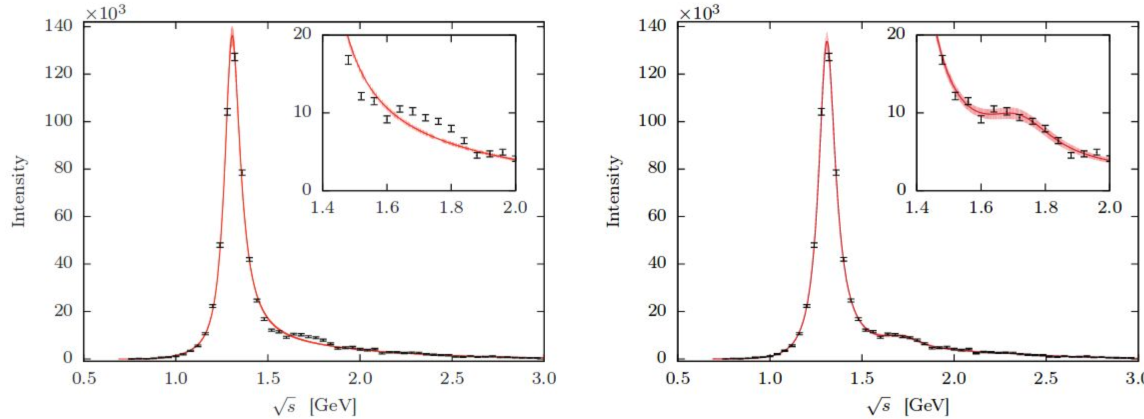
$$K^{-1}(s) = D_0(s)$$

Partial-wave analysis

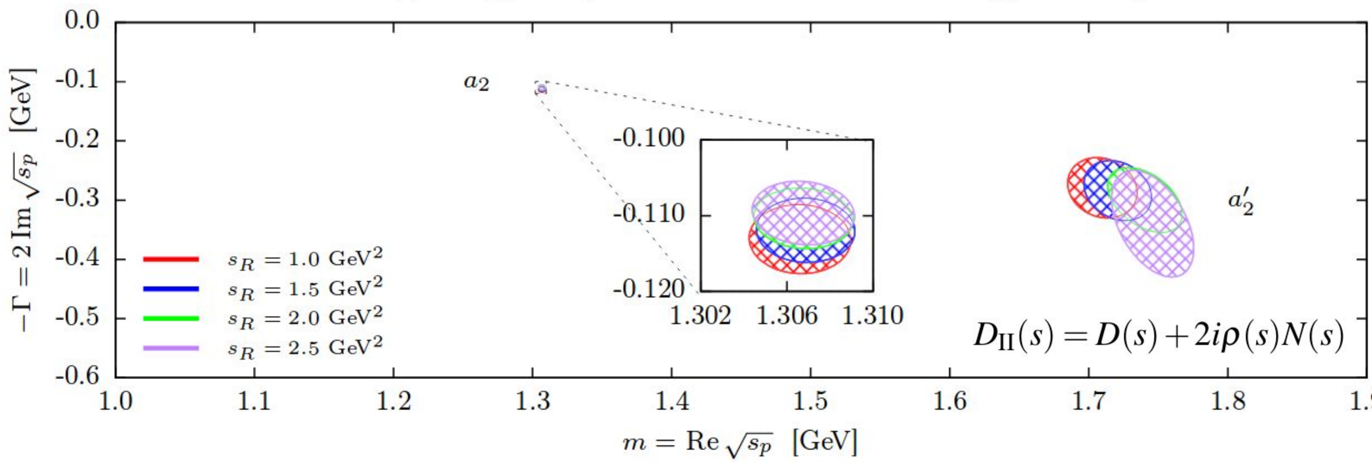








(a) CDD$_\infty$ pole only. (b) Two CDD poles.



$m(a_2) = (1307 \pm 1 \pm 6)$ MeV

$\Gamma(a_2) = (112 \pm 1 \pm 8)$ MeV

$m(a_2') = (1720 \pm 10 \pm 60)$ MeV

$\Gamma(a_2') = (280 \pm 10 \pm 70)$ MeV

[A.Jackura *et al.* (JPAC & COMPASS), PLB779 (2018)]

Exotic P-wave

LIGHT UNFLAVORED
$(S = C = B = 0)$

	$I^G(J^{PC})$		$I^G(J^{PC})$
• $\pi^{\pm}$	$1^-(0^-)$	• $\rho_3(1690)$	$1^+(3^{--})$
• π^0	$1^-(0^{-+})$	• $\rho(1700)$	$1^+(1^{--})$
• η	$0^+(0^{-+})$	$a_2(1700)$	$1^-(2^{++})$
• $f_0(500)$	$0^+(0^{++})$	• $f_0(1710)$	$0^+(0^{++})$
• $\rho(770)$	$1^+(1^{--})$	$\eta(1760)$	$0^+(0^{-+})$
• $\omega(782)$	$0^-(1^{--})$	• $\pi(1800)$	$1^-(0^{-+})$
• $\eta'(958)$	$0^+(0^{-+})$	$f_2(1810)$	$0^+(2^{++})$
• $f_0(980)$	$0^+(0^{++})$	$X(1835)$	$?^?(0^{-+})$
• $a_0(980)$	$1^-(0^{++})$	$X(1840)$	$?^?(?^{??})$
• $\phi(1020)$	$0^-(1^{--})$	$a_1(1420)$	$1^-(1^{++})$
• $h_1(1170)$	$0^-(1^{+-})$	• $\phi_3(1850)$	$0^-(3^{--})$
• $b_1(1235)$	$1^+(1^{+-})$	$\eta_2(1870)$	$0^+(2^{-+})$
• $a_1(1260)$	$1^-(1^{++})$	• $\pi_2(1880)$	$1^-(2^{-+})$
• $f_2(1270)$	$0^+(2^{++})$	$\rho(1900)$	$1^+(1^{--})$
• $f_1(1285)$	$0^+(1^{++})$	$f_2(1910)$	$0^+(2^{++})$
• $\eta(1295)$	$0^+(0^{-+})$	$a_0(1950)$	$1^-(0^{++})$
• $\pi(1300)$	$1^-(0^{-+})$	• $f_2(1950)$	$0^+(2^{++})$
• $a_2(1320)$	$1^-(2^{++})$	$\rho_3(1990)$	$1^+(3^{--})$
• $f_0(1370)$	$0^+(0^{++})$	• $f_2(2010)$	$0^+(2^{++})$
$h_1(1380)$	$?^-(1^{+-})$	$f_0(2020)$	$0^+(0^{++})$
• $\pi_1(1400)$	$1^-(1^{-+})$	• $a_4(2040)$	$1^-(4^{++})$
• $\eta(1405)$	$0^+(0^{-+})$	• $f_4(2050)$	$0^+(4^{++})$
• $f_1(1420)$	$0^+(1^{++})$	$\pi_2(2100)$	$1^-(2^{-+})$
• $\omega(1420)$	$0^-(1^{--})$	$f_0(2100)$	$0^+(0^{++})$
$f_2(1430)$	$0^+(2^{++})$	$f_2(2150)$	$0^+(2^{++})$
• $a_0(1450)$	$1^-(0^{++})$	$\rho(2150)$	$1^+(1^{--})$
• $\rho(1450)$	$1^+(1^{--})$	• $\phi(2170)$	$0^-(1^{--})$
• $\eta(1475)$	$0^+(0^{-+})$	$f_0(2200)$	$0^+(0^{++})$
• $f_0(1500)$	$0^+(0^{++})$	$f_J(2220)$	$0^+(2^{++}$
$f_1(1510)$	$0^+(1^{++})$		or $4^{++})$
• $f'_2(1525)$	$0^+(2^{++})$	$\eta(2225)$	$0^+(0^{-+})$
$f_2(1565)$	$0^+(2^{++})$	$\rho_3(2250)$	$1^+(3^{--})$
$\rho(1570)$	$1^+(1^{--})$	• $f_2(2300)$	$0^+(2^{++})$
$h_1(1595)$	$0^-(1^{+-})$	$f_4(2300)$	$0^+(4^{++})$
• $\pi_1(1600)$	$1^-(1^{-+})$	$f_0(2330)$	$0^+(0^{++})$
$a_1(1640)$	$1^-(1^{++})$	• $f_2(2340)$	$0^+(2^{++})$
$f_2(1640)$	$0^+(2^{++})$	$\rho_5(2350)$	$1^+(5^{--})$
• $\eta_2(1645)$	$0^+(2^{-+})$	$a_6(2450)$	$1^-(6^{++})$
• $\omega(1650)$	$0^-(1^{--})$	$f_6(2510)$	$0^+(6^{++})$
• $\omega_3(1670)$	$0^-(3^{--})$	OTHER LIGHT	
• $\pi_2(1670)$	$1^-(2^{-+})$	Further States	
• $\phi(1680)$	$0^-(1^{--})$		

$\pi_1(1400)$

$I^G(J^{PC}) = 1^-(1^{-+})$

See also the mini-review under non-$q\bar{q}$ candidates in PDG 06, Journal of Physics **G33** 1 (2006).

$\pi_1(1400)$ MASS

VALUE (MeV)			EVTS	DOCUMENT ID		TECN	CHG	COMMENT
1354	**±25**			**OUR AVERAGE** Error includes scale factor of 1.8. See the ideogram below.				
1257	±20	±25	23.5k	ADAMS	07B	B852		18 $\pi^- p \rightarrow \eta\pi^0 n$
1384	±20	±35	90k	SALVINI	04	OBLX		$\bar{p}p \rightarrow 2\pi^+ 2\pi^-$
1360	±25			ABELE	99	CBAR		0.0 $\bar{p}p \rightarrow \pi^0\pi^0\eta$
1400	±20	±20		ABELE	98B	CBAR		0.0 $\bar{p}n \rightarrow \pi^-\pi^0\eta$
1370	±16	$^{+50}_{-30}$		[1] THOMPSON	97	MPS		18 $\pi^- p \rightarrow \eta\pi^- p$
• • • We do not use the following data for averages, fits, limits, etc. • • •								
1323.1±	4.6			[2] AOYAGI	93	BKEI		$\pi^- p \rightarrow \eta\pi^- p$
1406	±20			[3] ALDE	88B	GAM4	0	100 $\pi^- p \rightarrow \eta\pi^0 n$

[1] Natural parity exchange, questioned by DZIERBA 03.
[2] Unnatural parity exchange.
[3] Seen in the P_0-wave intensity of the $\eta\pi^0$ system, unnatural parity exchange.

$\pi_1(1600)$

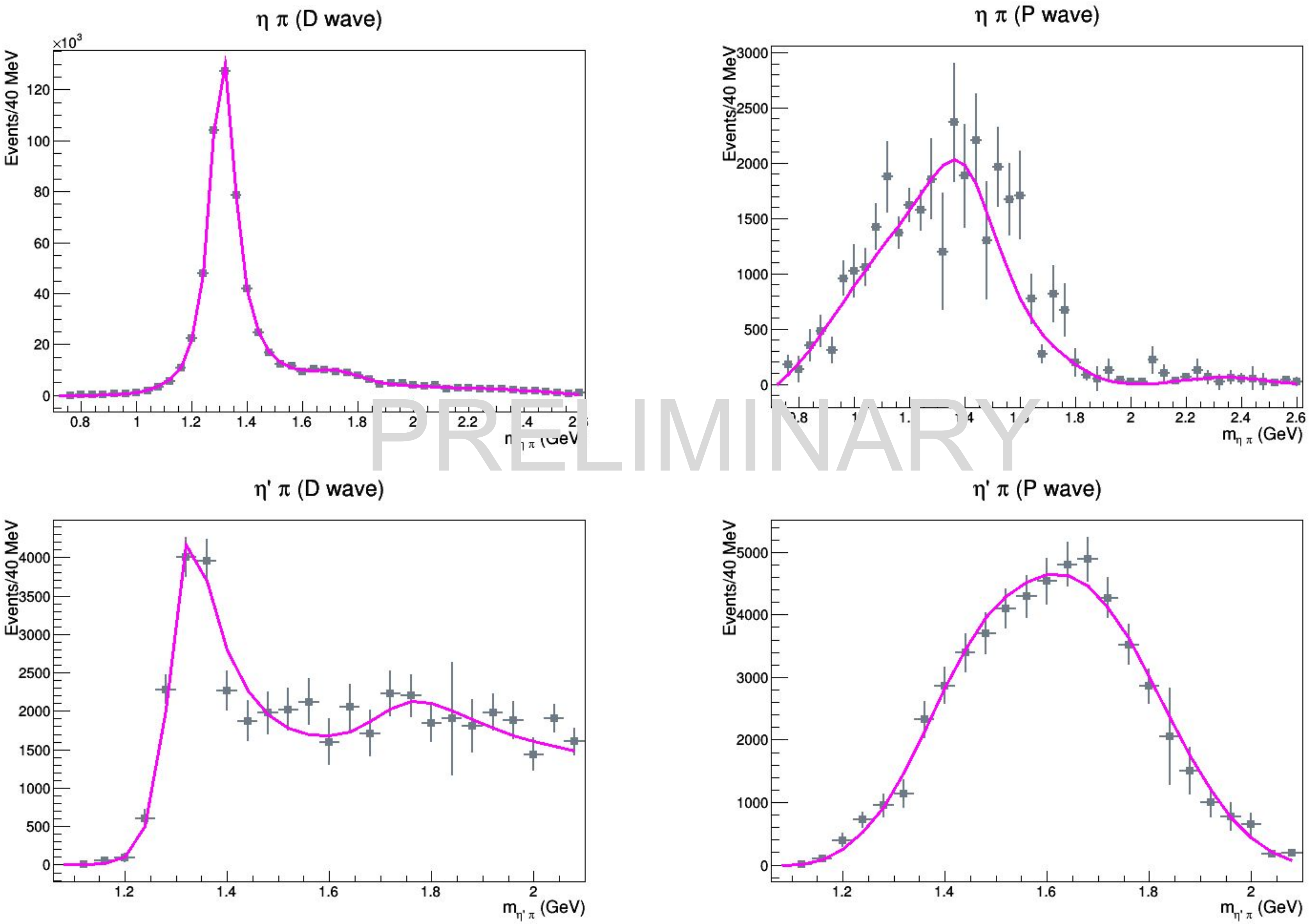
$I^G(J^{PC}) = 1^-(1^{-+})$

$\pi_1(1600)$ MASS

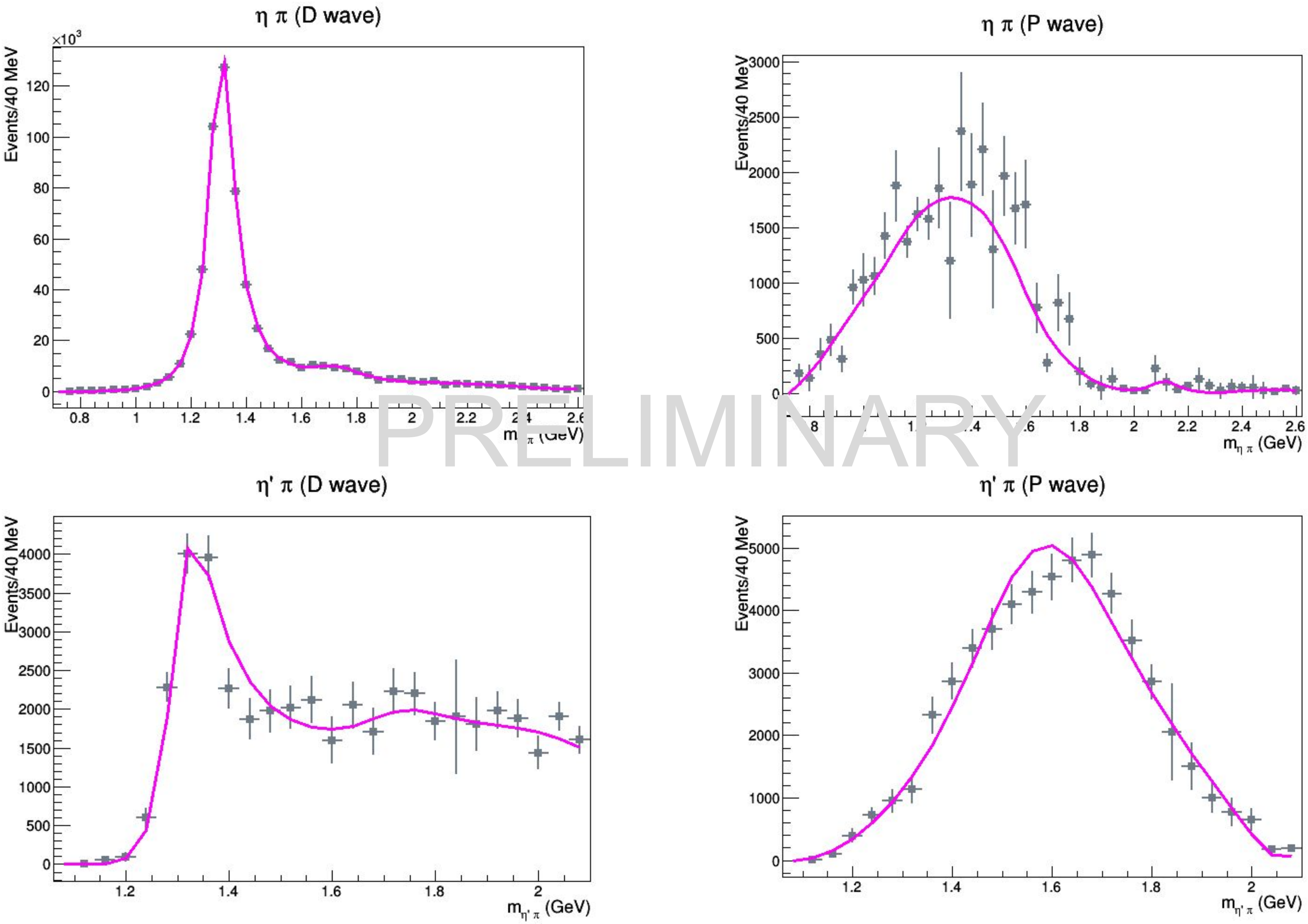
VALUE (MeV)	EVTS	DOCUMENT ID		TECN	COMMENT
$1662^{+\ 8}_{-\ 9}$		**OUR AVERAGE**			
$1660 \pm 10^{+\ 0}_{-64}$	420k	ALEKSEEV	10	COMP	190 $\pi^- Pb \rightarrow \pi^-\pi^-\pi^+ Pb'$
$1664 \pm\ 8 \pm 10$	145k	[1] LU	05	B852	18 $\pi^- p \rightarrow \omega\pi^-\pi^0 p$
$1709 \pm 24 \pm 41$	69k	[2] KUHN	04	B852	18 $\pi^- p \rightarrow \eta\pi^+\pi^-\pi^- p$
$1597 \pm 10^{+45}_{-10}$		[2] IVANOV	01	B852	18 $\pi^- p \rightarrow \eta'\pi^- p$
• • • We do not use the following data for averages, fits, limits, etc. • • •					
$1593 \pm\ 8^{+29}_{-47}$		[2,3] ADAMS	98B	B852	18.3 $\pi^- p \rightarrow \pi^+\pi^-\pi^- p$

[1] May be a different state: natural and unnatural parity exchanges.
[2] Natural parity exchange.
[3] Superseded by DZIERBA 06 excluding this state in a more refined PWA analysis, with 2.6 M events of $\pi^- p \rightarrow \pi^-\pi^-\pi^+ p$ and 3 M events of $\pi^- p \rightarrow \pi^-\pi^0\pi^0 p$ of E852 data.

Coupled channel: 2 poles in P

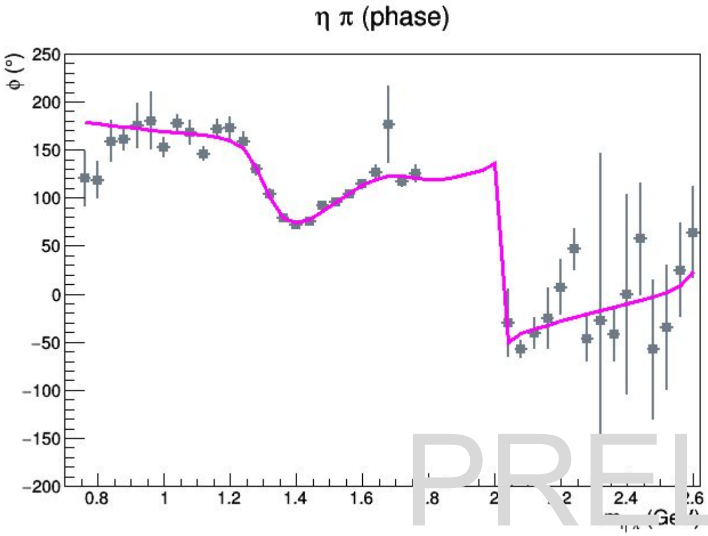


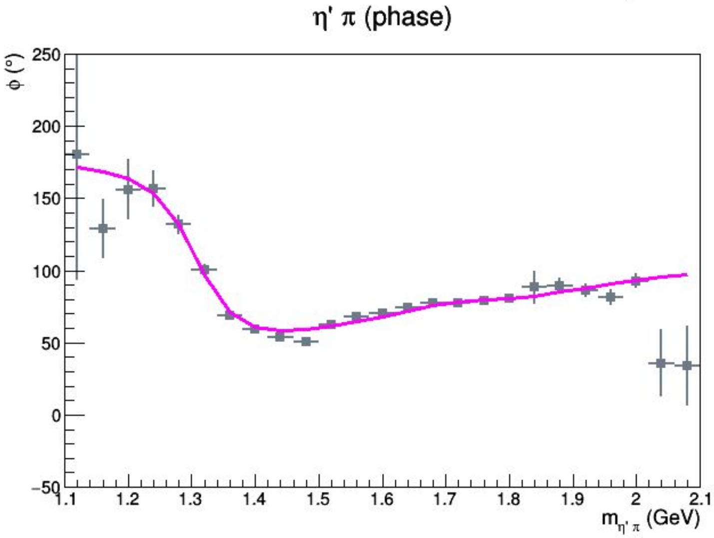
Coupled channel: 1 pole in P



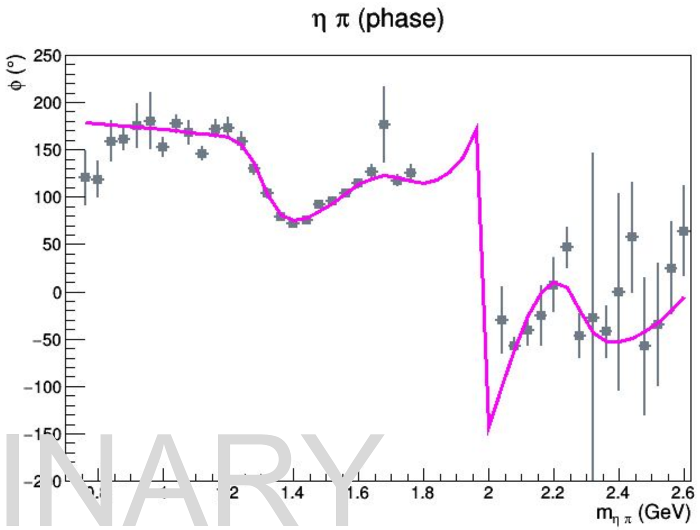
Coupled channel: relative phase (P,D)

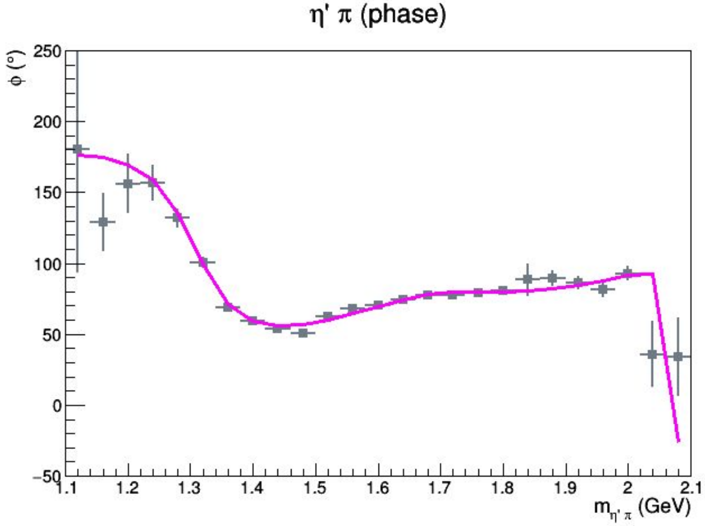
2 poles



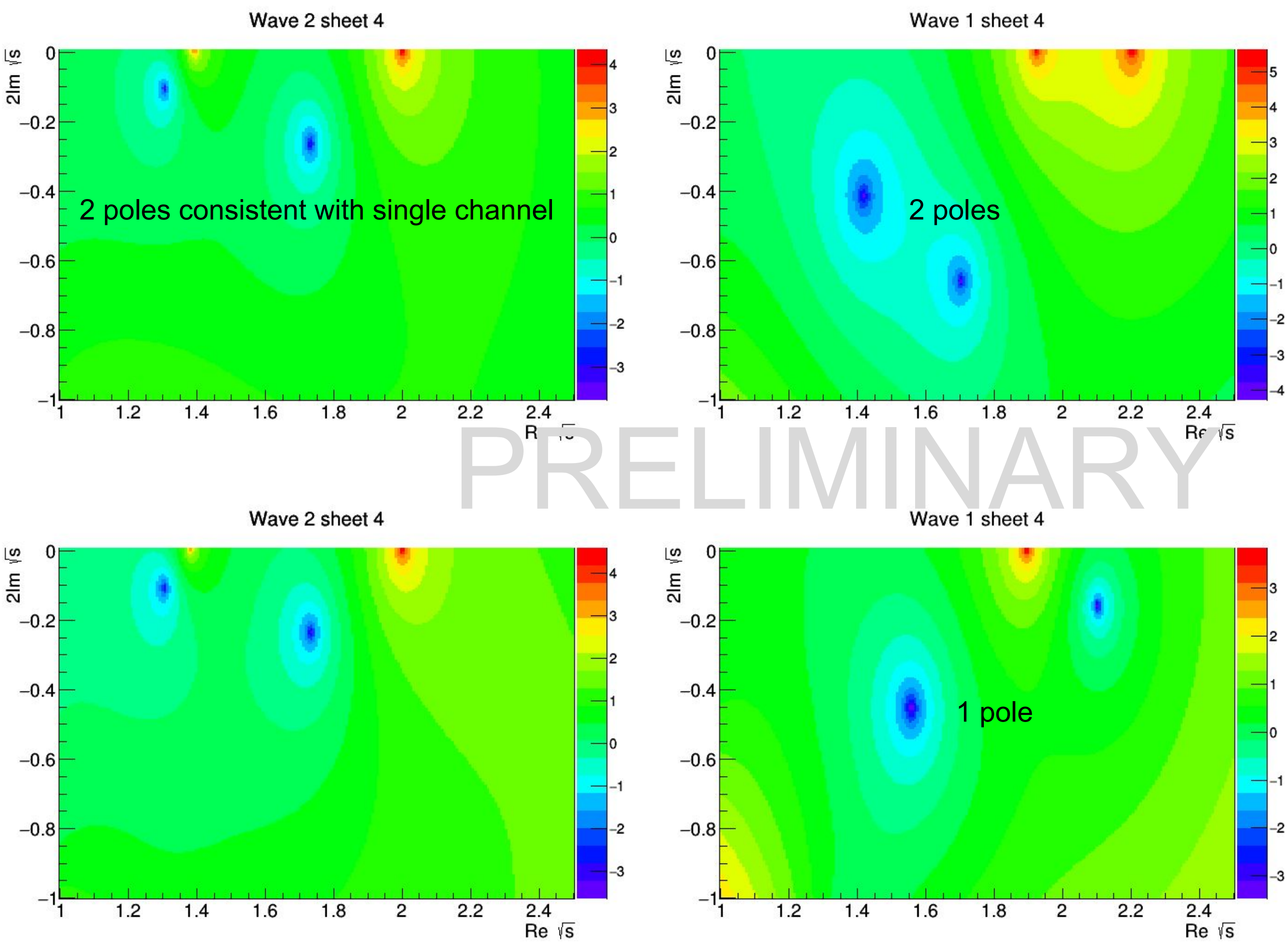


1 pole





Coupled channel: sheet structure



[JPAC, in preparation]

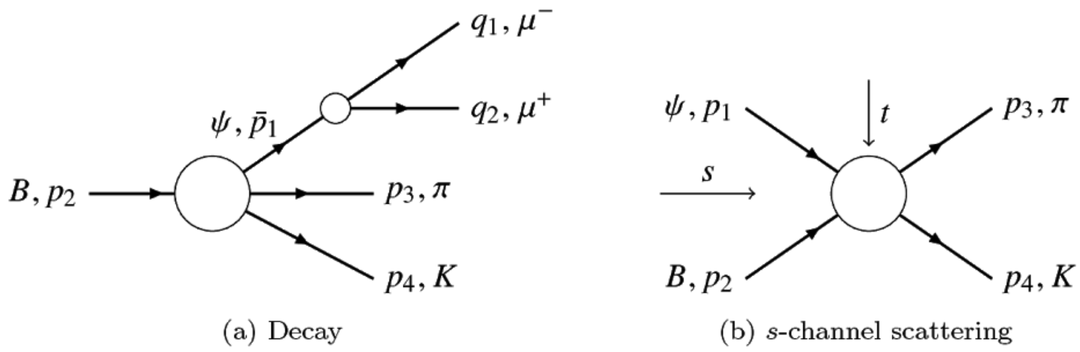
Summary

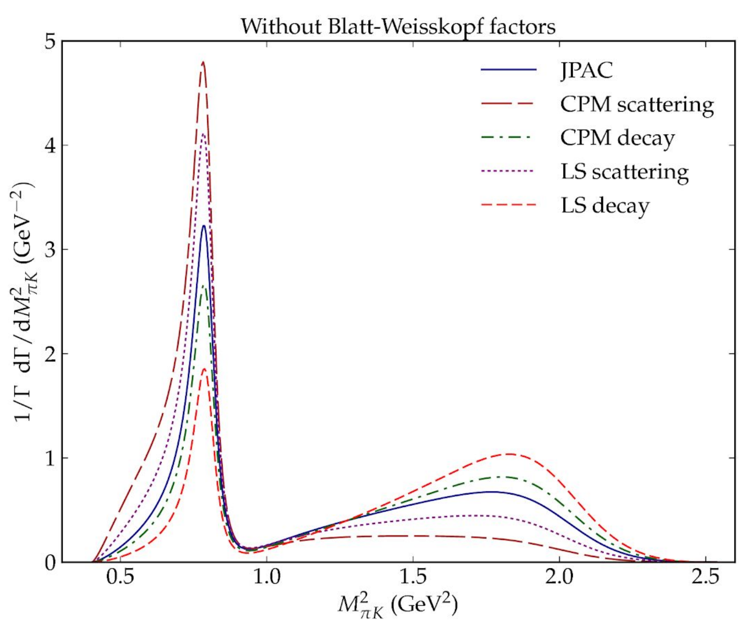
- Hunt for **exotic mesons** has started at Jefferson Lab
- Many analyses to understand the **production process** in photoproduction processes
- **Analyticity** constraints are necessary to predict the naturality of the exchanges
- **Kinematic singularities** must be removed properly before hunting for dynamic singularities: scheme dependent
- Analysis of the **exotic P-wave** in COMPASS data with a unitary and analytic model

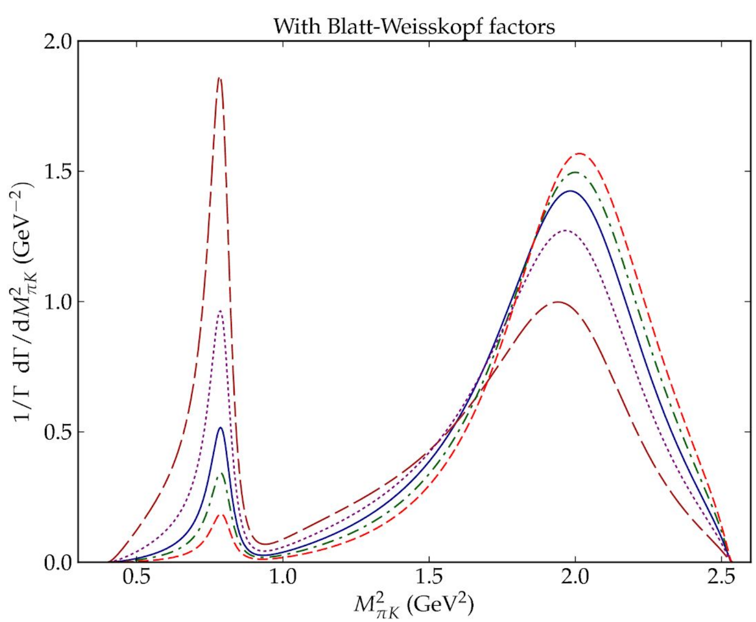
Ongoing study: sum rules for $\eta^{(\prime)}\pi$

Backup

Kinematic singularities







High-energy model

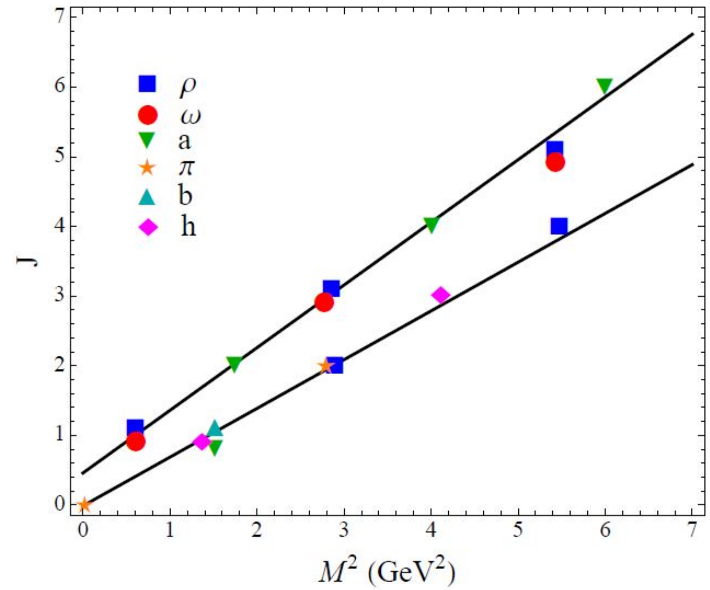
Regge pole model

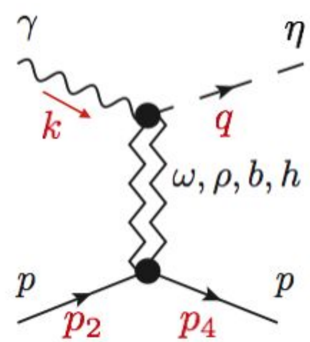
J(m^2)

$$A_i(\nu,t) = -\beta_i(t)\frac{\pm 1 + e^{-i\pi\alpha(t)}}{\sin \pi\alpha(t)}\nu^{\alpha(t)}$$

Dominant: vector exchanges

A_i	I^G	J^{PC}	η	Leading exchanges
A_1	$0^-, 1^+$	$(1,3,5,\ldots)^{--}$	$+1$	$\rho(770), \omega(782)$
A_2'	$0^-, 1^+$	$(1,3,5,\ldots)^{+-}$	-1	$h_1(1170), b_1(1235)$
A_3	$0^-, 1^+$	$(2,4,\ldots)^{--}$	-1	$\rho_2(??), \omega_2(??)$
A_4	$0^-, 1^+$	$(1,3,5,\ldots)^{--}$	$+1$	$\rho(770), \omega(782)$



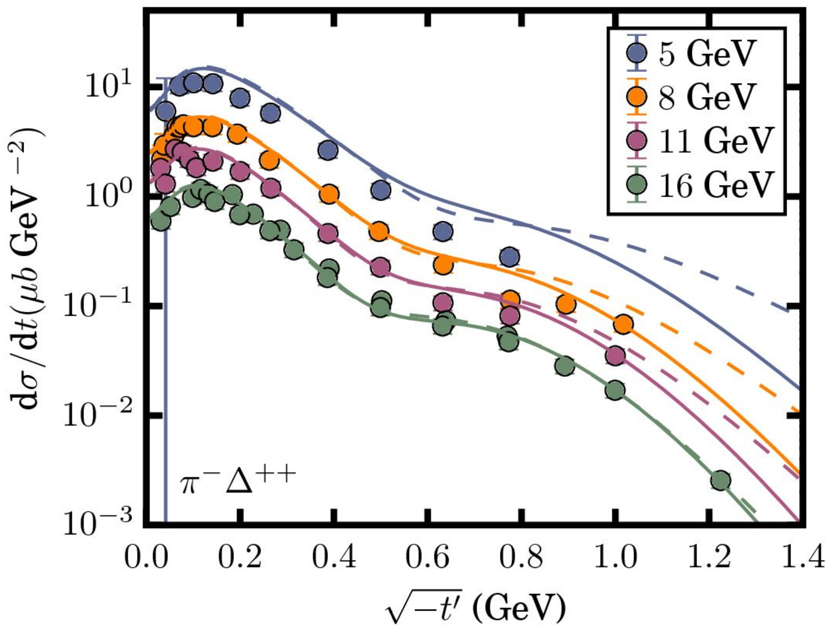


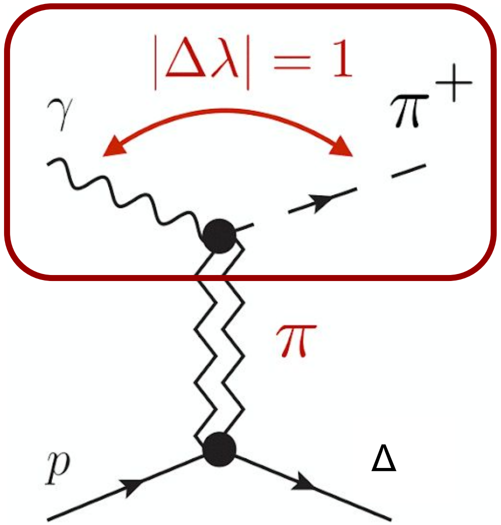
$$\gamma p \to \eta p\,, \qquad A = (\omega + h + \omega_2) + (\rho + b + \rho_2)$$

$$\gamma n \to \eta n\,, \qquad A = (\omega + h + \omega_2) - (\rho + b + \rho_2)$$

πΔ photoproduction

J.N *et al.* (JPAC) [arXiv:1710.09394]



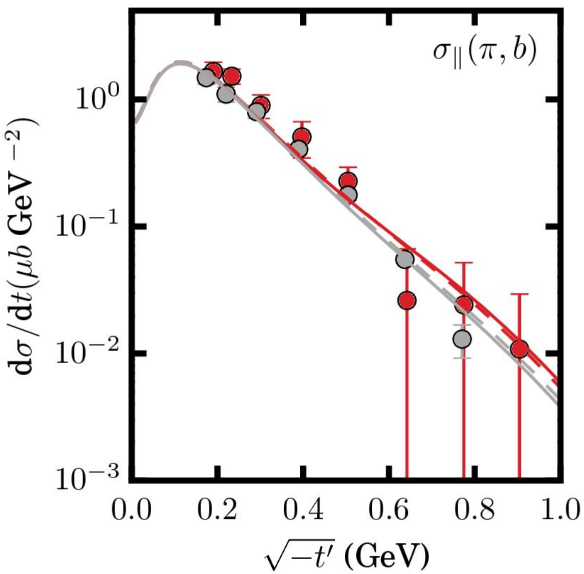


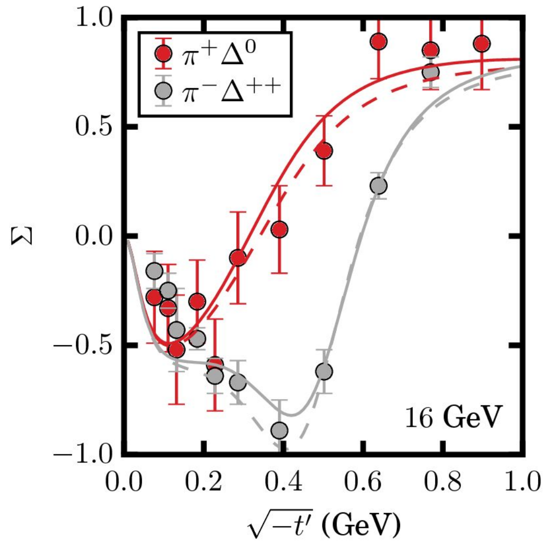
$$A^{10}_{-\frac{1}{2}\frac{1}{2}} \propto \frac{-t}{m_\pi^2 - t} \quad \rightarrow \frac{-m_\pi^2}{m_\pi^2 - t}$$

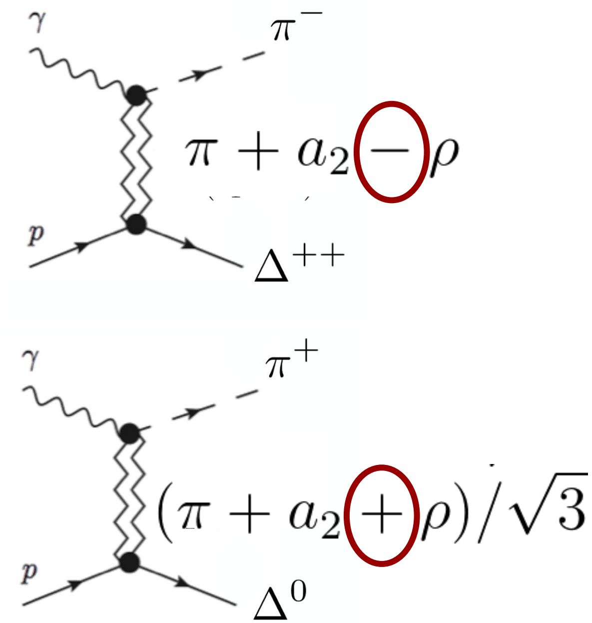
From residue factorization:
$\sqrt{-t}$ **for each helicity flip**
Not seen in data → contact term

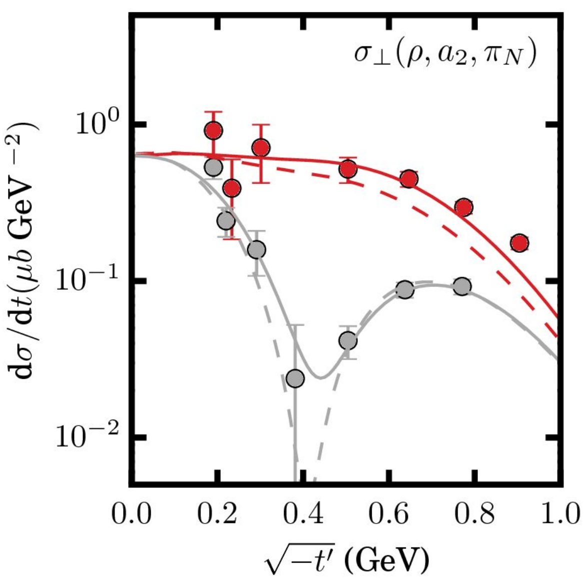
πΔ photoproduction

J.N. *et al.* (JPAC) [arXiv:1710.09394]







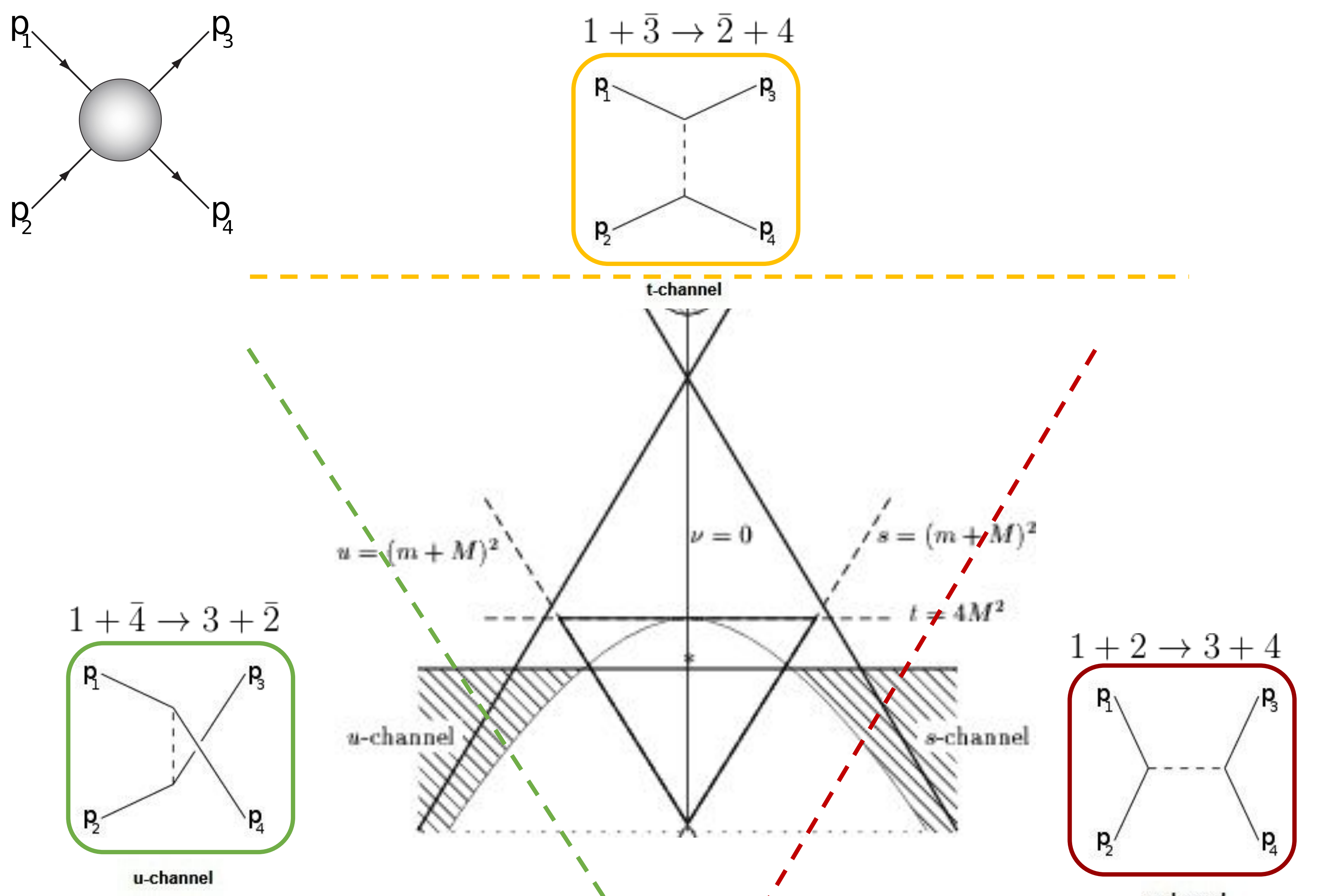


Data available at **16 GeV**

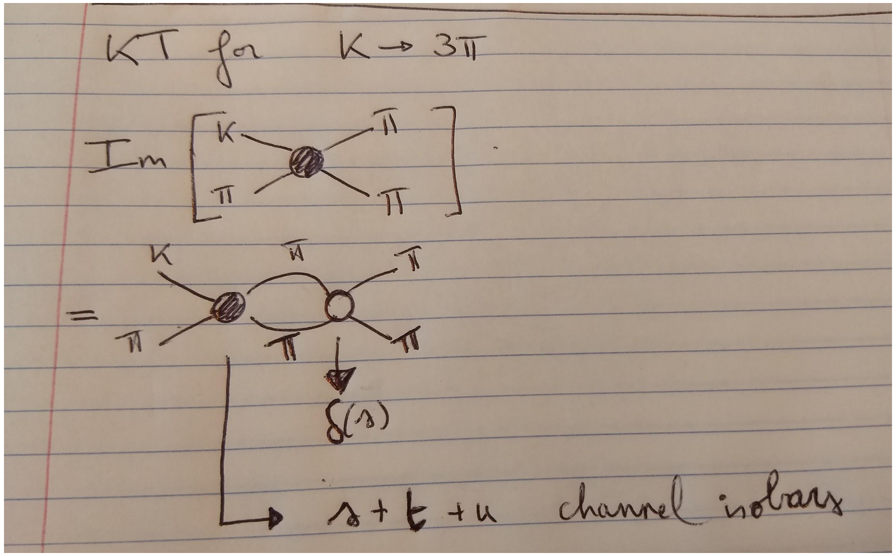
- π-exchange is featureless and entirely fixed
- Strong interference pattern in natural exchange sector
- Negligible role of b exchange

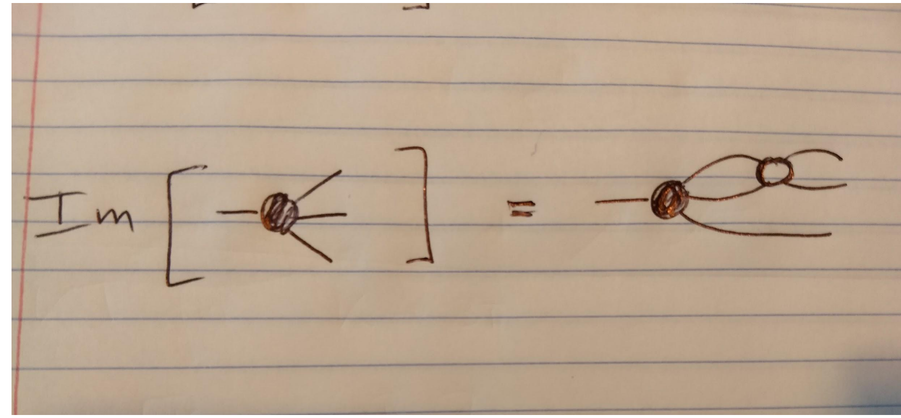
Fix t-dependence and extrapolate to JLab energies (**9 GeV**)

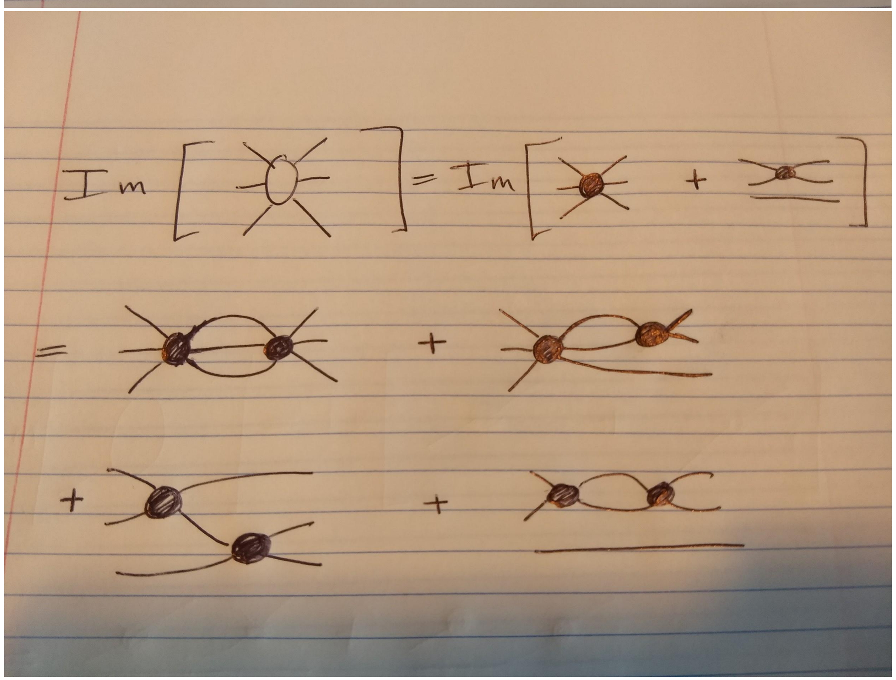
Using the right degrees of freedom



KT approach







$$\frac{1}{\sqrt{2s}}\left(A_{+,+1}+A_{-,-1}\right)=\sqrt{-t}A_4 \tag{19}$$

$$\frac{1}{\sqrt{2s}}\left(A_{+,-1}-A_{-,+1}\right)=A_1 \tag{20}$$

$$\frac{1}{\sqrt{2s}}\left(A_{+,+1}-A_{-,-1}\right)=\sqrt{-t}A_3 \tag{21}$$

$$\frac{1}{\sqrt{2s}}\left(A_{+,-1}+A_{-,+1}\right)=-A_2'=-(A_1+tA_2) \tag{22}$$

Thus, at high energies the invariants A_3 and A_4 (A_1 and A_2') correspond to the s-channel nucleon-helicity non-flip (flip), respectively. Combining Eqs. (20) and (22) we obtain

$$A_{-,+1}=-\frac{s}{\sqrt{2}}\left(A_2'+A_1\right) . \tag{23}$$

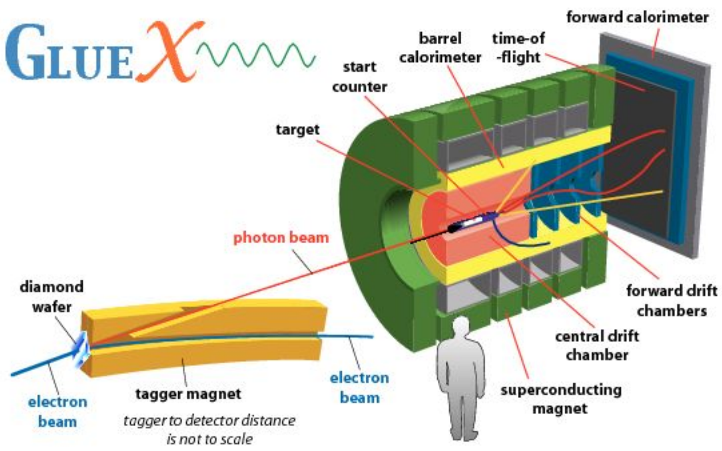
$$A_{\mu_f,\mu_i\,\mu_\gamma} \underset{t\to 0}{\sim} (-t)^{n/2} , \tag{17}$$

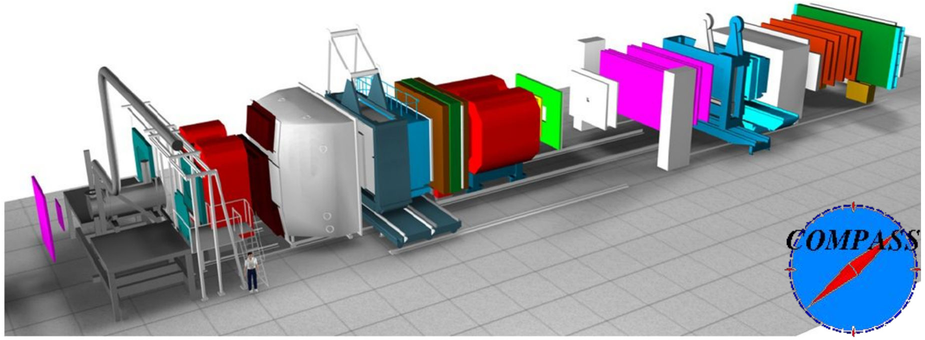
where $n=|(\mu_\gamma-\mu_i)-(-\mu_f)|\geq 0$ is the net s-channel helicity flip. This is a weaker condition than the one imposed by angular-momentum conservation on factorizable Regge amplitudes,

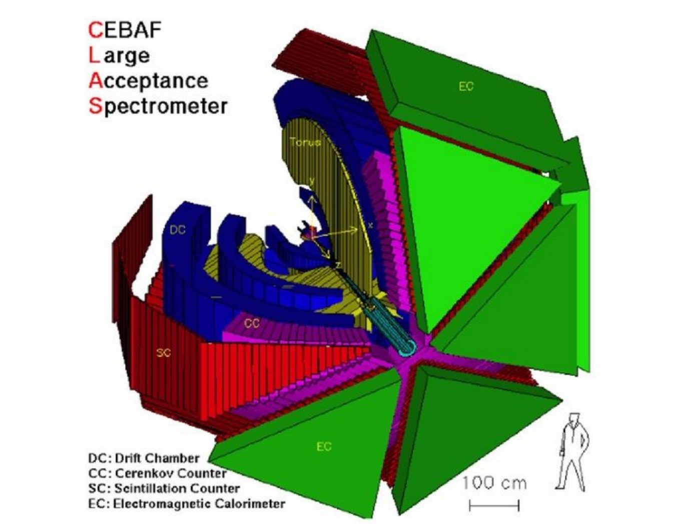
$$A_{\mu_f,\mu_i\,\mu_\gamma} \underset{t\to 0}{\sim} (-t)^{(n+x)/2} , \tag{18}$$

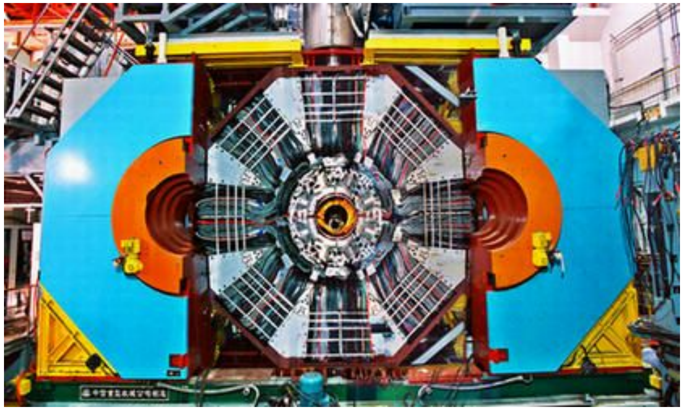
where $n+x=|\mu_\gamma|+|\mu_i-\mu_f|\geq 1$. We summarize the

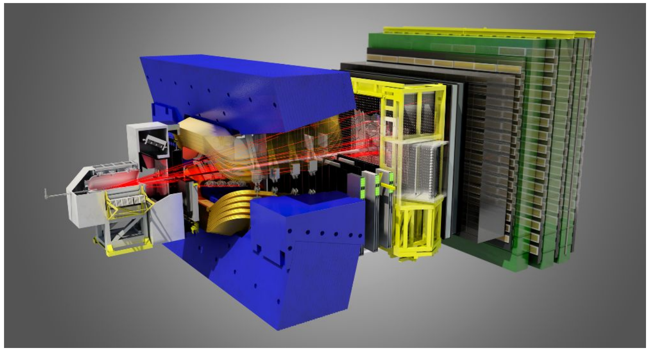
Spectroscopy (experiment)







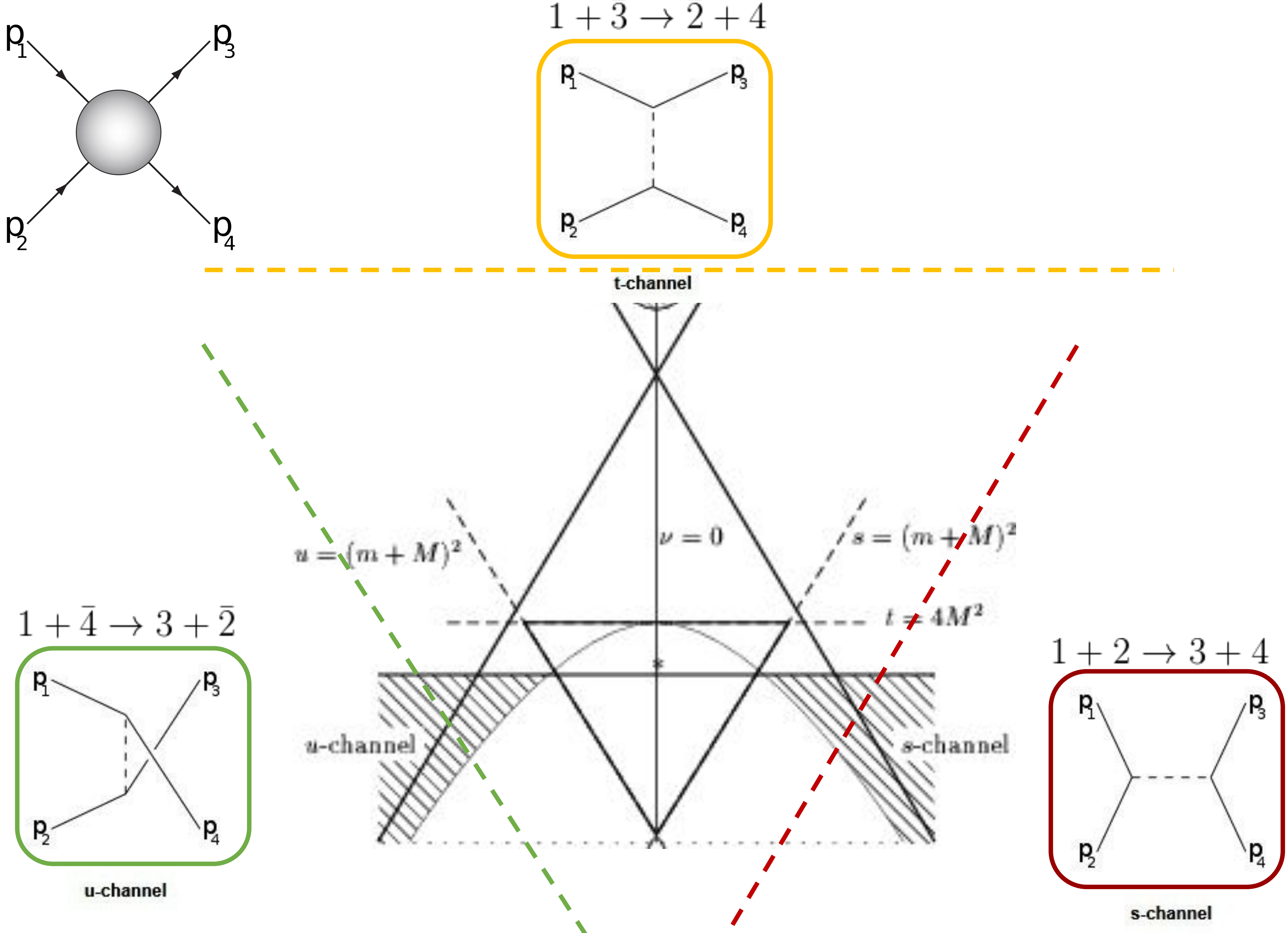




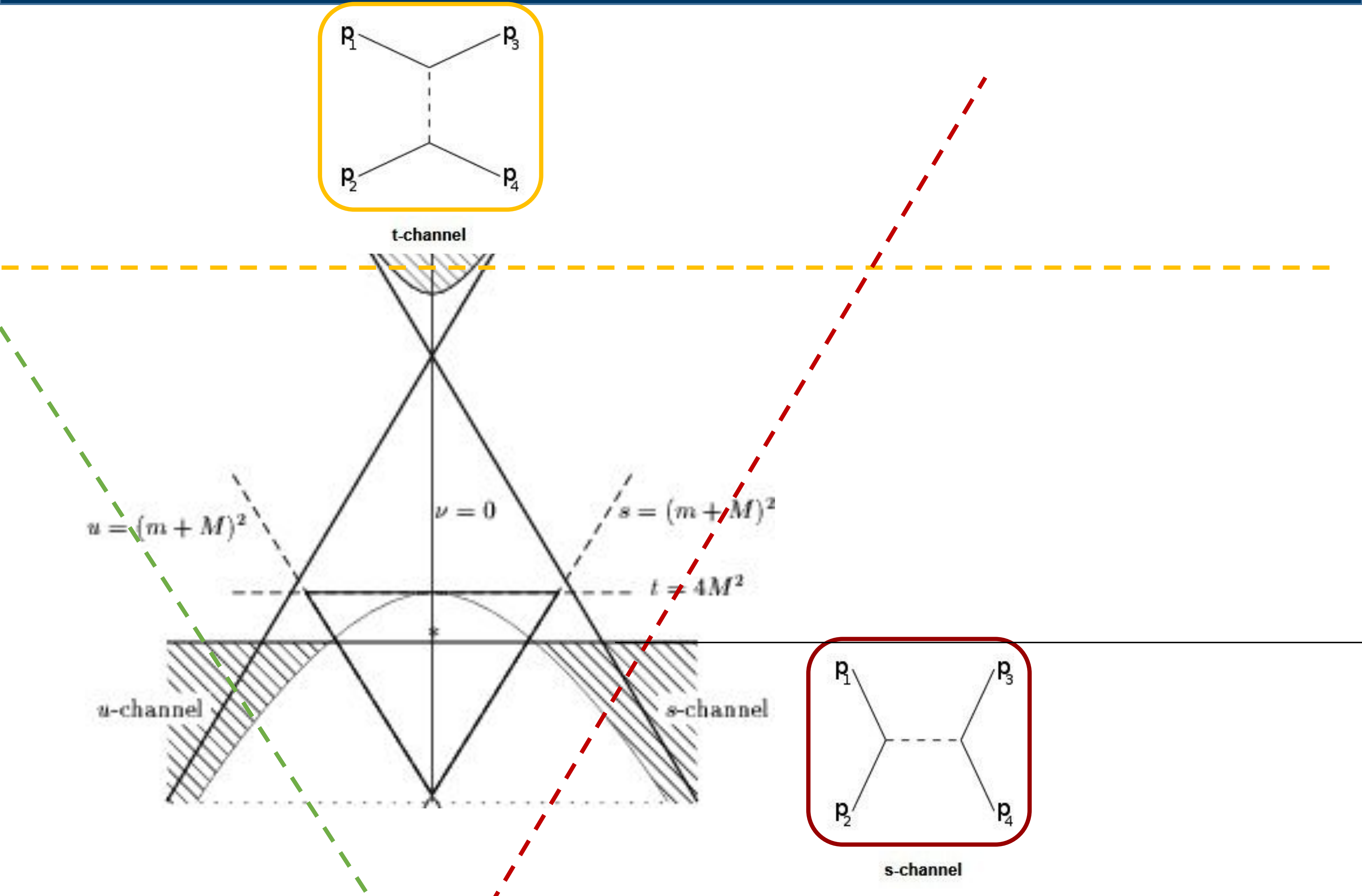
$$\alpha_{1,4}^{(\sigma)} \equiv \alpha_N(t) = 0.9(t - m_\rho^2) + 1$$

$$\alpha_{2,3}^{(\sigma)} \equiv \alpha_U(t) = 0.7(t - m_\pi^2) + 0$$

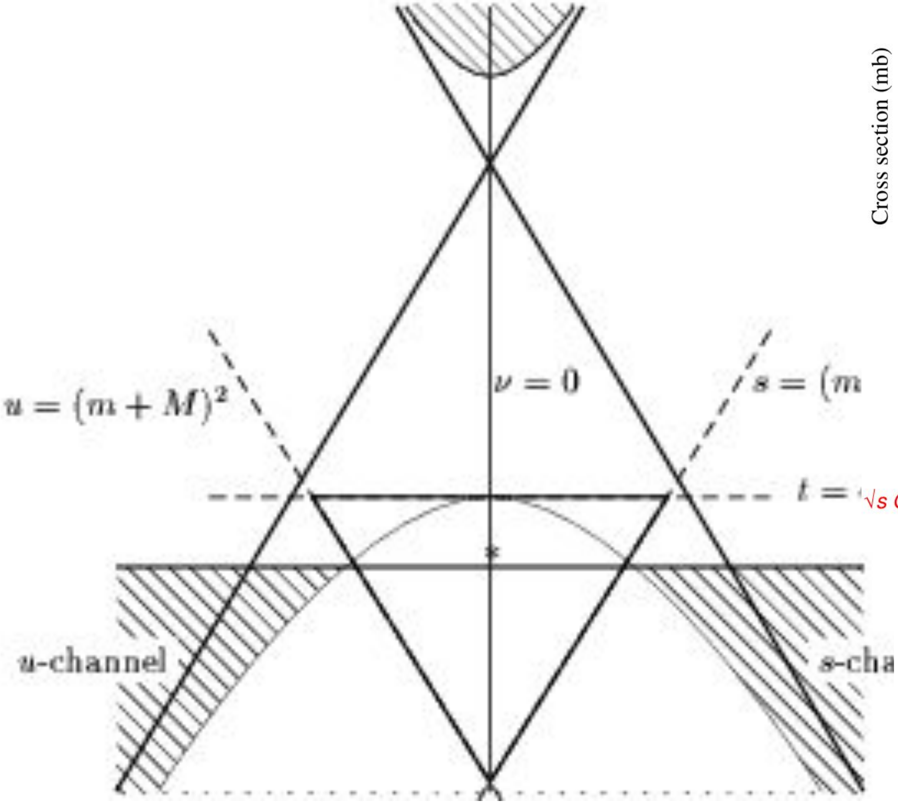
Using the right degrees of freedom

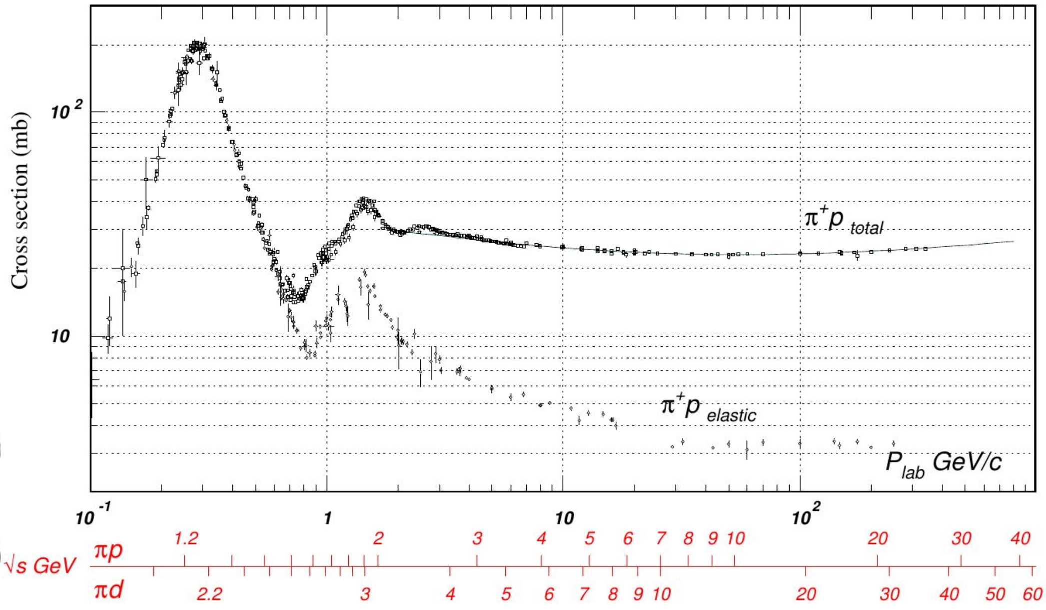


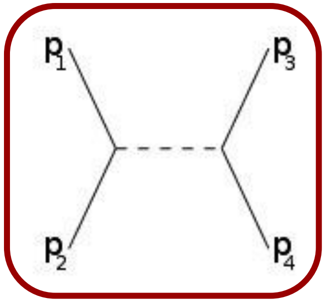
Using the right degrees of freedom



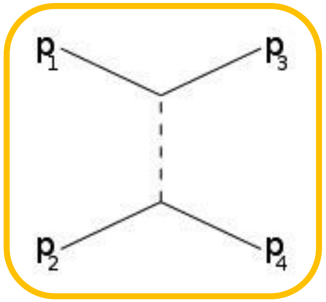
Using the right degrees of freedom







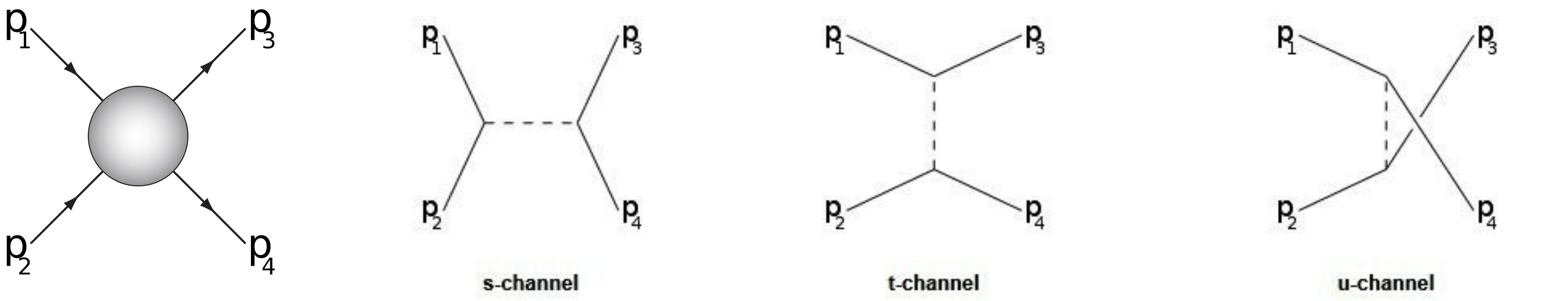
s-channel



t-channel

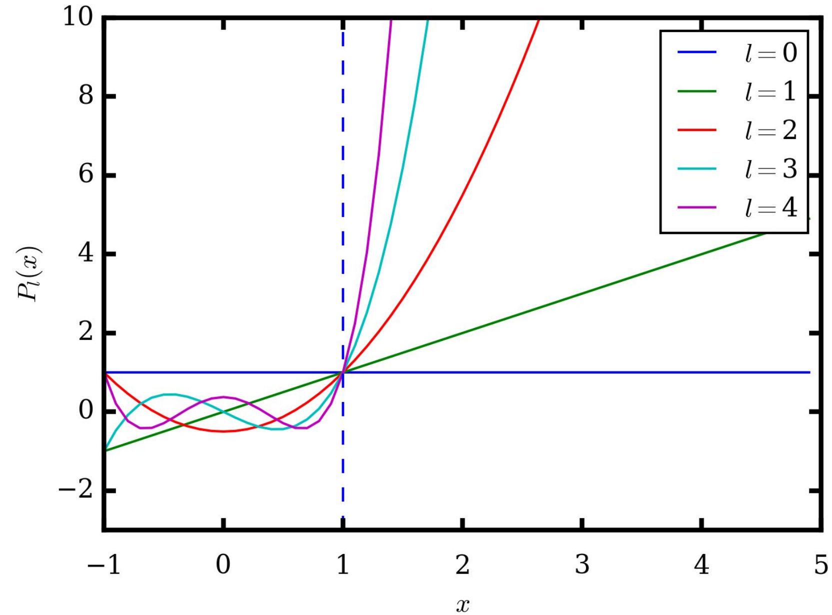
Partial-wave expansion in any channel

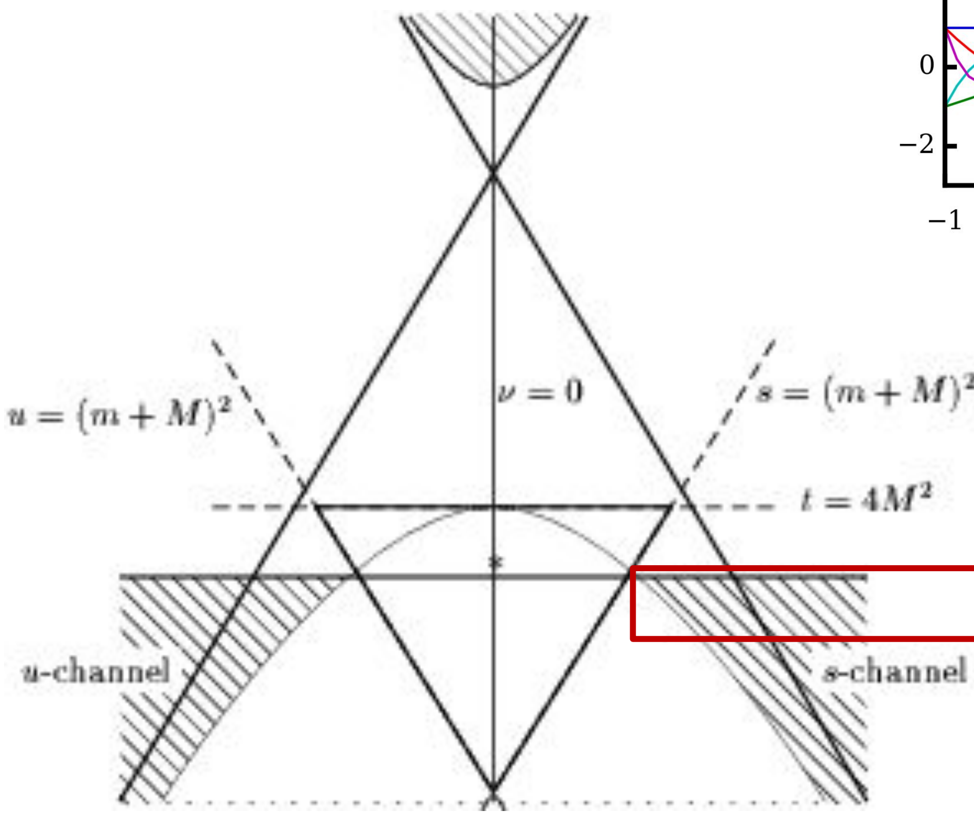
$$A(s,t)=\sum_{l=0}^{\infty}A_l(s)P_l(z_s)=\sum_{l=0}^{\infty}A_l(t)P_l(z_t)=\sum_{l=0}^{\infty}A_l(u)P_l(z_u)$$



$$A_l(s)\sim q_s^l \qquad (q_s\to 0 \text{ for } s\to s_{thr})$$

Truncated partial-wave expansion





$$\sum_{l=0}^{\infty} A_l(s) P_l(z_s)$$

$$\sum_{l=0}^{\infty} A_l(t) P_l(z_t)$$

s-channel: *truncated* partial-wave analysis

$$\sum_{l=0}^{\not\infty} A_l(s) P_l(z_s)$$

- Various models available for extracting baryon resonances (**W < 2 GeV**)
  - SAID
  - MAID
  - Bonn-Gatchina
  - Julich-Bonn
  - ...

s-channel: *truncated* partial-wave analysis

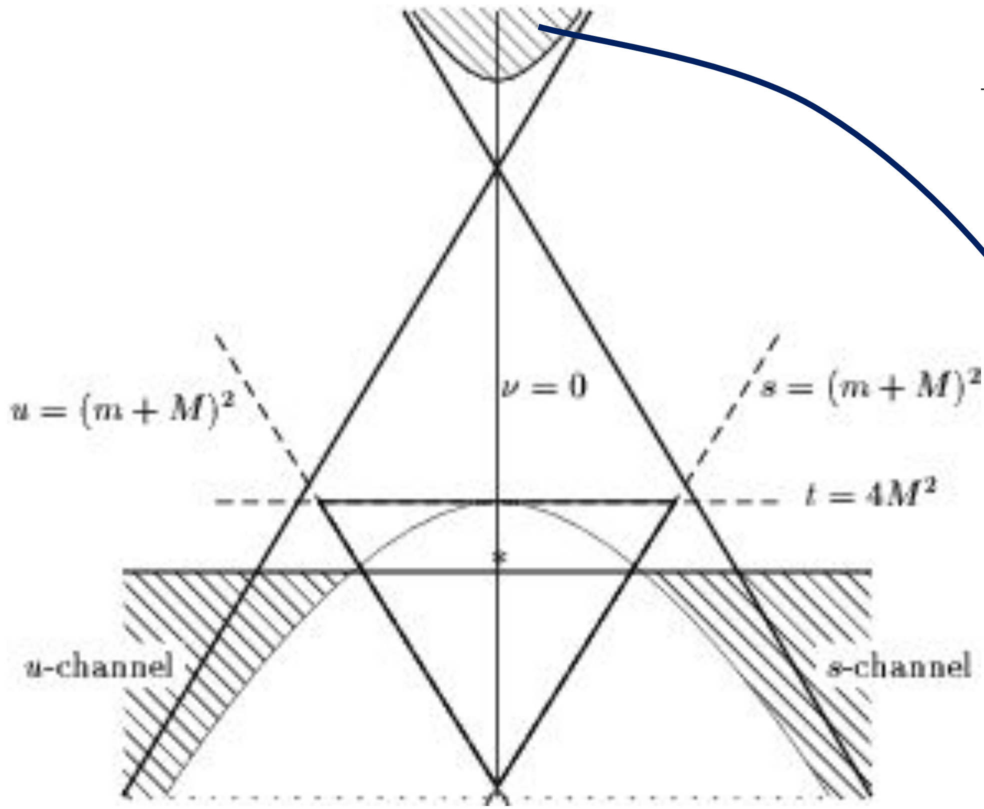
$$\sum_{l=0}^{\infty} A_l(s) P_l(z_s)$$

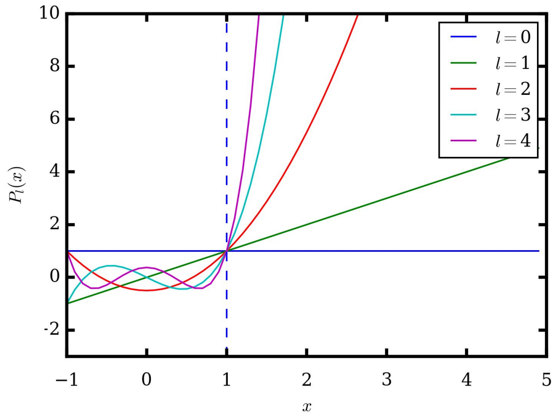
- Analyticity, Unitarity, Crossing symmetry
- Look for poles on the second Riemann sheet
- Cutoff L increases as s increases

t-channel: *no truncation possible*

$$\sum_{l=0}^{\infty} A_l(t) P_l(z_t)$$

Convergent here





Not convergent here

t-channel: *no truncation possible*

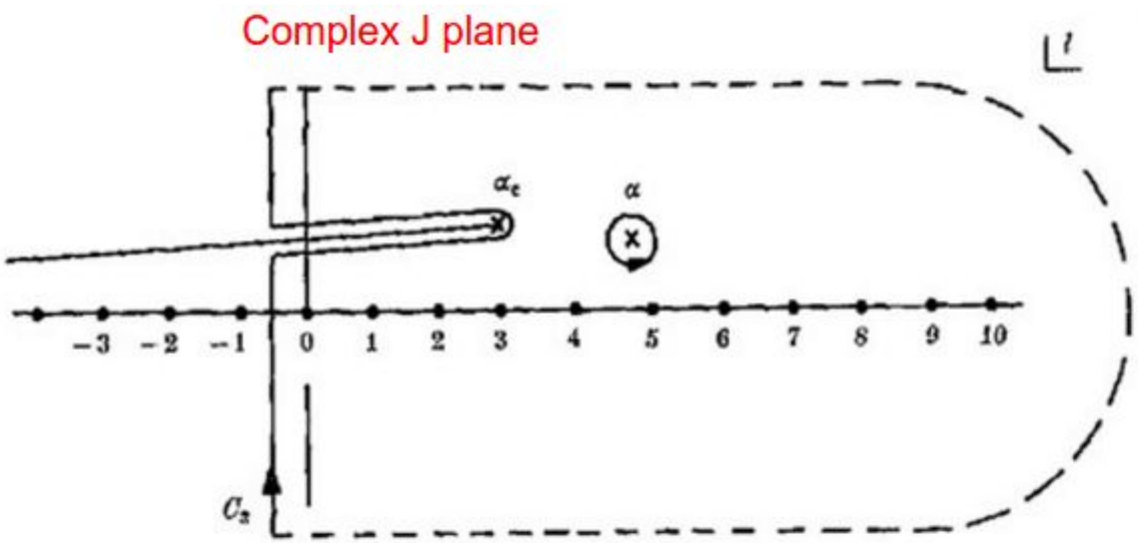
$$\sum_{l=0}^{\infty} A_l(t) P_l(z_t)$$

$$A(s,t) = -\frac{1}{2i}\oint_C \frac{A(t,J)P_J(-z_t)}{\sin \pi J} dJ$$

$$A(t,J) = \frac{\beta(t)}{J - \alpha(t)}$$

$$\oint_C A(z)dz = 2\pi i \sum_i \mathrm{Res}_{z \to z_i} A(z)$$

but z=J

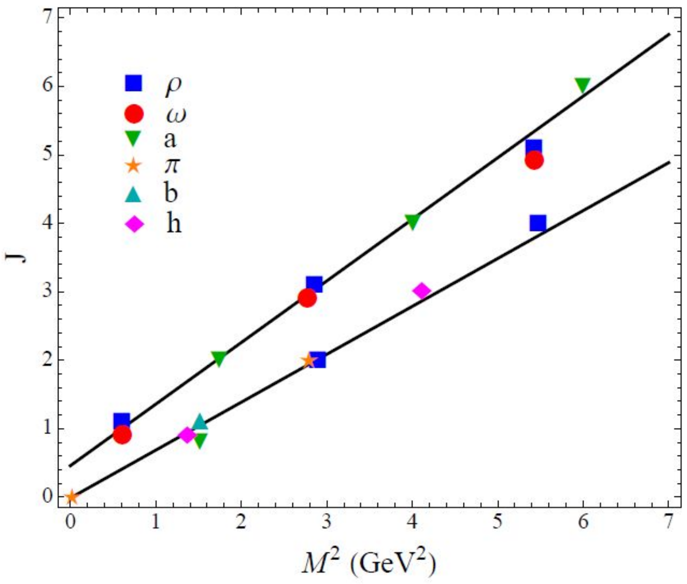


$$\frac{1}{\sin \pi J}$$

t-channel: *no truncation possible*

$$\sum_{l=0}^{\infty} A_l(t) P_l(z_t)$$

Using $A(t, J) = \dfrac{\beta(t)}{J - \alpha(t)}$

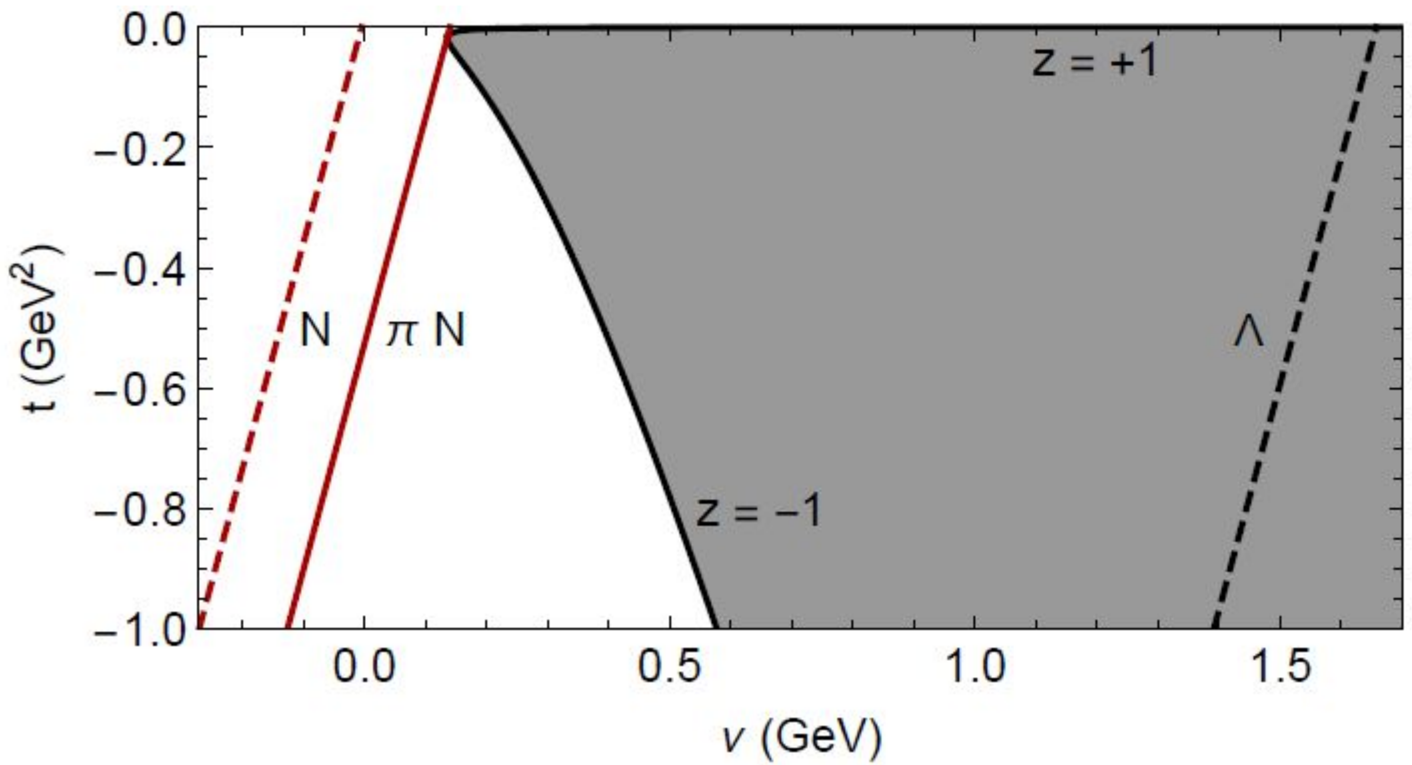


so
$$A_0(t) \sim \frac{1}{m_0^2 - t}$$
$$A_2(t) \sim \frac{1}{m_2^2 - t}$$
$$A_4(t) \sim \frac{1}{m_4^2 - t}$$

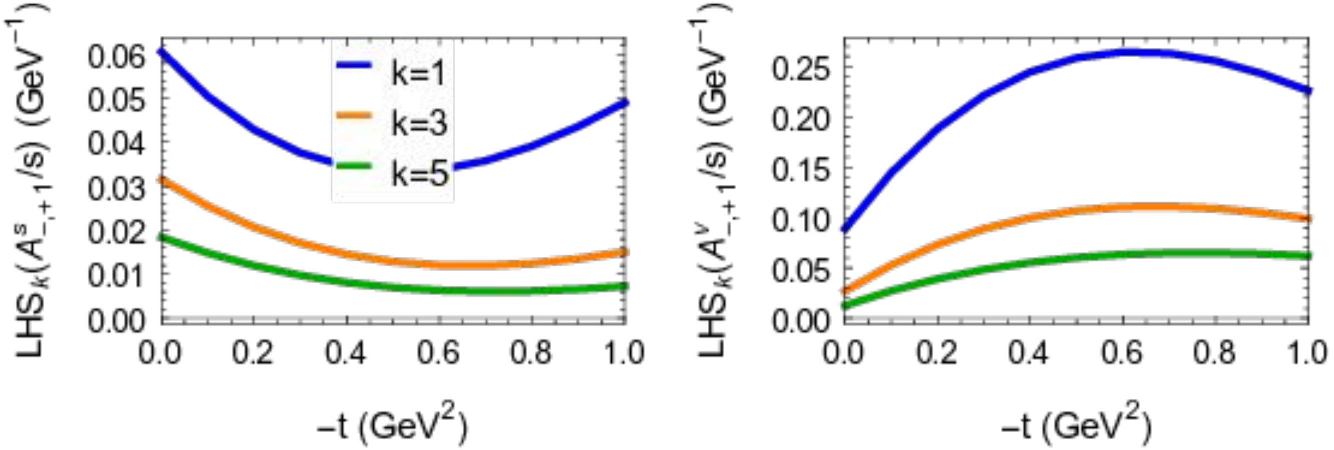
Solution $\alpha(t) = \alpha'(t - m_0^2)$

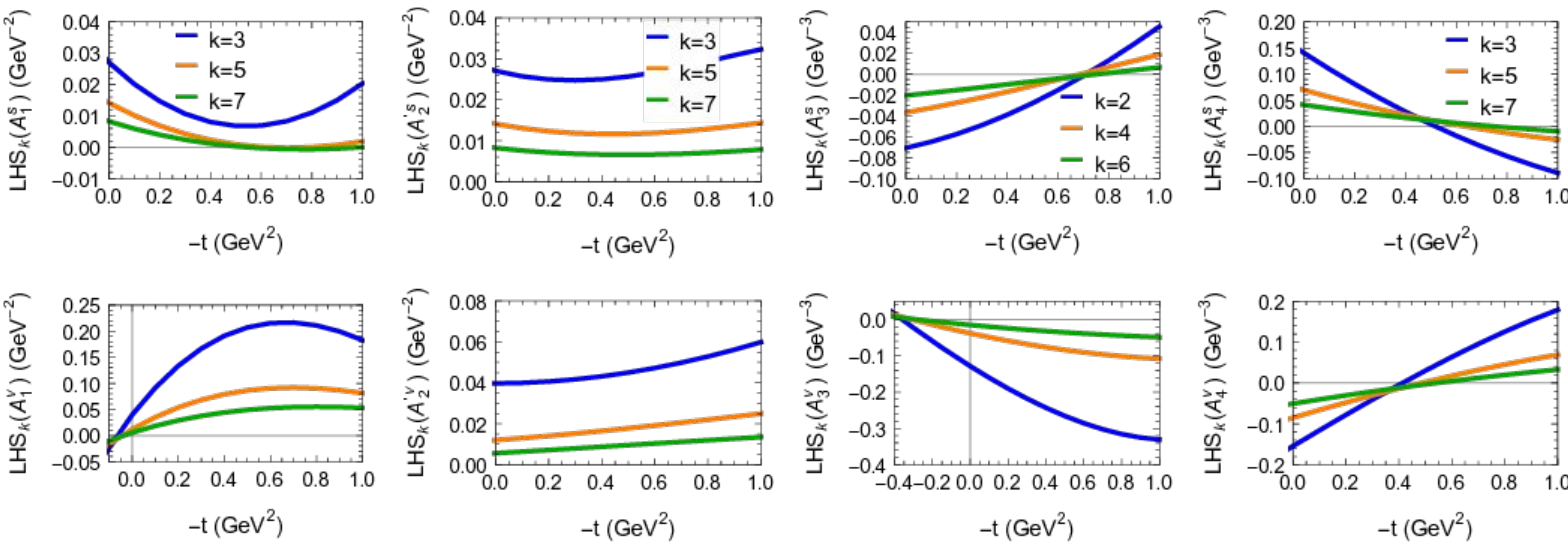
$$P_J(z_t \to +\infty) \to z_t^J$$

$$A(s,t) = \beta(t) \frac{\tau + e^{-i\pi\alpha(t)}}{\sin \pi\alpha(t)} s^{\alpha(t)}$$

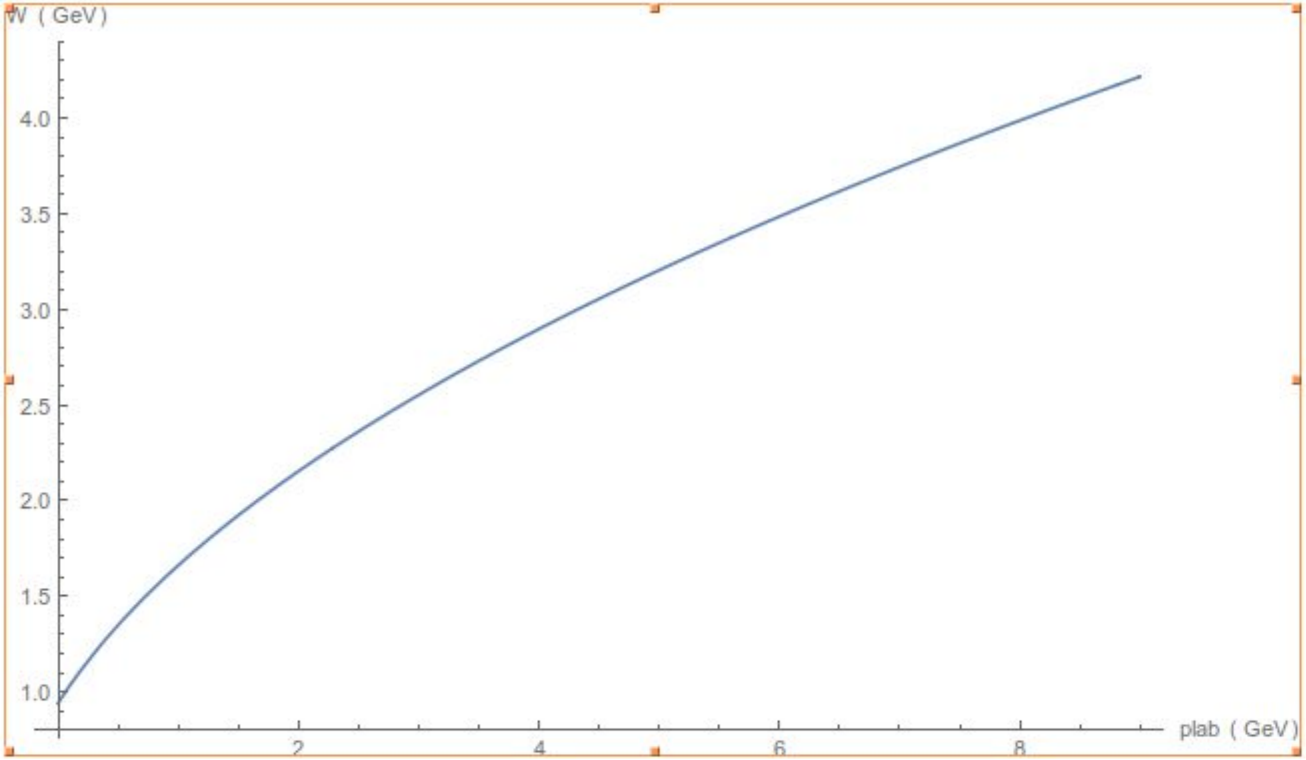


0.0
−0.2
−0.4
−0.6
−0.8
−1.0
t (GeV²)
0.0
0.5
1.0
1.5
ν (GeV)
N
π N
z = +1
z = −1
Λ

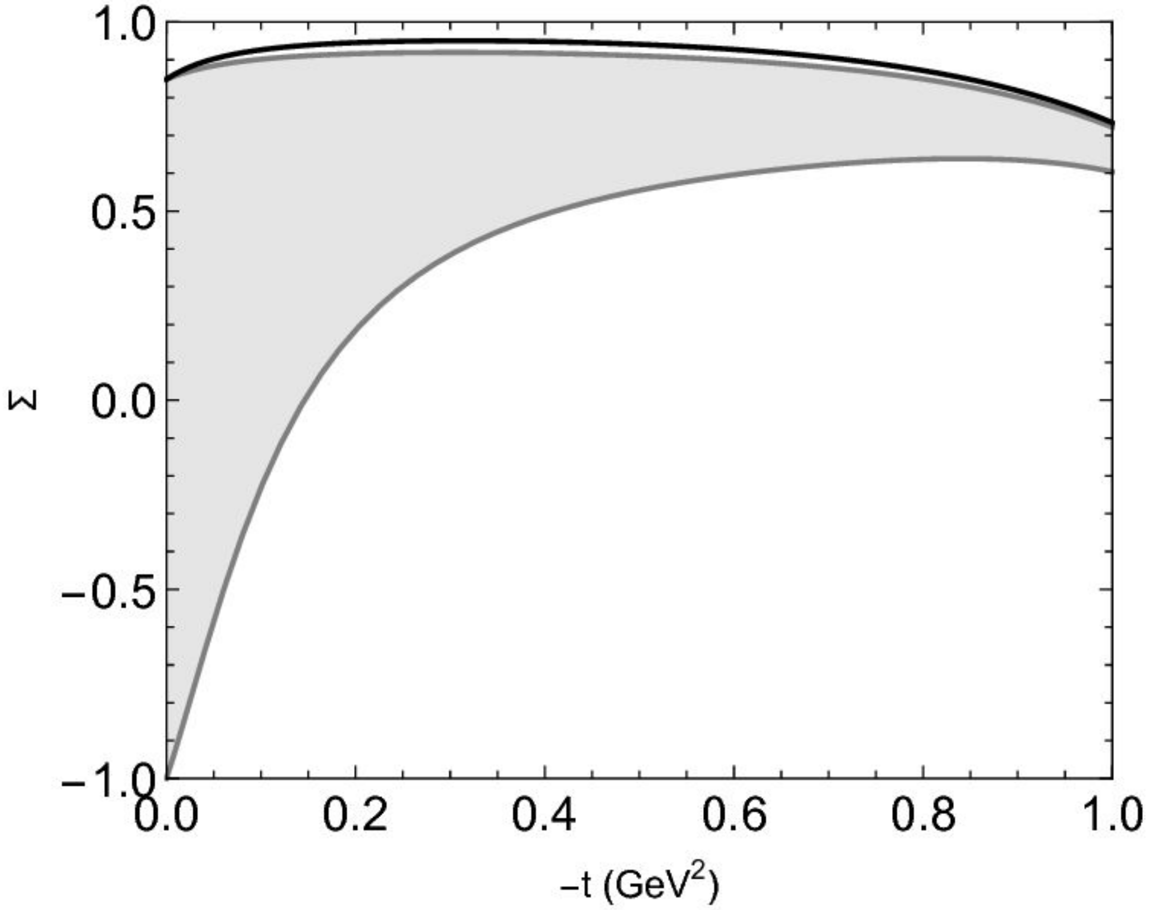

LHS$_k$($A^s_{-,+1}$/s) (GeV^{-1})
0.06
0.05
0.04
0.03
0.02
0.01
0.00
0.0 0.2 0.4 0.6 0.8 1.0
−t (GeV^2)
k=1
k=3
k=5
LHS$_k$($A^v_{-,+1}$/s) (GeV^{-1})
0.25
0.20
0.15
0.10
0.05
0.00
0.0 0.2 0.4 0.6 0.8 1.0
−t (GeV^2)



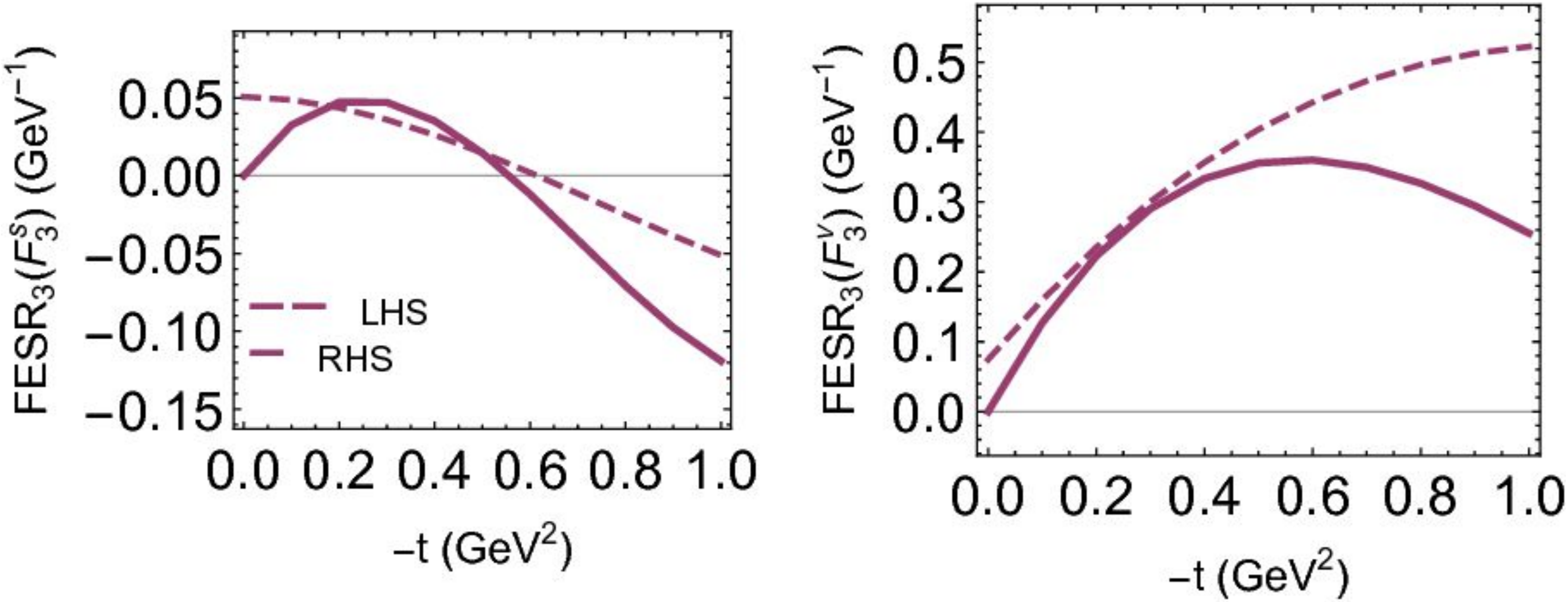
LHS$_k$(A_1^s) (GeV^{-2})
0.04
0.03
0.02
0.01
0.00
-0.01
0.0 0.2 0.4 0.6 0.8 1.0
k=3
k=5
k=7
−t (GeV^2)
LHS$_k$($A_2'^s$) (GeV^{-2})
0.04
0.03
0.02
0.01
0.00
0.0 0.2 0.4 0.6 0.8 1.0
k=3
k=5
k=7
−t (GeV^2)
LHS$_k$(A_3^s) (GeV^{-3})
0.04
0.02
0.00
−0.02
−0.04
−0.06
−0.08
−0.10
0.0 0.2 0.4 0.6 0.8 1.0
k=2
k=4
k=6
−t (GeV^2)
LHS$_k$(A_4^s) (GeV^{-3})
0.20
0.15
0.10
0.05
0.00
−0.05
−0.10
0.0 0.2 0.4 0.6 0.8 1.0
k=3
k=5
k=7
−t (GeV^2)
LHS$_k$(A_1^v) (GeV^{-2})
0.25
0.20
0.15
0.10
0.05
0.00
-0.05
0.0 0.2 0.4 0.6 0.8 1.0
−t (GeV^2)
LHS$_k$($A_2'^v$) (GeV^{-2})
0.08
0.06
0.04
0.02
0.00
0.0 0.2 0.4 0.6 0.8 1.0
−t (GeV^2)
LHS$_k$(A_3^v) (GeV^{-3})
0.0
−0.1
−0.2
−0.3
−0.4
−0.4 −0.2 0.0 0.2 0.4 0.6 0.8 1.0
−t (GeV^2)
LHS$_k$(A_4^v) (GeV^{-3})
0.2
0.1
0.0
−0.1
−0.2
0.0 0.2 0.4 0.6 0.8 1.0
−t (GeV^2)



W (GeV)
4.0
3.5
3.0
2.5
2.0
1.5
1.0
plab (GeV)
2
4
6
8



1.0
0.5
0.0
−0.5
−1.0
Σ
0.0
0.2
0.4
0.6
0.8
1.0
−t (GeV2)



$$F_3 = 2M_N A_1 - tA_4$$
$$F_4 = A_3\,.$$

Overview

- Intro
- Dispersion relations
- Low-energy amplitudes (PWA)
- High-energy amplitudes
- Applications to π,η photoproduction:
  Finite-Energy Sum Rules

Dispersion relations - FESR

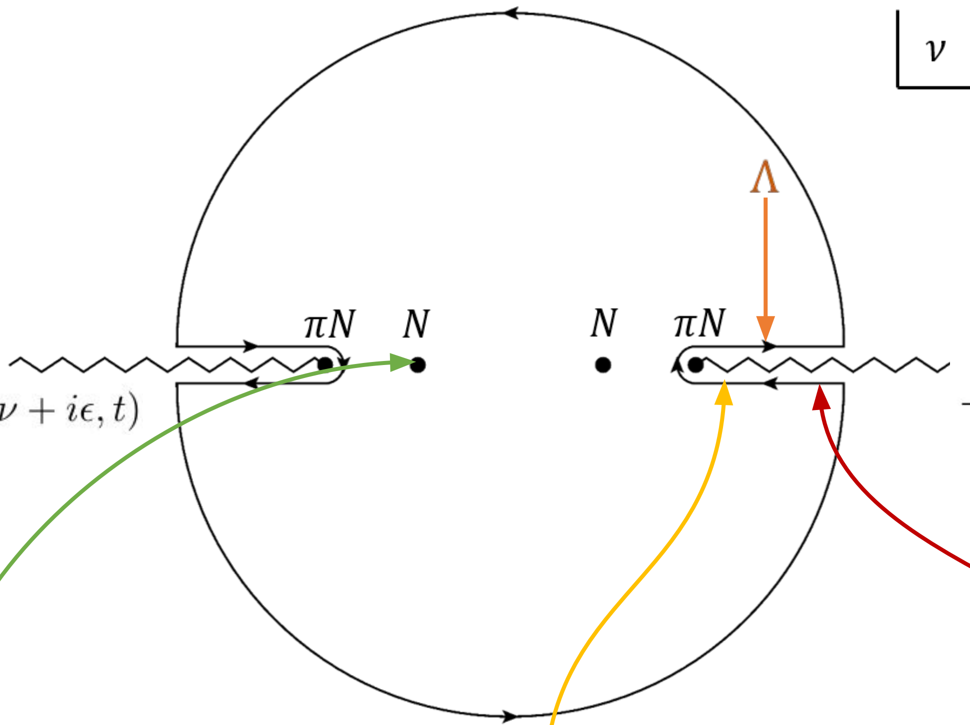
$A_i(\nu, t)$

Fixed t

Crossing variable: $\nu = \frac{s-u}{4\,M}$

s-channel: $+\,\nu$

u-channel: $-\,\nu$



$A_i^\sigma(-\nu - i\epsilon, t) = \xi_i A_i^\sigma(\nu + i\epsilon, t)$

$-\beta_i(t)\frac{\tau + e^{-i\pi\alpha(t)}}{\sin\pi\alpha(t)}(r_i\nu)^{\alpha(t)-1}$

$$\pi B_i^\sigma(t)\left(\frac{\nu_M}{\Lambda}\right)^k + \int_{\nu_\pi}^{\Lambda} \mathrm{Im}\, A_i^\sigma(\nu', t)\left(\frac{\nu'}{\Lambda}\right)^k d\nu' = \beta_i^\sigma(t)\frac{(\Lambda/\nu_0)^{\alpha(t)}}{\alpha(t)+k}\nu_0$$

Nucleon pole — Low-energy model — Regge pole

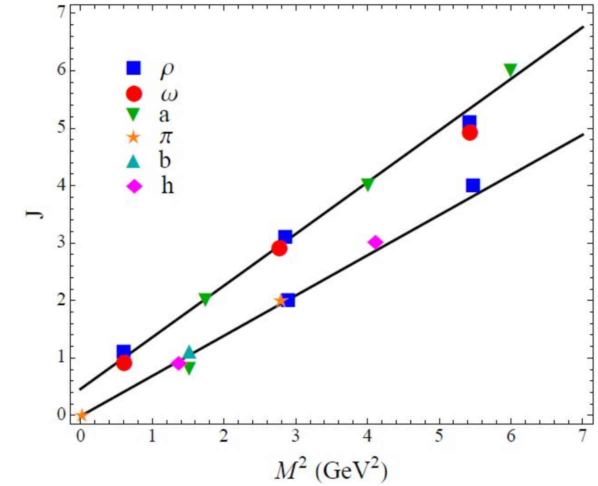
Analyticity results in Finite-Energy Sum Rules.

High energies

Regge pole model

$$A_{i,R}(\nu,t) = -\beta_i(t)\frac{\tau + e^{-i\pi\alpha(t)}}{\sin\pi\alpha(t)}(r_i\nu)^{\alpha(t)-1}$$

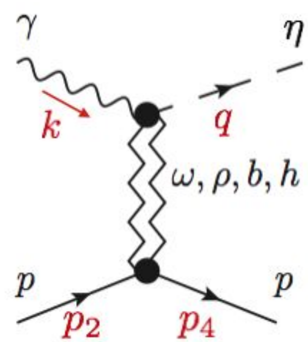
$J(m^2)$



Dominant: vector exchanges

A_i	I^G	J^{PC}	η	Leading exchanges
A_1	$0^-, 1^+$	$(1,3,5,...)^{--}$	$+1$	$\rho(770), \omega(782)$
A_2'	$0^-, 1^+$	$(1,3,5,...)^{+-}$	-1	$h_1(1170), b_1(1235)$
A_3	$0^-, 1^+$	$(2,4,...)^{--}$	-1	$\rho_2(??), \omega_2(??)$
A_4	$0^-, 1^+$	$(1,3,5,...)^{--}$	$+1$	$\rho(770), \omega(782)$

$A_2' = A_1 + tA_2$



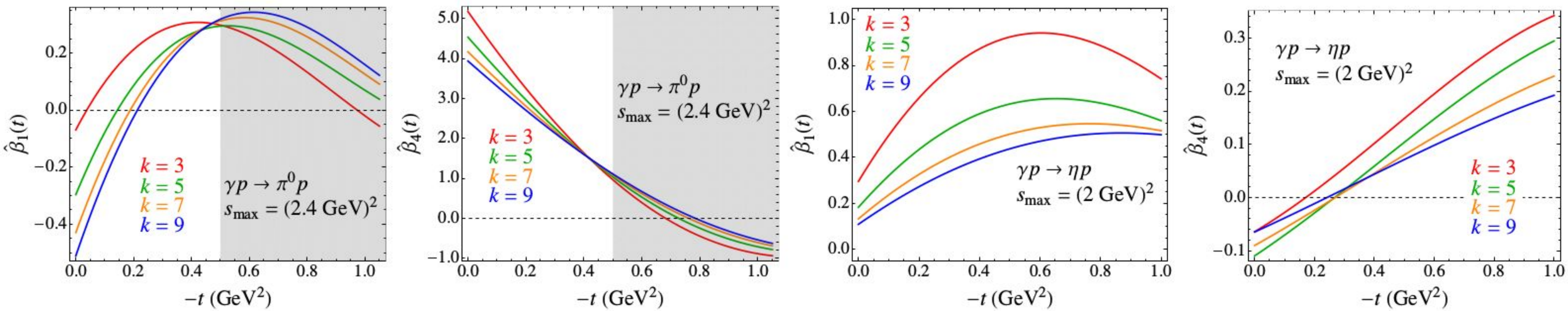
$$\gamma p \to \eta p\,, \qquad A = (\omega + h + \omega_2) + (\rho + b + \rho_2)$$

$$\gamma n \to \eta n\,, \qquad A = (\omega + h + \omega_2) - (\rho + b + \rho_2)$$

Sensitivity to k

$$\pi B_i^\sigma(t)\left(\frac{\nu_M}{\Lambda}\right)^k + \int_{\nu_\pi}^{\Lambda} \mathrm{Im}\ A_i^\sigma(\nu', t)\left(\frac{\nu'}{\Lambda}\right)^k d\nu' = \beta_i^\sigma(t)\frac{(\Lambda/\nu_0)^{\alpha(t)}}{\alpha(t)+k}\nu_0$$

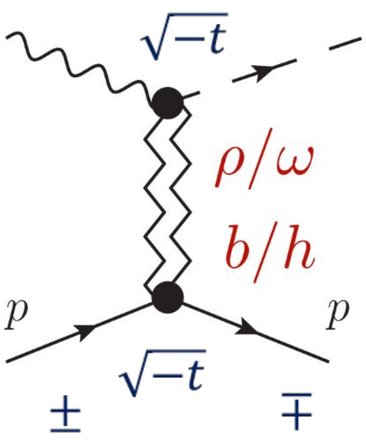
$$\widehat{\beta}_i(t) = \frac{\alpha(t)+k}{\Lambda^{\alpha(t)+k}}\int_0^{\Lambda} \mathrm{Im}\, A_i^{\mathrm{PWA}}(\nu, t)\, \nu^k\, \mathrm{d}\nu$$



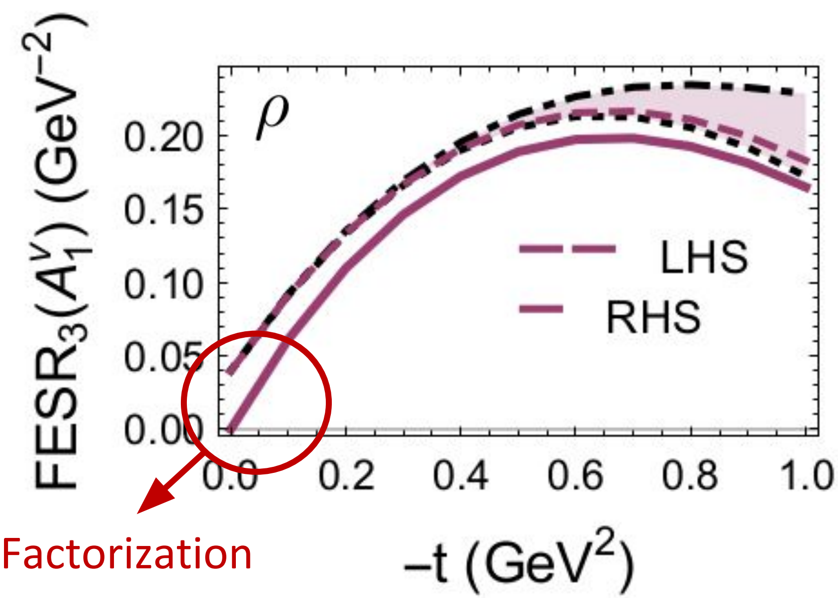
Matching: natural exchanges

$$\pi B_i^\sigma(t)\left(\frac{\nu_M}{\Lambda}\right)^k + \int_{\nu_\pi}^{\Lambda} \mathrm{Im}\ A_i^\sigma(\nu',t)\left(\frac{\nu'}{\Lambda}\right)^k d\nu' = \beta_i^\sigma(t)\frac{(\Lambda/\nu_0)^{\alpha(t)}}{\alpha(t)+k}\nu_0$$

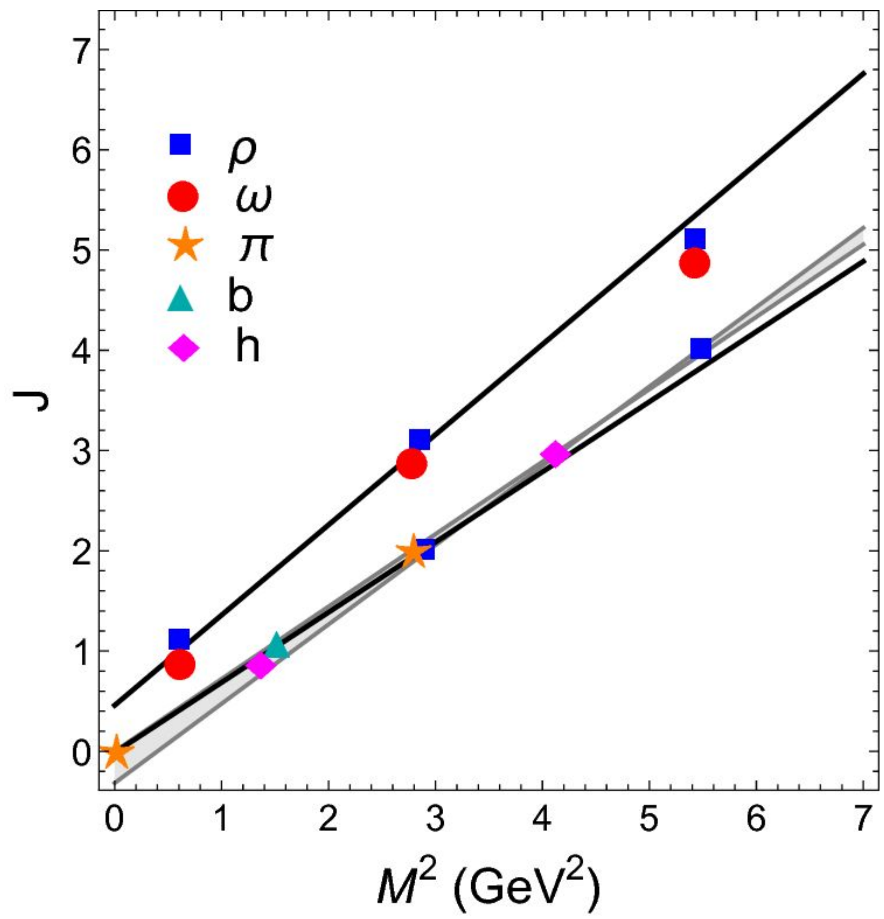
Nucleon pole | Low-energy model | Regge pole



$$\text{ang. mom.}: A_1 \sim 1$$
$$\text{single pole}: A_1 \sim t$$



$F_3 = 2\,M_N A_1 - tA_4$


ρ
ω
π
b
h
J
M^2 (GeV^2)
0
1
2
3
4
5
6
7

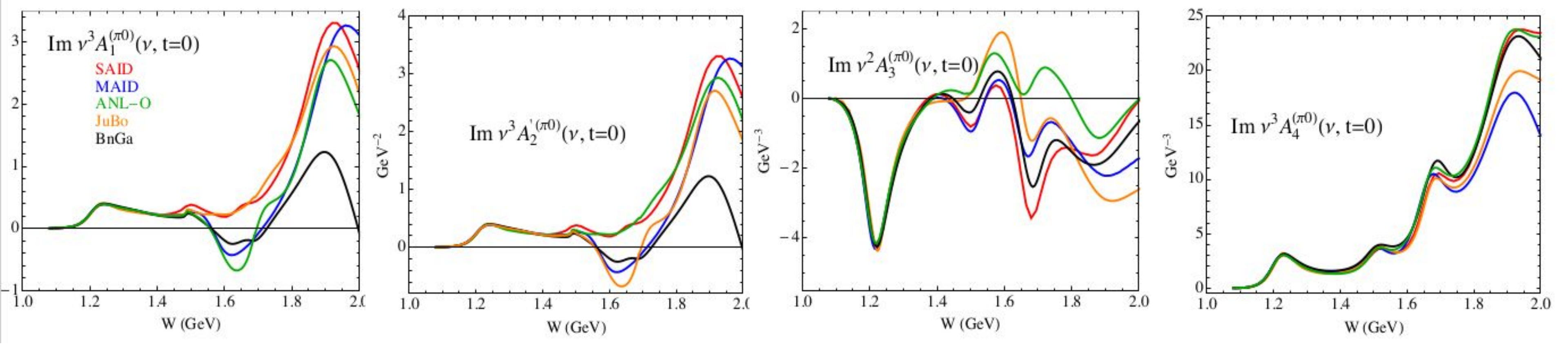
Low energies

$$\int_{\nu_\pi}^{\Lambda} \text{Im } A_i^\sigma(\nu', t) \left(\frac{\nu'}{\Lambda}\right)^k d\nu'$$

Low energy models

- BnGa, Julich-Bonn, ANL-Osaka, SAID, MAID,...

$\gamma N \to \pi^0 N$

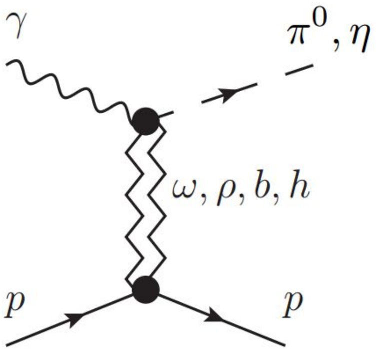


Formalism

$\pi^0, \eta(0^{-+})$ **have**

same production as

$\pi_1^0, \eta_1(1^{-+})$



$$M_k \equiv M_k(s, t, \lambda_\gamma)$$

$$A_{\lambda';\lambda\lambda_\gamma}(s,t) = \overline{u}_{\lambda'}(p') \left(\sum_{k=1}^{4} A_k(s,t) M_k \right) u_\lambda(p)$$

$$M_1 = \frac{1}{2}\gamma_5\gamma_\mu\gamma_\nu F^{\mu\nu},$$

$$M_2 = 2\gamma_5 q_\mu P_\nu F^{\mu\nu},$$

$$M_3 = \gamma_5\gamma_\mu q_\nu F^{\mu\nu},$$

$$M_4 = \frac{i}{2}\epsilon_{\alpha\beta\mu\nu}\gamma^\alpha q^\beta F^{\mu\nu}.$$

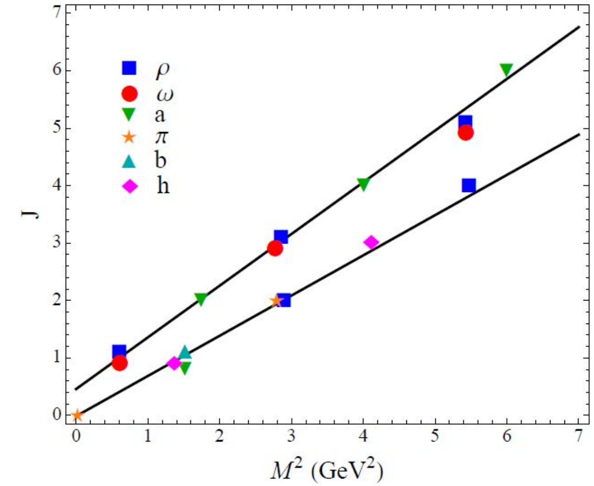
- No kinematic singularities
- No kinematic zeros
- Discontinuities:
  - Unitarity cut
  - Nucleon pole

High energies

Regge pole model

$J(m^2)$

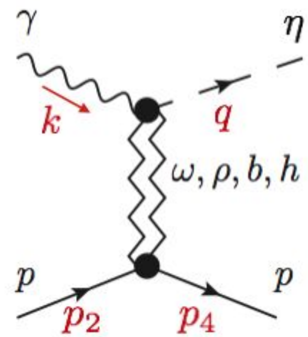
$$A_{i,R}(\nu,t) = -\beta_i(t)\frac{\tau + e^{-i\pi\alpha(t)}}{\sin \pi\alpha(t)}(r_i\nu)^{\alpha(t)-1}$$



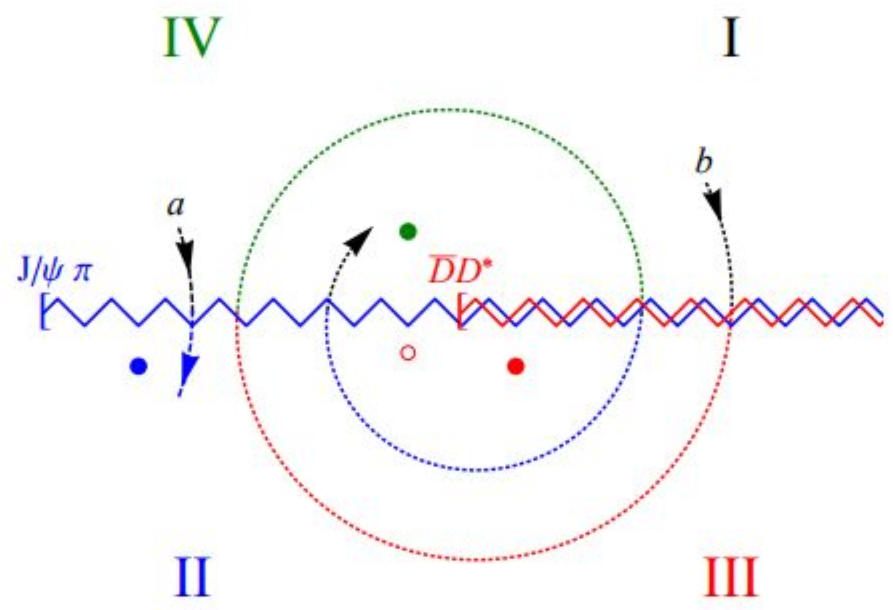
Dominant: vector exchanges

A_i	I^G	J^{PC}	η	Leading exchanges
A_1	$0^-, 1^+$	$(1,3,5,...)^{--}$	$+1$	$\rho(770), \omega(782)$
A_2'	$0^-, 1^+$	$(1,3,5,...)^{+-}$	-1	$h_1(1170), b_1(1235)$
A_3	$0^-, 1^+$	$(2,4,...)^{--}$	-1	$\rho_2(??), \omega_2(??)$
A_4	$0^-, 1^+$	$(1,3,5,...)^{--}$	$+1$	$\rho(770), \omega(782)$

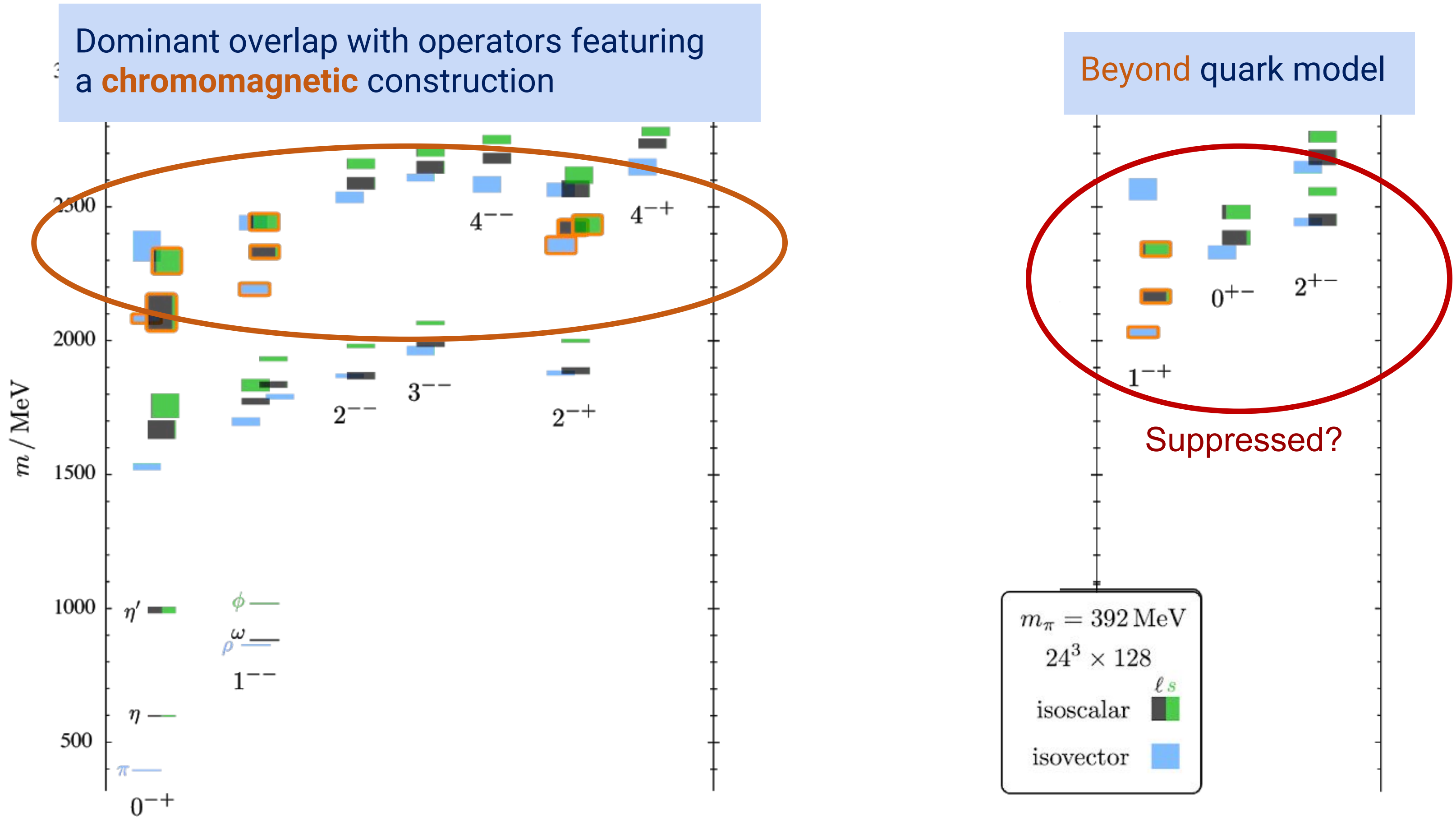
$A_2' = A_1 + tA_2$



$$\gamma p \to \eta p\,, \qquad A = (\omega + h + \omega_2) + (\rho + b + \rho_2)$$
$$\gamma n \to \eta n\,, \qquad A = (\omega + h + \omega_2) - (\rho + b + \rho_2)$$

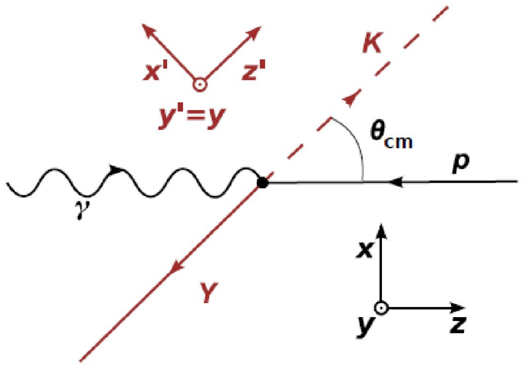

IV
I
b
a
J/ψ π
$\bar{D}D^*$
II
III

Spectroscopy from QCD



Static spectrum from Lattice QCD [Phys.Rev. D88 (2013) no.9, 094505]

Completely determined system



$$\mathcal{M}^{\lambda_\gamma}_{\lambda_p,\lambda_\Lambda} \to \mathcal{M}_{i=1,2,3,4}$$

$$\begin{aligned}
b_1 &\equiv {}_y\langle +|\, J_y \,|+\rangle_y \\
b_2 &\equiv {}_y\langle -|\, J_y \,|-\rangle_y \\
b_3 &\equiv {}_y\langle +|\, J_x \,|-\rangle_y \\
b_4 &\equiv {}_y\langle -|\, J_x \,|+\rangle_y
\end{aligned}$$

Relation with experiments

	$(\mathcal{B}_1, \mathcal{T}_1, \mathcal{R}_1)$	$(\mathcal{B}_2, \mathcal{T}_2, \mathcal{R}_2)$	Transversity expression
Σ	$(y, 0, 0)$	$(x, 0, 0)$	$r_1^2 + r_2^2 - r_3^2 - r_4^2$
T	$(0, +y, 0)$	$(0, -y, 0)$	$r_1^2 - r_2^2 - r_3^2 + r_4^2$
P	$(0, 0, +y)$	$(0, 0, -y)$	$r_1^2 - r_2^2 + r_3^2 - r_4^2$
C_x	$(+, 0, +x)$	$(+, 0, -x)$	$-2\,\mathrm{Im}(a_1 a_4^* + a_2 a_3^*)$
C_z	$(+, 0, +z)$	$(+, 0, -z)$	$+2\,\mathrm{Re}(a_1 a_4^* - a_2 a_3^*)$
O_x	$(+\frac{\pi}{4}, 0, +x)$	$(+\frac{\pi}{4}, 0, -x)$	$+2\,\mathrm{Re}(a_1 a_4^* + a_2 a_3^*)$
O_z	$(+\frac{\pi}{4}, 0, +z)$	$(+\frac{\pi}{4}, 0, -z)$	$+2\,\mathrm{Im}(a_1 a_4^* - a_2 a_3^*)$
E	$(+, -z, 0)$	$(+, +z, 0)$	$+2\,\mathrm{Re}(a_1 a_3^* - a_2 a_4^*)$
F	$(+, +x, 0)$	$(+, -x, 0)$	$-2\,\mathrm{Im}(a_1 a_3^* + a_2 a_4^*)$
G	$(+\frac{\pi}{4}, +z, 0)$	$(+\frac{\pi}{4}, -z, 0)$	$-2\,\mathrm{Im}(a_1 a_3^* - a_2 a_4^*)$
H	$(+\frac{\pi}{4}, +x, 0)$	$(+\frac{\pi}{4}, -x, 0)$	$+2\,\mathrm{Re}(a_1 a_3^* + a_2 a_4^*)$
T_x	$(0, +x, +x)$	$(0, +x, -x)$	$+2\,\mathrm{Re}(a_1 a_2^* + a_3 a_4^*)$
T_z	$(0, +x, +z)$	$(0, +x, -z)$	$+2\,\mathrm{Im}(a_1 a_2^* + a_3 a_4^*)$
L_x	$(0, +z, +x)$	$(0, +z, -x)$	$-2\,\mathrm{Im}(a_1 a_2^* - a_3 a_4^*)$
L_z	$(0, +z, +z)$	$(0, +z, -z)$	$+2\,\mathrm{Re}(a_1 a_2^* - a_3 a_4^*)$

- $\frac{d\sigma}{d\Omega}^{(\mathcal{B},\mathcal{T},\mathcal{R})}$: cross section for given beam ($\mathcal{B}$), target ($\mathcal{T}$), recoil ($\mathcal{R}$) polarization
- Asymmetries

$$\mathcal{A} = \frac{\frac{d\sigma}{d\Omega}^{(\mathcal{B}_1,\mathcal{T}_1,\mathcal{R}_1)} - \frac{d\sigma}{d\Omega}^{(\mathcal{B}_2,\mathcal{T}_2,\mathcal{R}_2)}}{\frac{d\sigma}{d\Omega}^{(\mathcal{B}_1,\mathcal{T}_1,\mathcal{R}_1)} + \frac{d\sigma}{d\Omega}^{(\mathcal{B}_2,\mathcal{T}_2,\mathcal{R}_2)}}$$

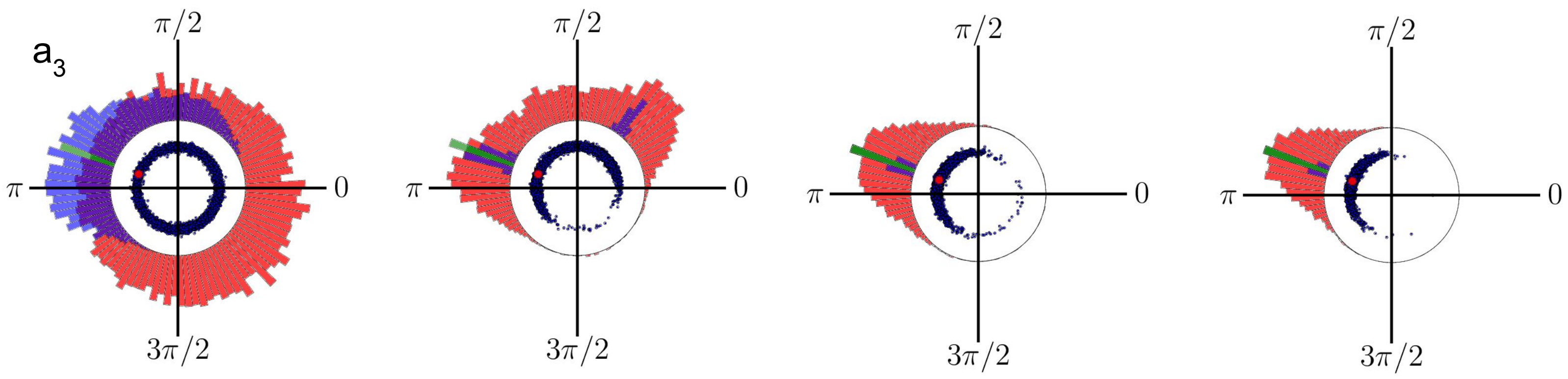
- $\frac{d\sigma}{d\Omega}^{(0,0,0)} = \frac{\rho}{4} \sum_{i=1}^{4} |b_i|^2$

$$a_i = \frac{b_i}{\sqrt{|b_1|^2 + |b_2|^2 + |b_3|^2 + |b_4|^2}} = r_i e^{i\alpha_i}$$

A ‘complete set’ is a minimum set of observables from which one can determine the underlying amplitudes (b_i) unambiguously:

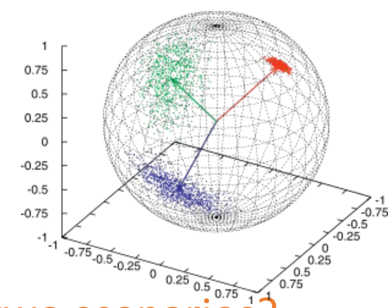
8 well-chosen observables [Chiang *et al.* PRC55 (1997)]

Impact of polarization observables



'mathematically complete'

'practically complete'



Questions:

1) Given 2 models, which measurement would help me **distinguish** the two scenarios?
2) Given the currently available data, what is the **highest impact** measurement?

Approach 1:
'Completeness is related to a certain information content in amplitude space'

$$H = -\int p(\{x_i\}) \log p(\{x_i\}) d\{x_i\},$$

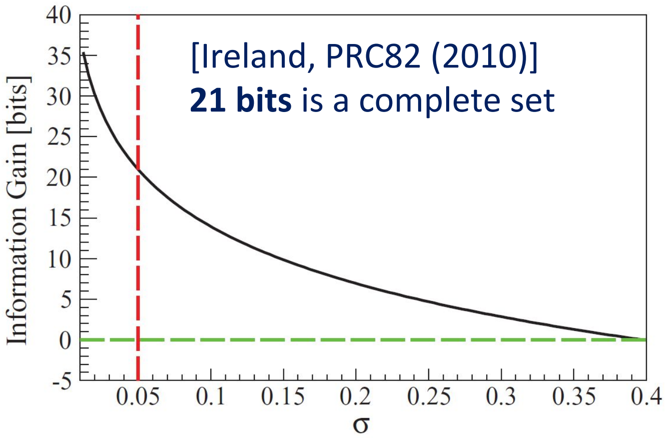
Approach 2:
'Completeness is related to model distance'

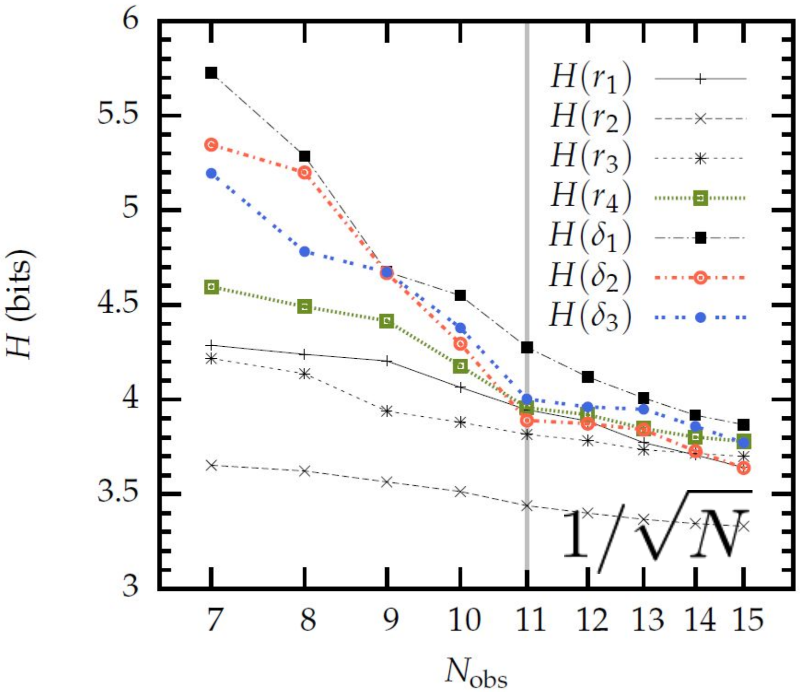
$$\Delta\mathcal{M} = \sqrt{\left\langle \mathcal{D}\left[\mathcal{M}_0, \mathcal{M}\right]^2 \right\rangle_{P(\mathcal{M}|\{A_i^{\exp}\})}},$$

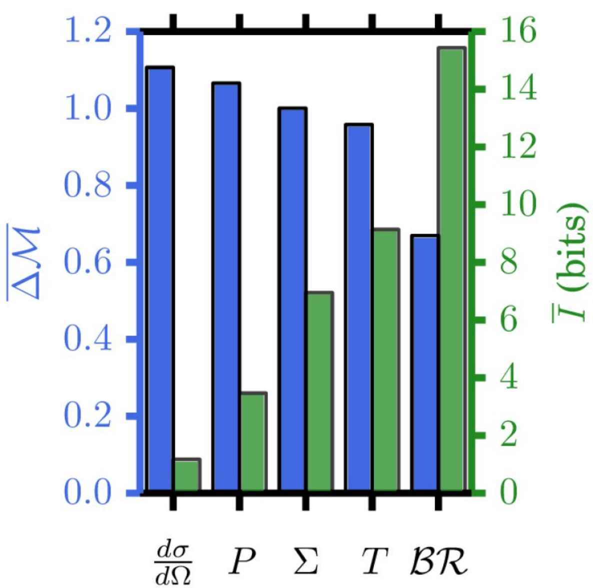
Approach 1: information content

- Map the posterior in amplitude and observable space
- Evaluate the entropy

$$H(P) = \int P(\boldsymbol{\mathcal{M}}|\{A_i^{\exp}\}) \log_2 P(\boldsymbol{\mathcal{M}}|\{A_i^{\exp}\})\, \mathrm{d}\boldsymbol{\mathcal{M}}$$



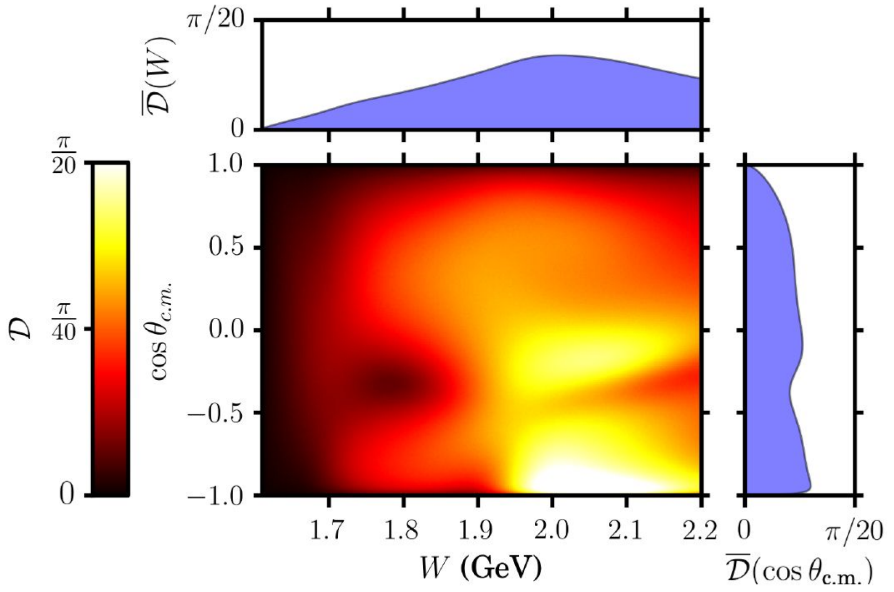


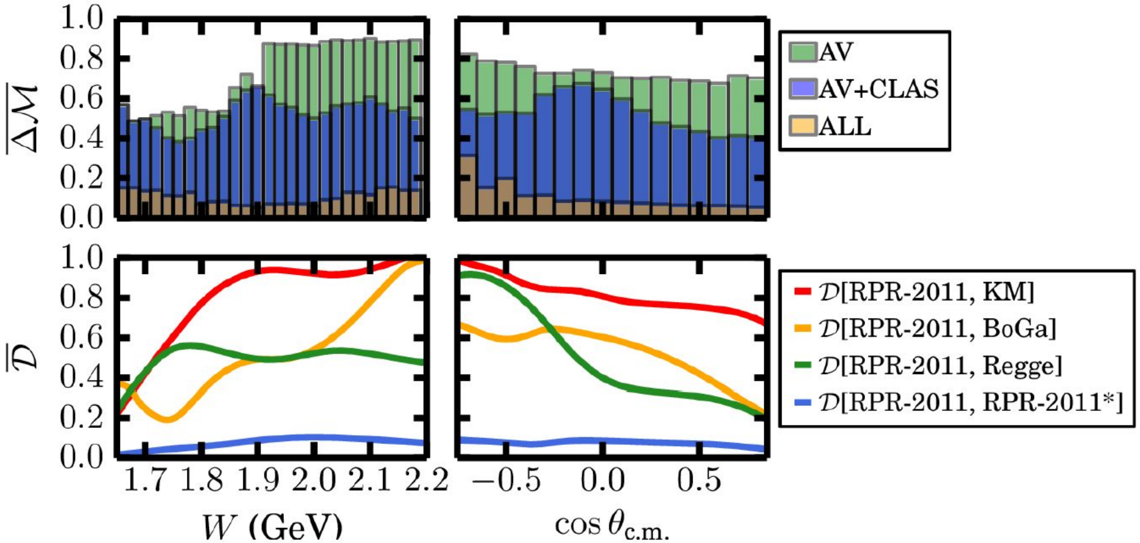


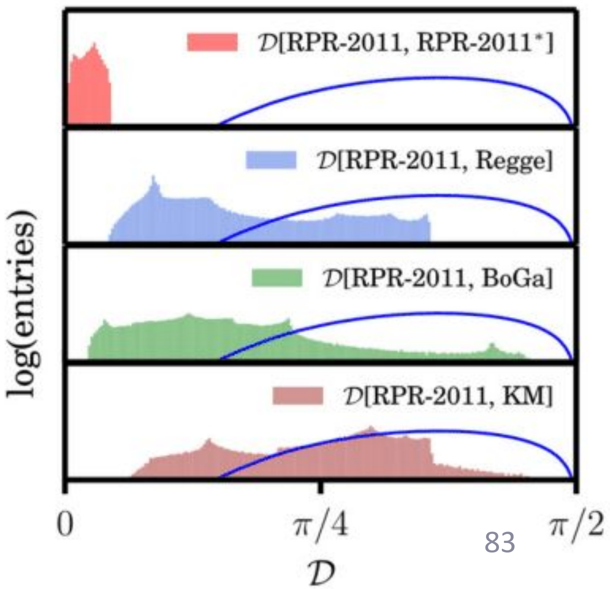
Approach 2: model distance

- Map the model distance
- Map the data resolution
- Data resolution must be much lower than model distance

(Rayleigh statement)



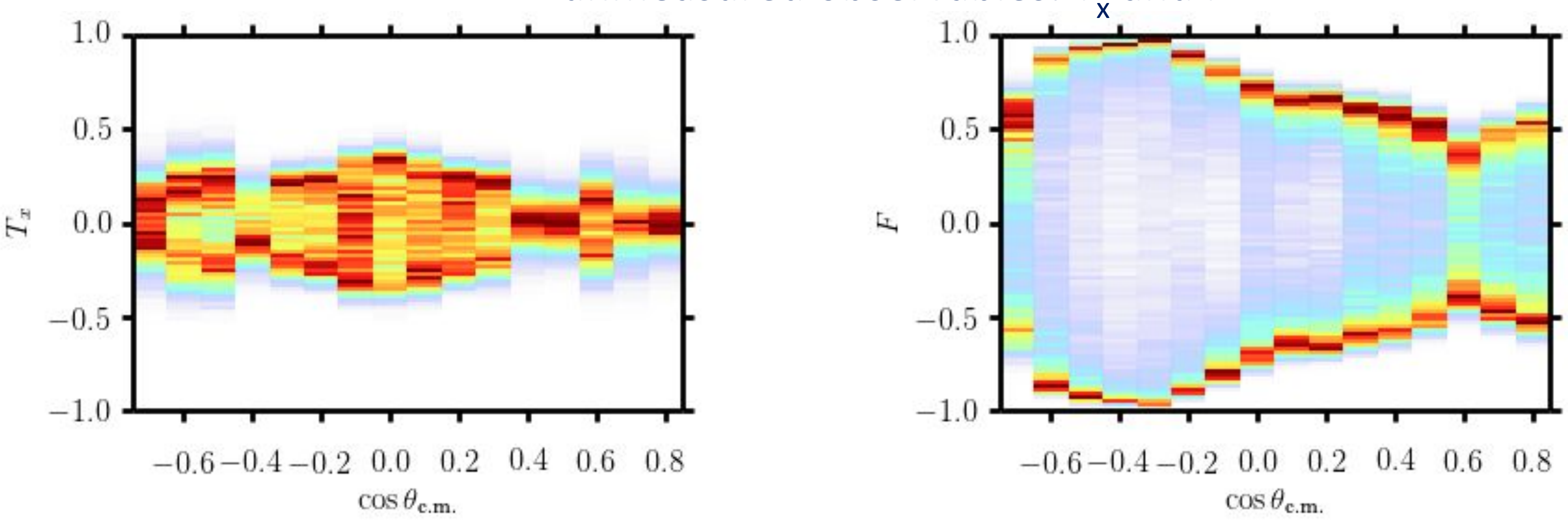




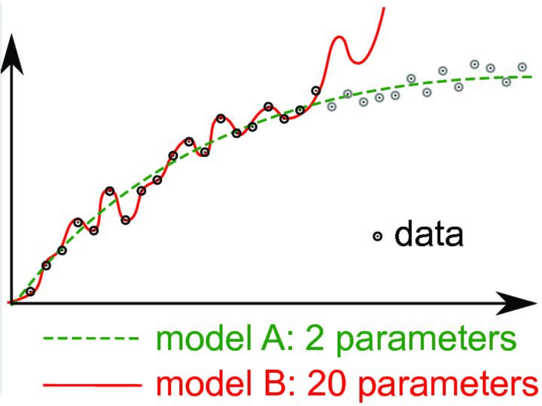
Concrete: **model-independent** predictions

- Project information in amplitude space onto observable space
- Clear effect of measurements by comparing posterior to prior

2 unmeasured observables: T_x and F

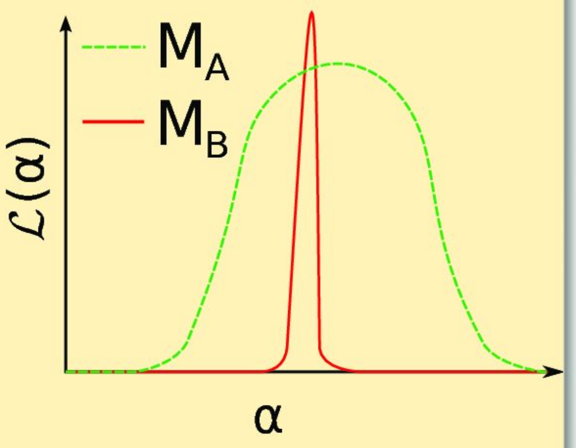


Classical analysis: point estimates



Conventional way of discriminating between models M_A en M_B

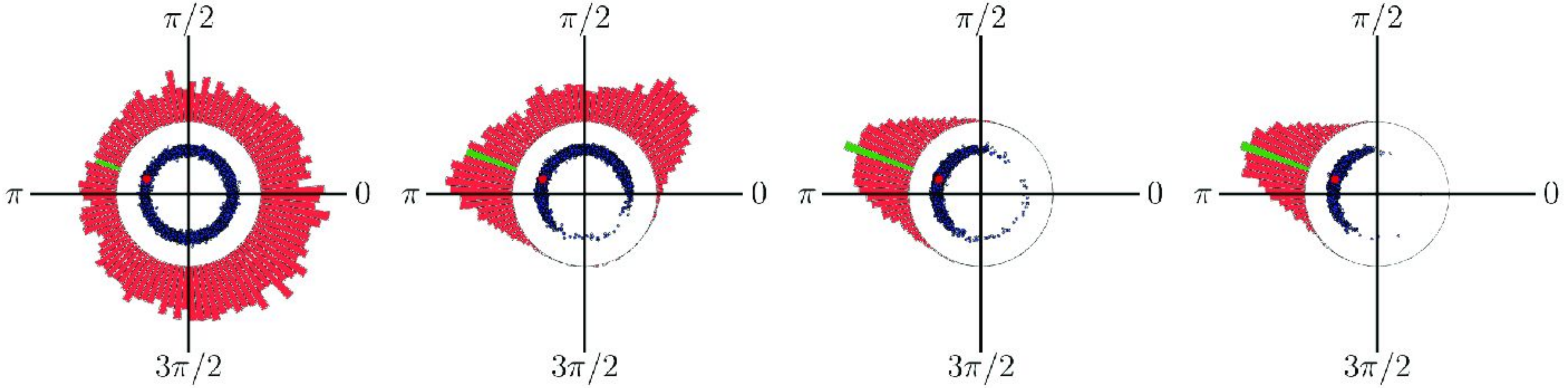
- χ^2 = measure for model-to-data distances
- $\mathcal{L}(\alpha) = \chi^2(\alpha)$-distribution
- $\max\left[\mathcal{L}(\alpha)\right] \Leftrightarrow \min\left[\chi^2(\alpha)\right]$
- Model selection on the basis of minimal χ^2 values
- Adding parameters often reduces the χ^2: the most complex model wins



+ Let's go beyond **point estimates**
+ Including **'naturality'** of parameter values (prior)

Extract $r_3 e^{i\delta_3^4}$ at $(W = 1.8\,\text{GeV}, \theta_{\text{c.m.}} = -0.1)$ from data

Extract the amplitudes from synthetic data with a realistic 10% error



(i) "mathematically complete set" $\{A_i^{\exp}\}_1 = \{\frac{d\sigma}{d\Omega}, \Sigma, T, P, C_x, O_x, E, F\}$

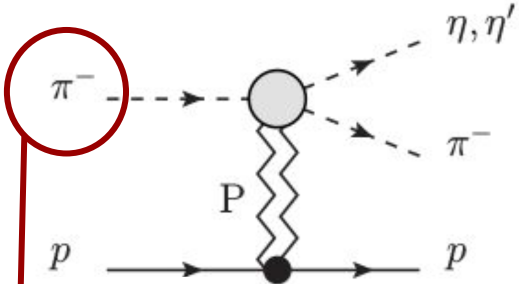
(ii) $\{A_i^{\exp}\}_2 = \{A_i^{\exp}\}_1 + \{C_z, O_z, G\}$

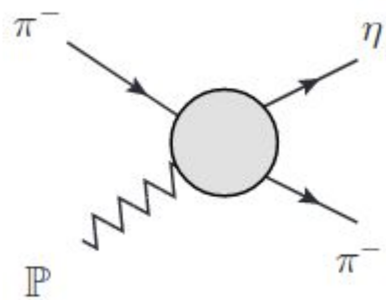
(iii) $\{A_i^{\exp}\}_3 = \{A_i^{\exp}\}_2 + \{H\}$

(iv) $\{A_i^{\exp}\}_4 = \{A_i^{\exp}\}_3 + \{T_x, T_z, L_x, L_z\}$

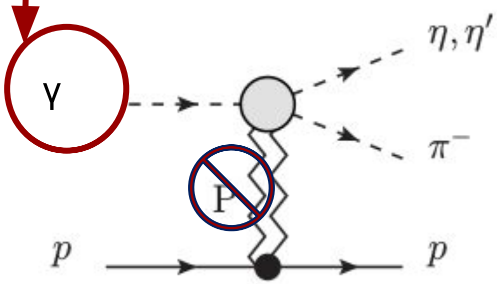
Totality has no Limits! Mathematical Completeness does not imply Practical Completeness!

Production process

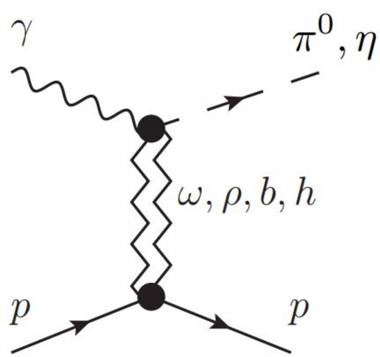




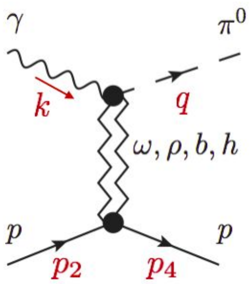
Jefferson Lab



$\pi^0, \eta(0^{-+})$ **have same production as** $\pi^0_1, \eta_1(1^{-+})$

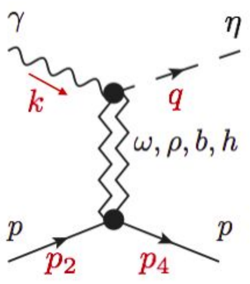


Neutral meson production

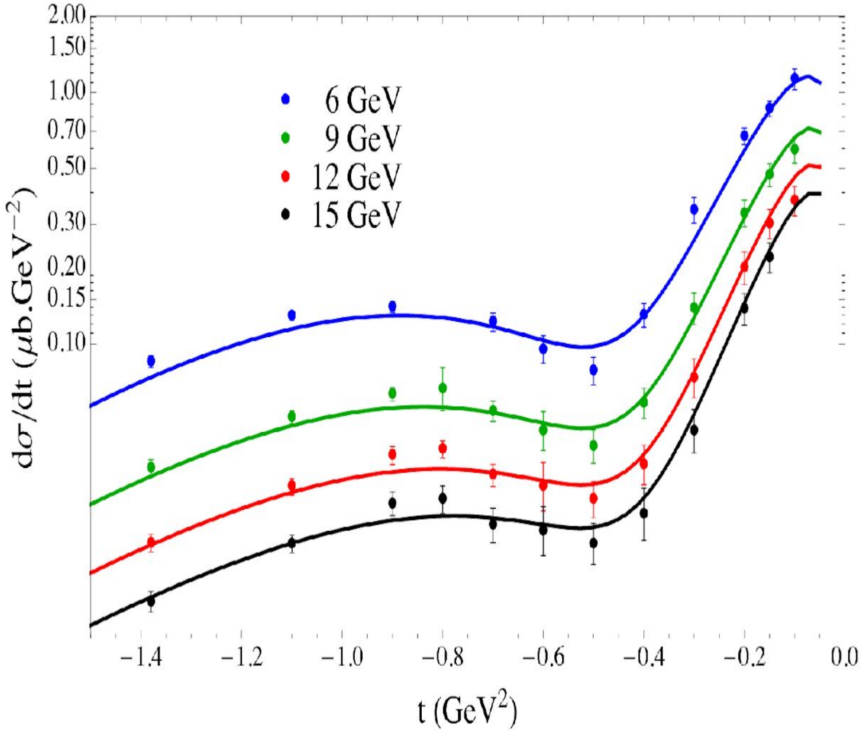


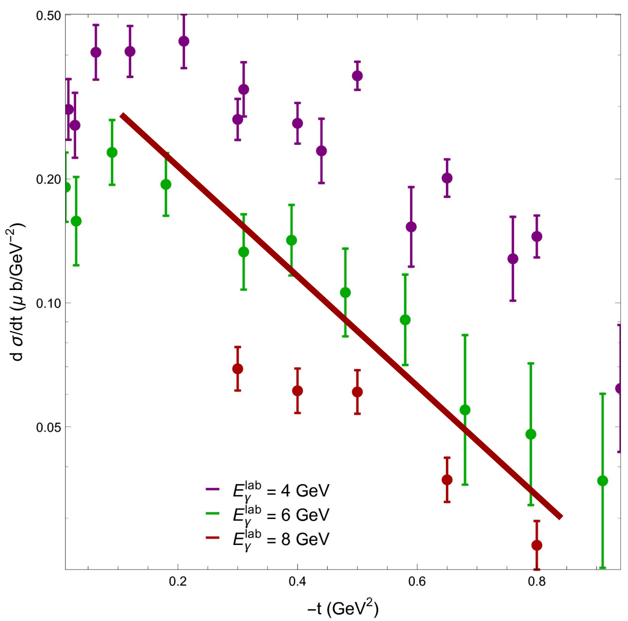
$$\gamma N \to \pi^0 N$$

$$\gamma N \to \eta N$$



Need information on t-dependence

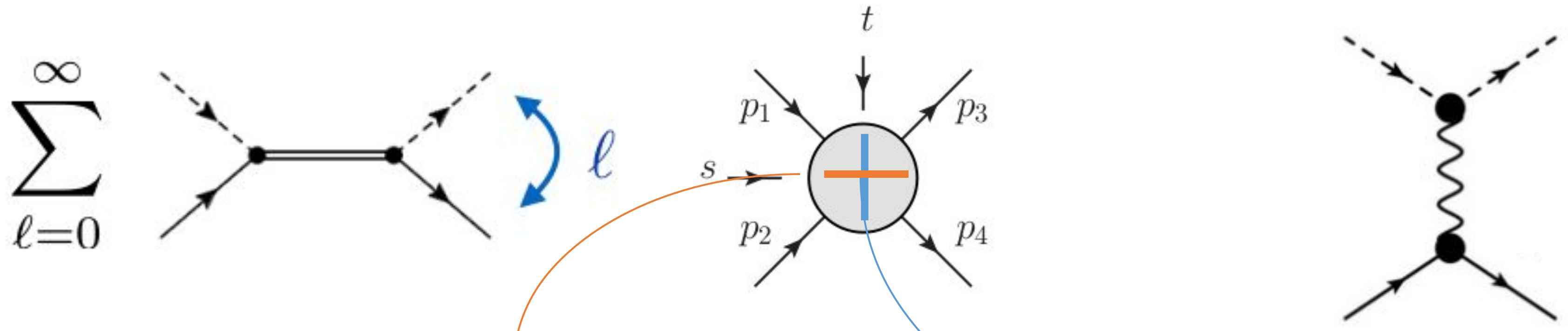


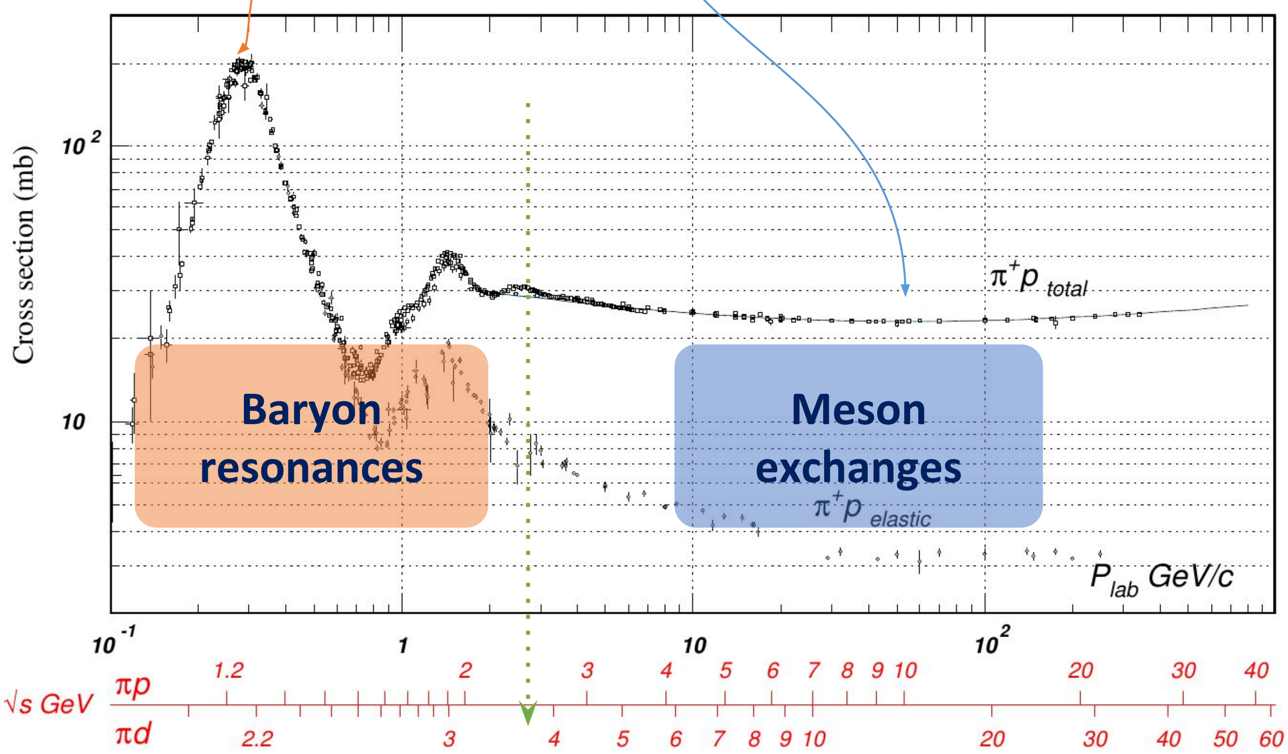


[V. Mathieu *et al.*, PRD92 (2015) 074013]

[J.N. *et al.*, PRD95 (2017) 034014]

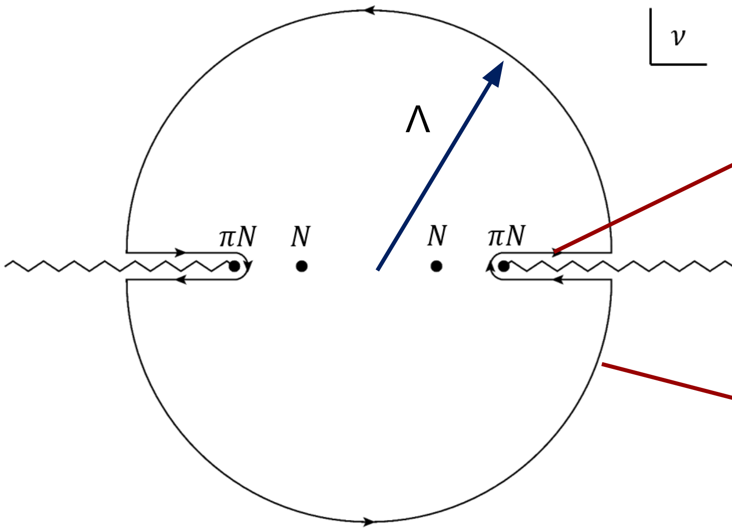
Analytic constraints





Connect low- and high-energy dynamics.

Finite-Energy Sum Rules

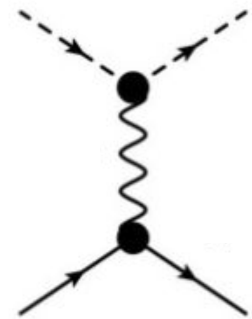


$$\mathrm{Im}\, A_i^{\mathrm{PWA}}(\nu, t)$$

INPUT

$$\beta_i(t)\, \nu^{\alpha_i(t)} \quad (\nu \approx s)$$

OUTPUT



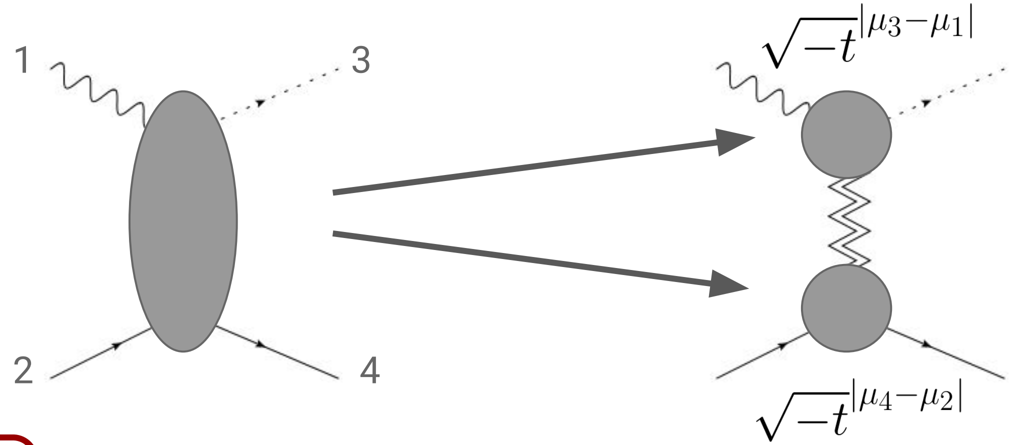
High energies: Regge theory (meson exchanges)

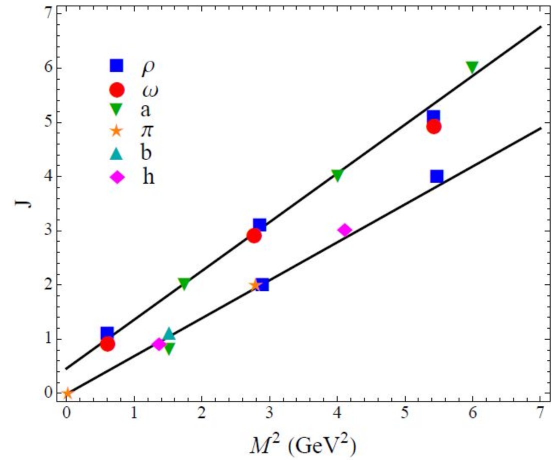
Low energies: partial-wave analyses (baryon resonances)

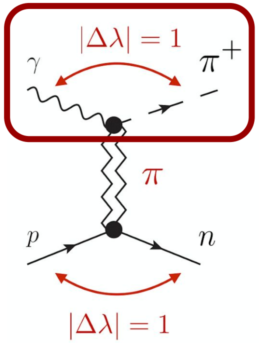
- SAID, MAID, Bonn-Gatchina, Julich-Bonn,...

High-energy model

- Contribution from photon and baryon vertex
- Suppresses amplitudes in forward direction (t=0)







$$A_{\mu_4\mu_3\mu_2\mu_1}(s,t) = \beta^{\text{top}}_{\mu_3\mu_1}(t)\beta^{\text{bottom}}_{\mu_4\mu_2}(t)R(s,t)$$

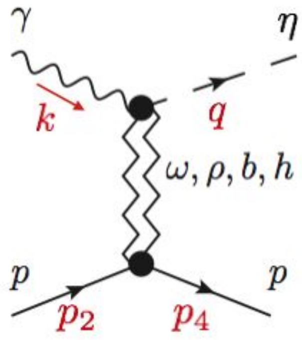
$s^{\alpha(t)}$

Choice of amplitudes

$$A_{\lambda';\lambda\,\lambda_\gamma}(s,t) = \overline{u}_{\lambda'}(p')\left(\sum_{k=1}^{4} A_k(s,t) M_k\right) u_\lambda(p)$$

- No kinematic singularities
- No kinematic zeros
- Discontinuities:
  - Unitarity cut
  - Nucleon pole

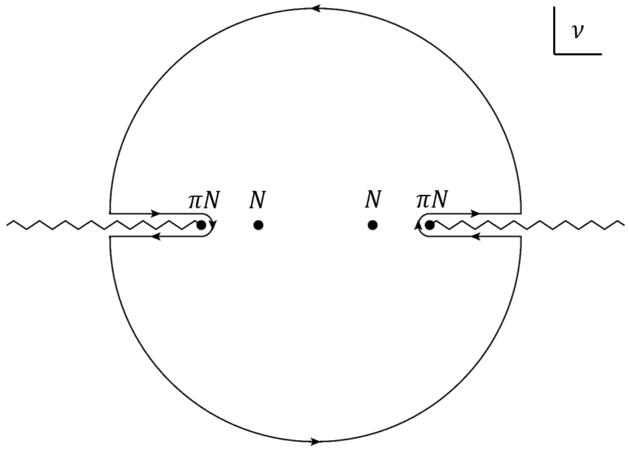
$$M_1 = \frac{1}{2}\gamma_5\gamma_\mu\gamma_\nu F^{\mu\nu}\,,$$
$$M_2 = 2\gamma_5 q_\mu P_\nu F^{\mu\nu}\,,$$
$$M_3 = \gamma_5\gamma_\mu q_\nu F^{\mu\nu}\,,$$
$$M_4 = \frac{i}{2}\epsilon_{\alpha\beta\mu\nu}\gamma^\alpha q^\beta F^{\mu\nu}\,.$$

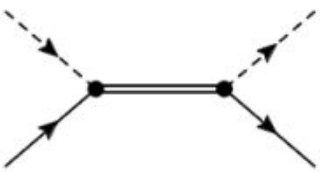


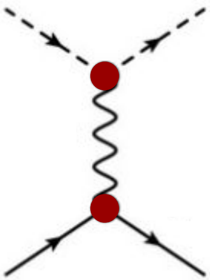
A_i	I^G	J^{PC}	η	Leading exchanges
A_1	$0^-, 1^+$	$(1,3,5,...)^{--}$	$+1$	$\rho(770), \omega(782)$
A_2'	$0^-, 1^+$	$(1,3,5,...)^{+-}$	-1	$h_1(1170), b_1(1235)$
A_3	$0^-, 1^+$	$(2,4,...)^{--}$	-1	$\rho_2(??), \omega_2(??)$
A_4	$0^-, 1^+$	$(1,3,5,...)^{--}$	$+1$	$\rho(770), \omega(782)$

$$\gamma p \to \eta p\,, \qquad A = (\omega + h + \omega_2) + (\rho + b + \rho_2)$$
$$\gamma n \to \eta n\,, \qquad A = (\omega + h + \omega_2) - (\rho + b + \rho_2)$$

Finite-Energy Sum Rules

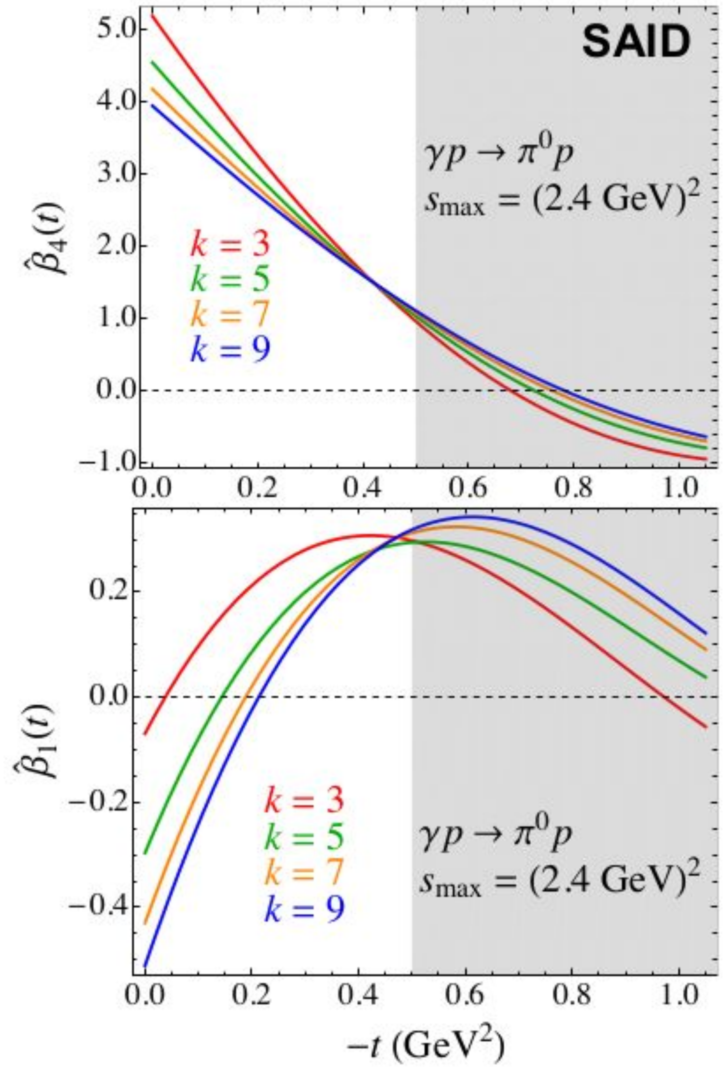




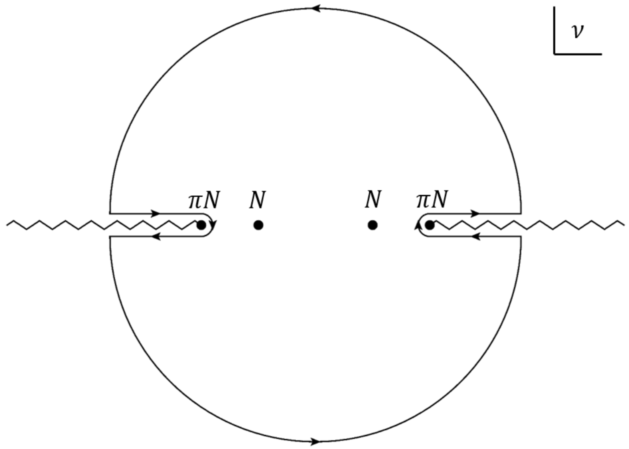


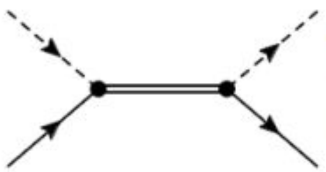
$$\int_0^{\Lambda} \mathrm{Im}\ A_i(\nu,t)\nu^k d\nu = \beta_i(t) \frac{\Lambda^{\alpha(t)+k}}{\alpha(t)+k}$$

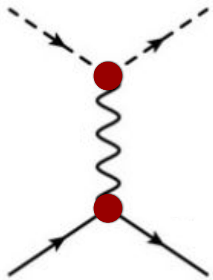
$$\beta_i(t) = \frac{\alpha(t)+k}{\Lambda^{\alpha(t)+k}} \int_0^{\Lambda} \mathrm{Im}\ A_i(\nu,t)\nu^k d\nu$$



Finite-Energy Sum Rules







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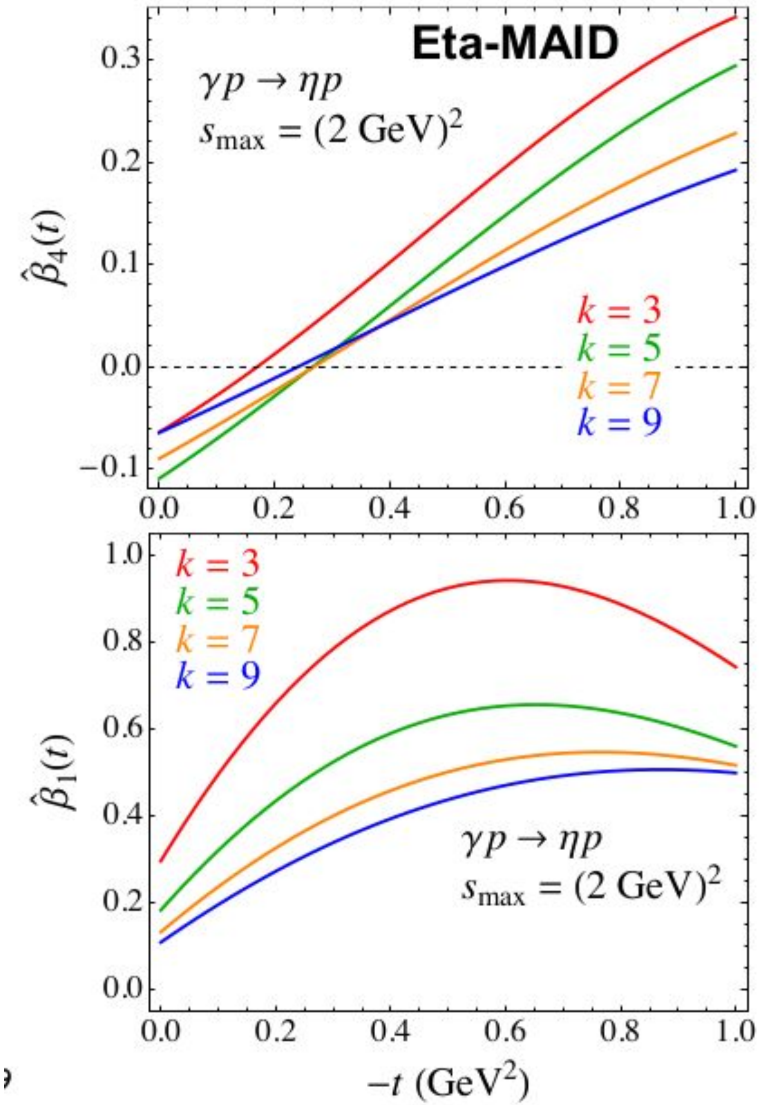
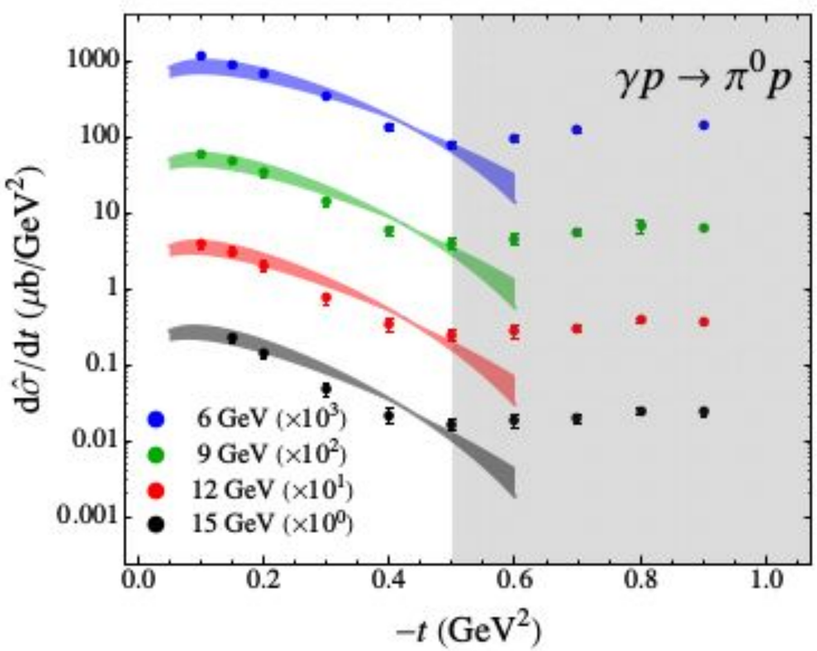
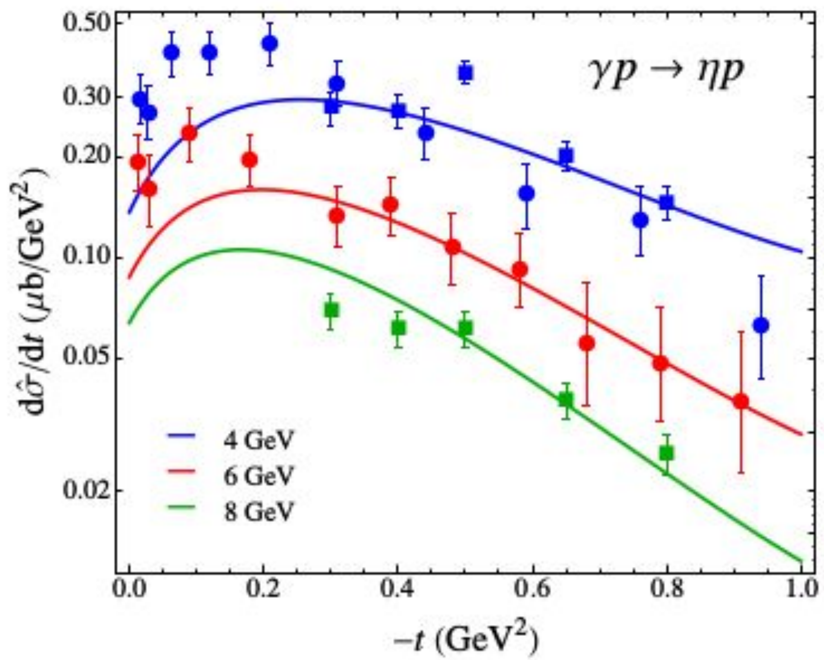


Finite-Energy Sum Rules

[V. Mathieu, J.N. *et al.* (JPAC) 1708.07779 (2017)]





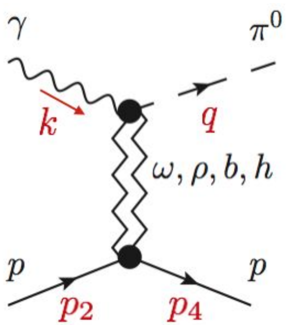
Combine energy regimes

- Low-energy model
- Predict high-energy observables

Two applications

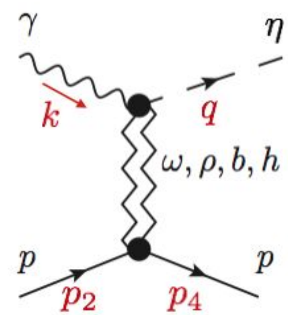
- Understand high-energy dynamics
- Constraining low-energy models

Photoproduction of neutral mesons

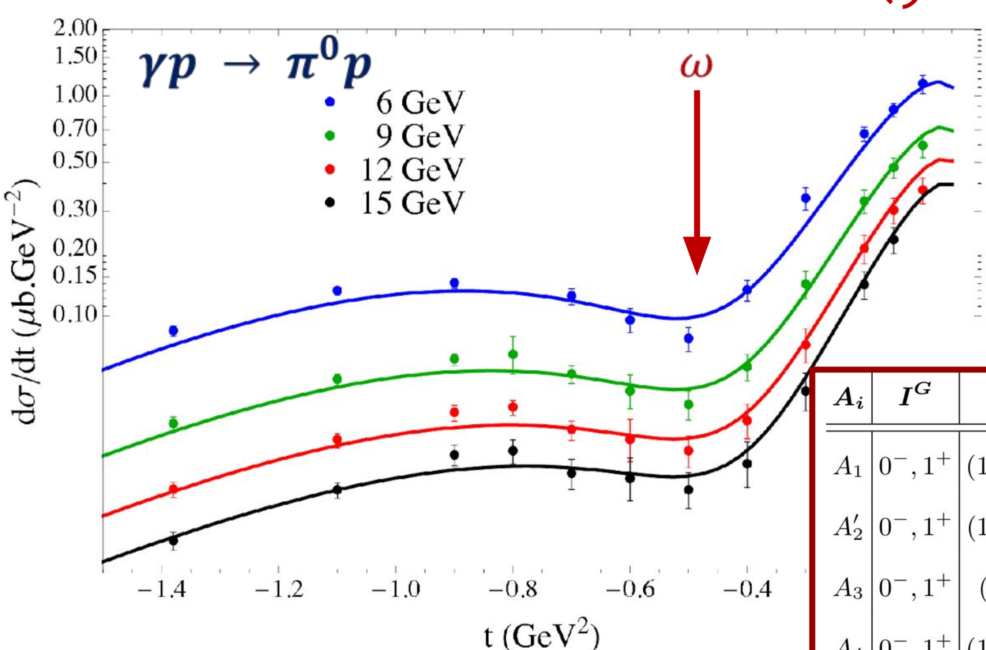


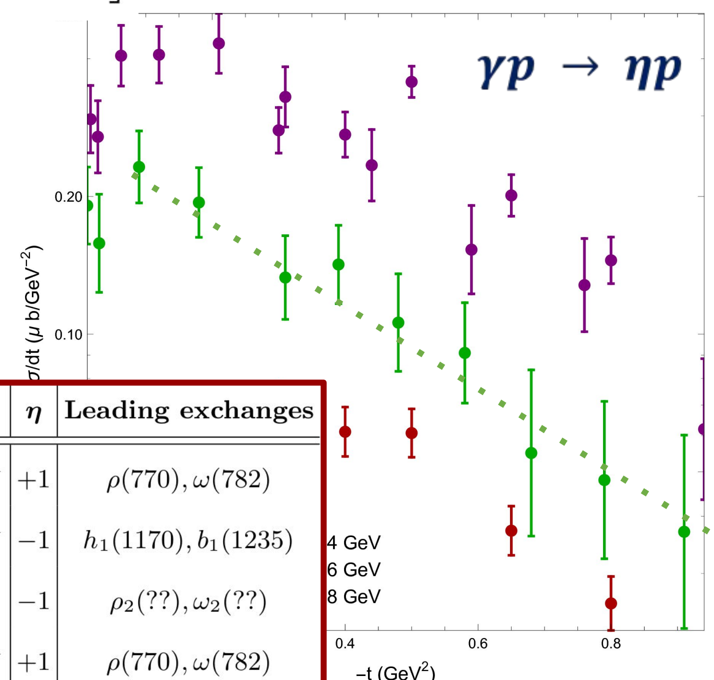
π^0 and η photoproduction

- Same exchanges
  - *isovector* more important for η
- Very different cross section
  - π^0 : dip (ω)
  - η : featureless (ρ)



$$A(\eta) = \sqrt{3}\mathcal{A}\left[A_\rho(\pi^0) + A_b(\pi^0) + \frac{1}{9}\left(A_\omega(\pi^0) + A_h(\pi^0)\right)\right]$$

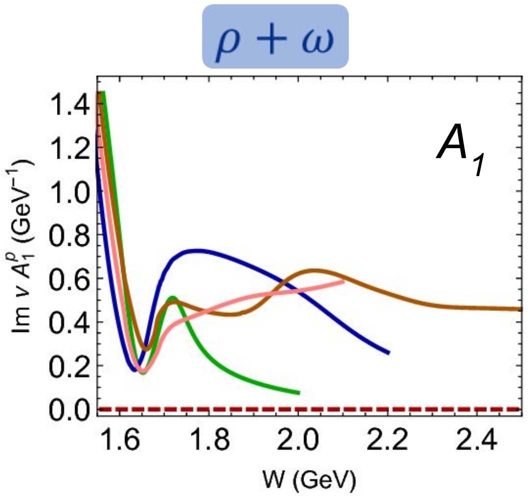


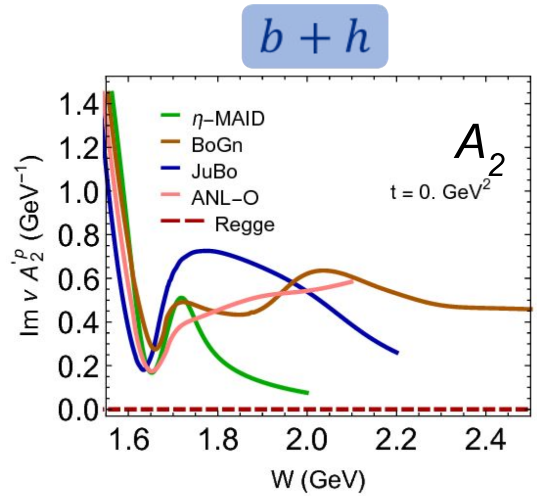


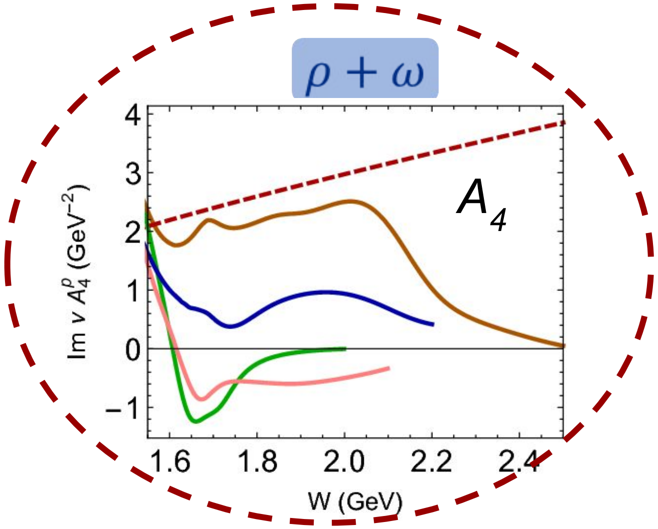
A_i	I^G	J^{PC}	η	Leading exchanges
A_1	$0^-, 1^+$	$(1,3,5,...)^{--}$	$+1$	$\rho(770), \omega(782)$
A_2'	$0^-, 1^+$	$(1,3,5,...)^{+-}$	-1	$h_1(1170), b_1(1235)$
A_3	$0^-, 1^+$	$(2,4,...)^{--}$	-1	$\rho_2(??), \omega_2(??)$
A_4	$0^-, 1^+$	$(1,3,5,...)^{--}$	$+1$	$\rho(770), \omega(782)$

Low-energy models (η)

[J.N. *et al.*, PRD95 (2017) 034014]



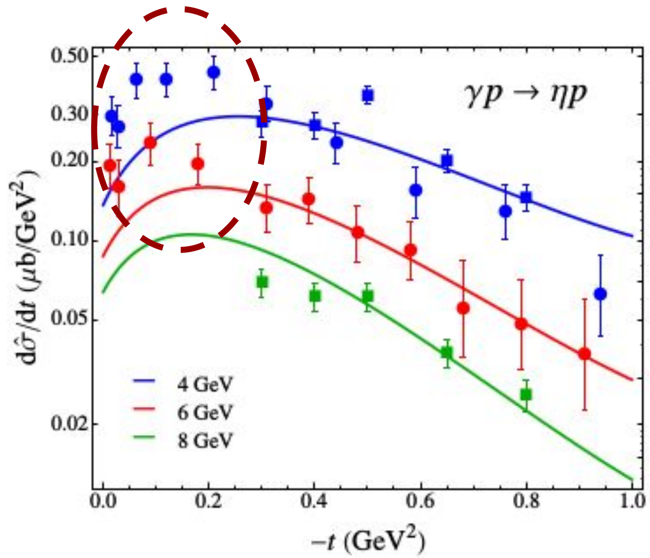




Ambiguities in the low-energy model (η-MAID)
→ Mismatch with high-energy data

Possibilities

- Low-energy model inconsistent
- Cut-off not high enough
  - High mass resonances!

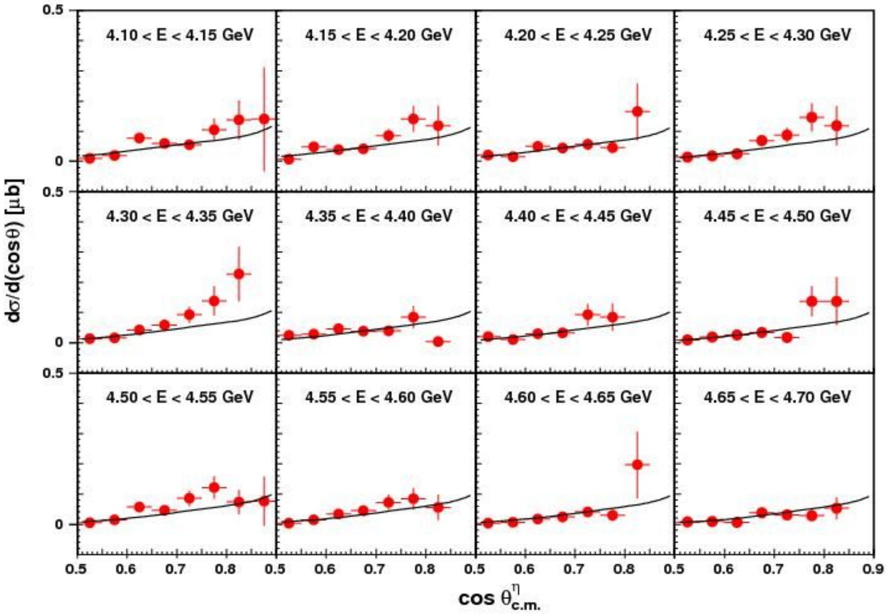


*Pre*dictions for GlueX & CLAS

$$\Sigma = \frac{\sigma_\perp - \sigma_\parallel}{\sigma_\perp + \sigma_\parallel} = \frac{|\rho+\omega|^2 - |b+h|^2}{|\rho+\omega|^2 + |b+h|^2}$$

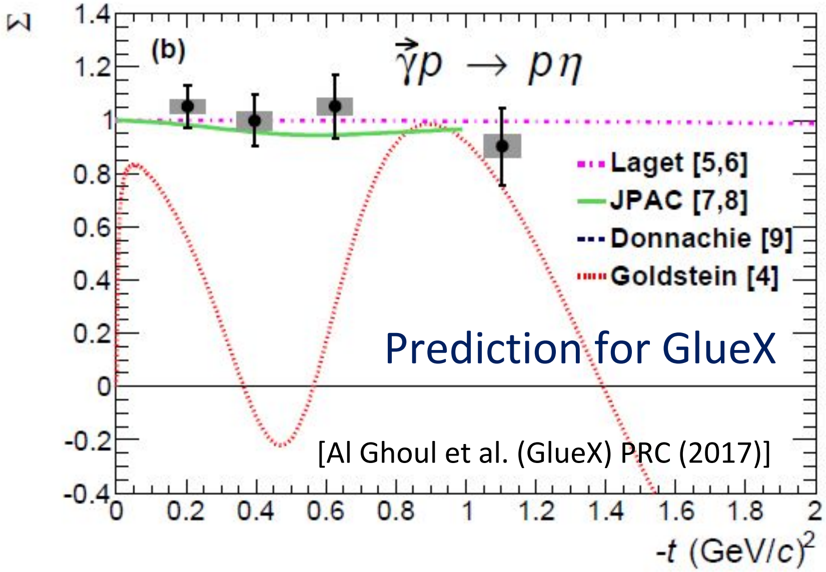
$$\Sigma = +1 \quad : \quad \rho, \omega$$

$$\Sigma = -1 \quad : \quad b, h$$



Preliminary (transition region)
[Courtesy of Zulkaida Akbar (**CLAS**)]

Natural dominant: $\Sigma = +1$
Unnatural dominant: $\Sigma = -1$



Fill up the dip with natural contribution: ρ

η' photoproduction

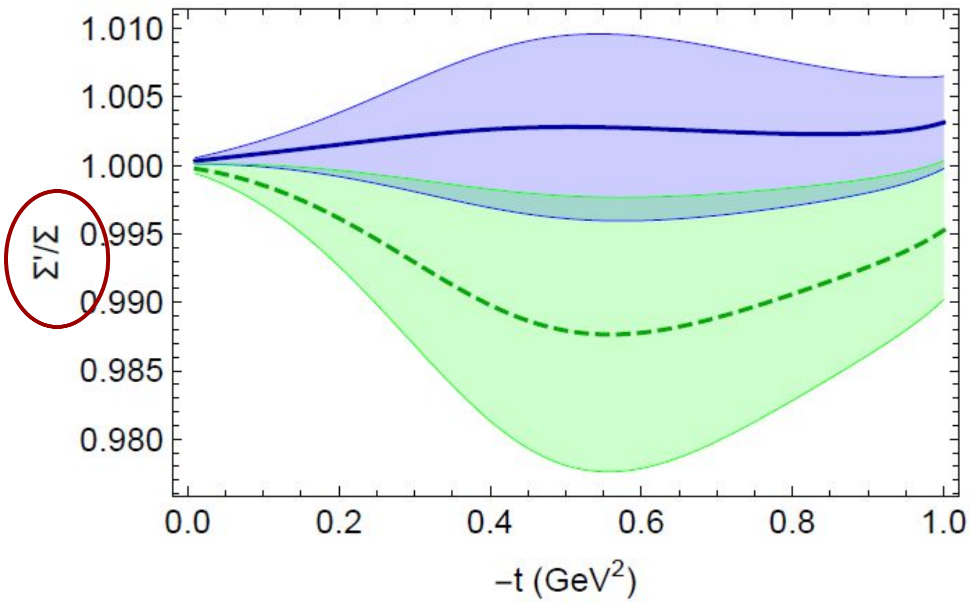
$$\Sigma = \frac{d\sigma_\perp - d\sigma_\parallel}{d\sigma_\perp + d\sigma_\parallel} \qquad (\gamma p \to \eta p)$$

$$\Sigma = \frac{|\rho+\omega|^2 - |b+h|^2}{|\rho+\omega|^2 + |b+h|^2} = \Sigma'$$

$$\Sigma' = \frac{d\sigma'_\perp - d\sigma'_\parallel}{d\sigma'_\perp + d\sigma'_\parallel} \qquad (\gamma p \to \eta' p)$$

$$\Sigma = \frac{|\rho+\omega+\boxed{\phi}|^2 - |b+h+\boxed{h'}|^2}{|\rho+\omega+\phi|^2 + |b+h+h'|^2} \neq \Sigma'$$

V.Mathieu, J.N. *et al.* (JPAC) [PLB774 (2017) 362]



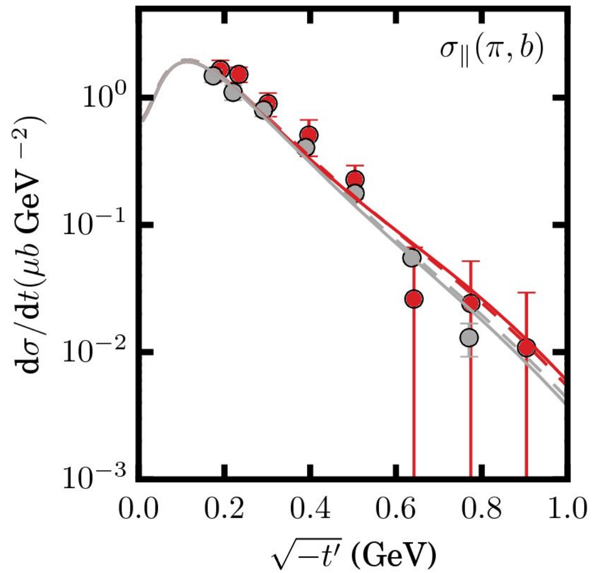
Based on the FESR for η:

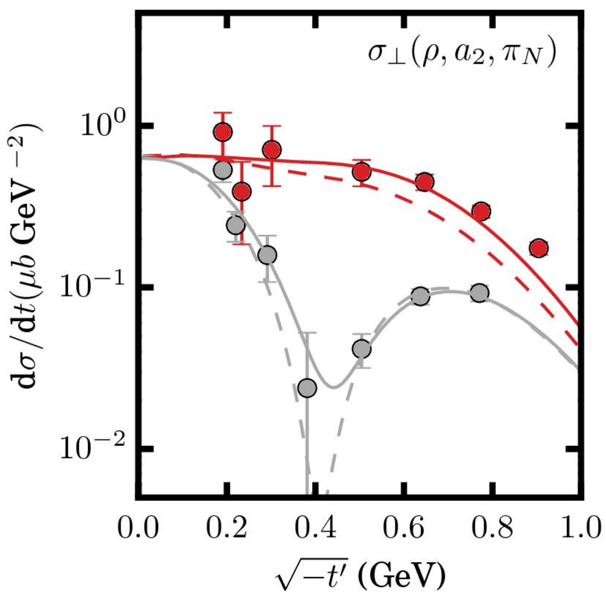
predict beam asymmetry for η'

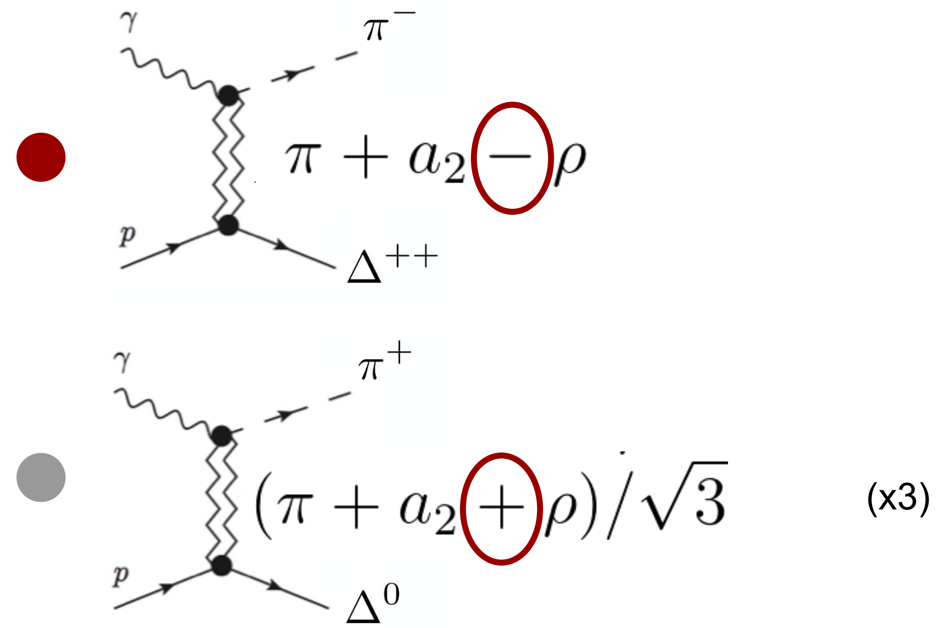
- Same exchanges
- Natural exchanges (ρ,ω) dominant
  - Couplings from radiative decays
  - Mixing angle cancels in ratio
- Unknown behavior of
  - ϕ exchange
  - unnatural exchanges (b,h)

Prediction: **≈ same beam asymmetry**

πΔ photoproduction







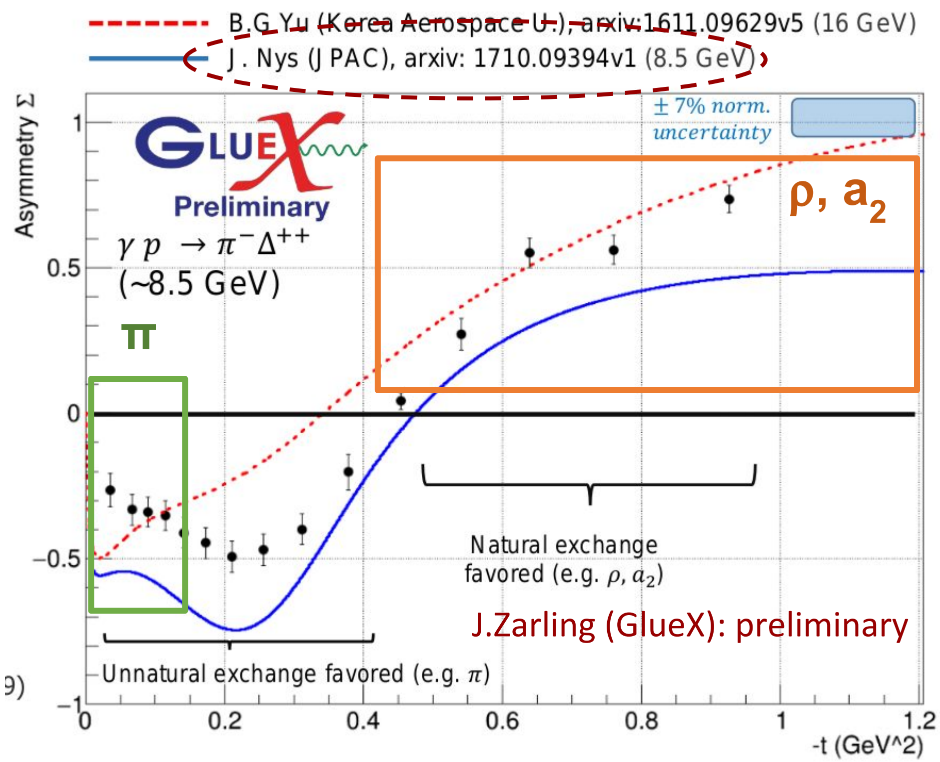
Data available at **16 GeV**

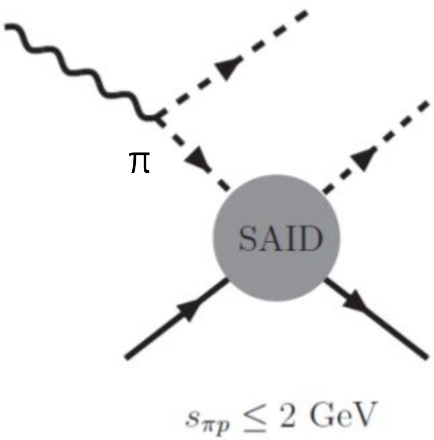
- π-exchange is featureless and entirely fixed
- Strong interference pattern in natural exchange sector
- Negligible role of b exchange

Fix t-dependence and extrapolate to JLab energies (**9 GeV**)

[J.N. *et al.*, PLB779 (2018)]

πΔ photoproduction





Łukasz Bibrzycki (Cracow)

Comparison to GlueX data

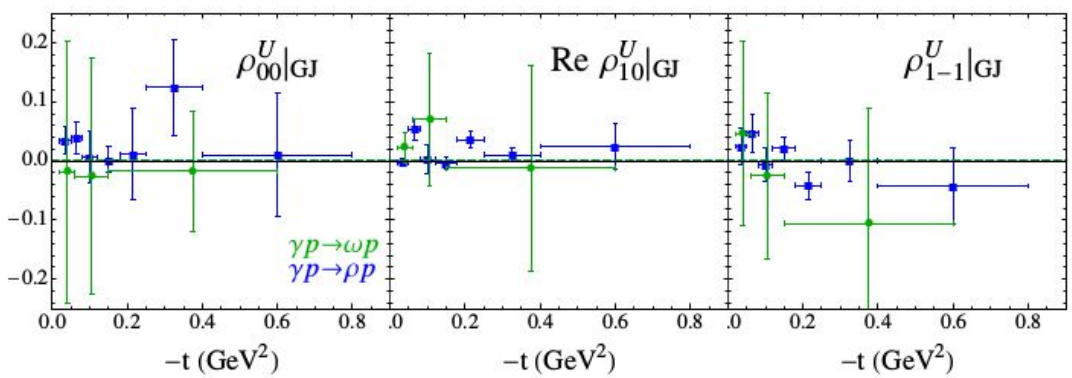
- Confirmation of interference pattern
- High -t: natural, low -t: unnatural
- Mismatch: oddly behaved π exchange
  - Ongoing analysis
  - Experimental or theoretical?

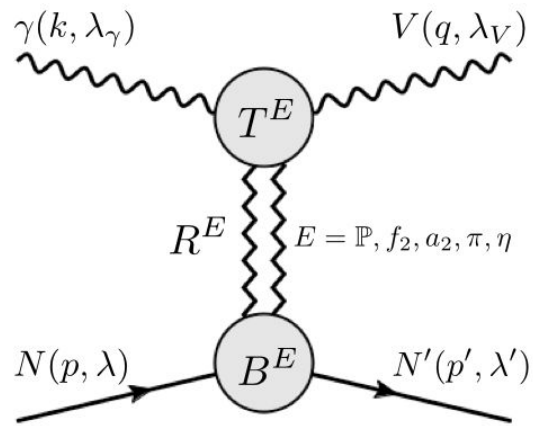
Neutral vector mesons

- Pomeron dominates at high energies
- Isoscalar exchanges dominantly helicity non-flip (λ=λ')
- Unnatural exchanges: only helicity flip (|λ-λ'|=1)

$$\mathcal{M}_{\substack{\lambda_V,\lambda_\gamma\\ \lambda',\lambda}}(s,t) = \sum_{E=\pi,\eta,\mathbb{P},f_2,a_2} \mathcal{M}^{E}_{\substack{\lambda_V,\lambda_\gamma\\ \lambda',\lambda}}(s,t)$$

$$\mathcal{M}^{\substack{N\\U}}_{\substack{-\lambda_\gamma,-\lambda_V\\ \lambda,\lambda'}} = \pm(-1)^{\lambda_\gamma-\lambda_V}\mathcal{M}^{\substack{N\\U}}_{\substack{\lambda_\gamma,\lambda_V\\ \lambda,\lambda'}}$$





$$\rho^{\substack{N\\U}}_{00} = \frac{1}{2}\left(\rho^0_{00} \mp \rho^1_{00}\right),$$

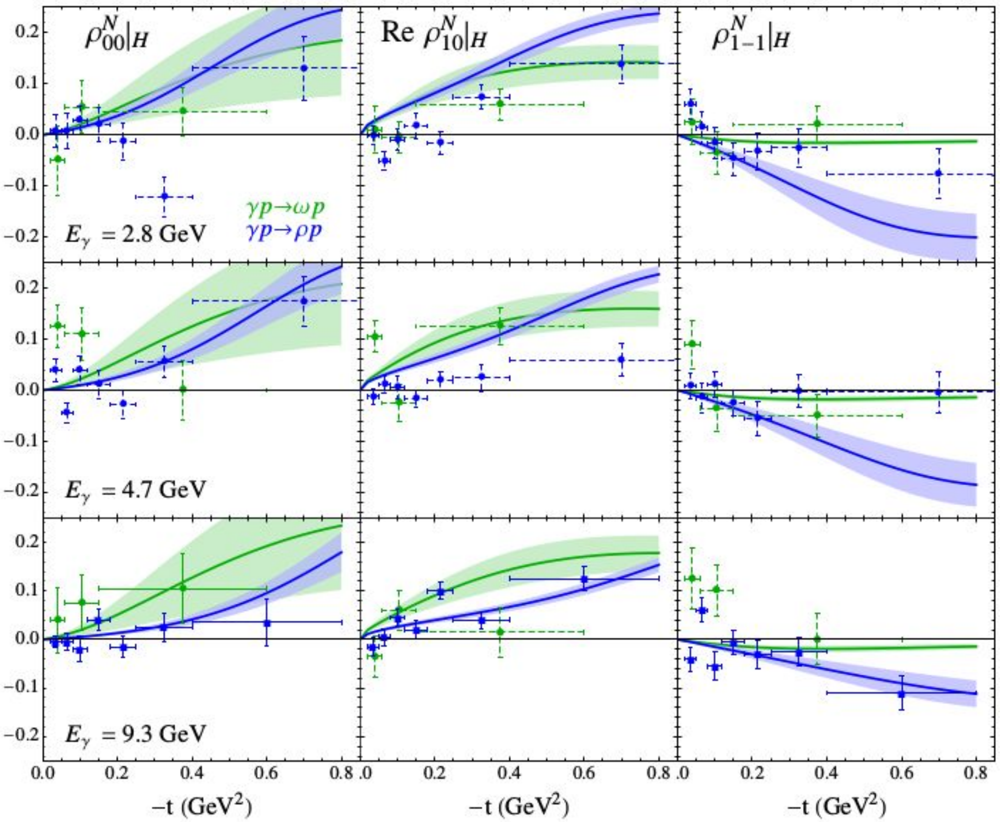
$$\mathrm{Re}\,\rho^{\substack{N\\U}}_{10} = \frac{1}{2}\left(\mathrm{Re}\,\rho^0_{10} \mp \mathrm{Re}\,\rho^1_{10}\right),$$

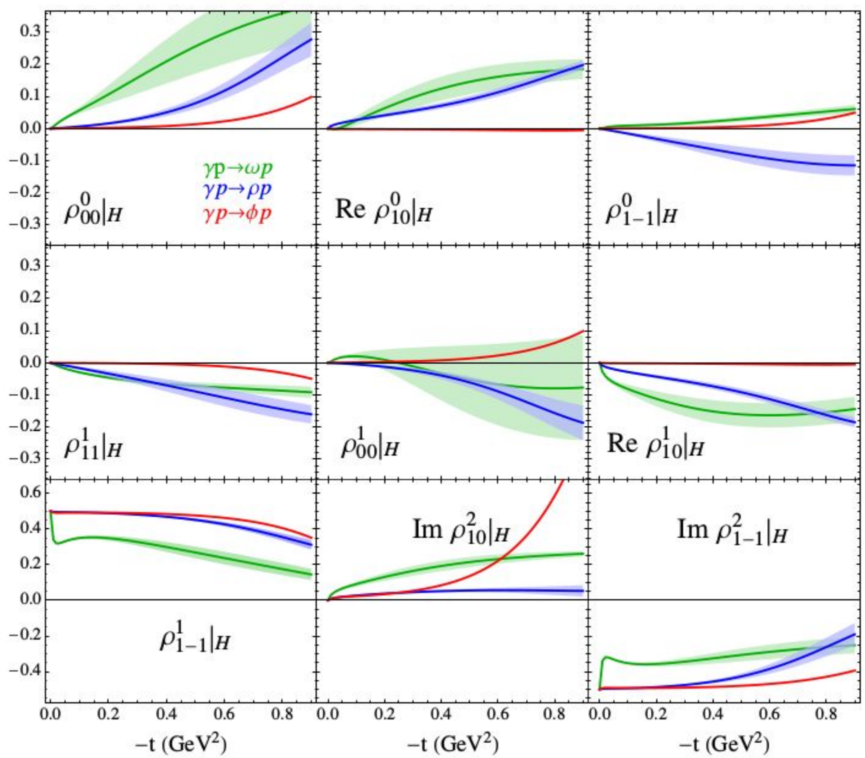
$$\rho^{\substack{N\\U}}_{1-1} = \frac{1}{2}\left(\rho^1_{1-1} \pm \rho^1_{11}\right).$$

[V.Mathieu, J.N. *et al.*, (2018) arXiv:1802.09403]

Neutral vector mesons

$$\mathcal{M}_{\substack{\lambda_V,\lambda_\gamma \\ \lambda',\lambda}}(s,t) = \sum_{E=\pi,\eta,\mathbb{P},f_2,a_2} \mathcal{M}^E_{\substack{\lambda_V,\lambda_\gamma \\ \lambda',\lambda}}(s,t)$$





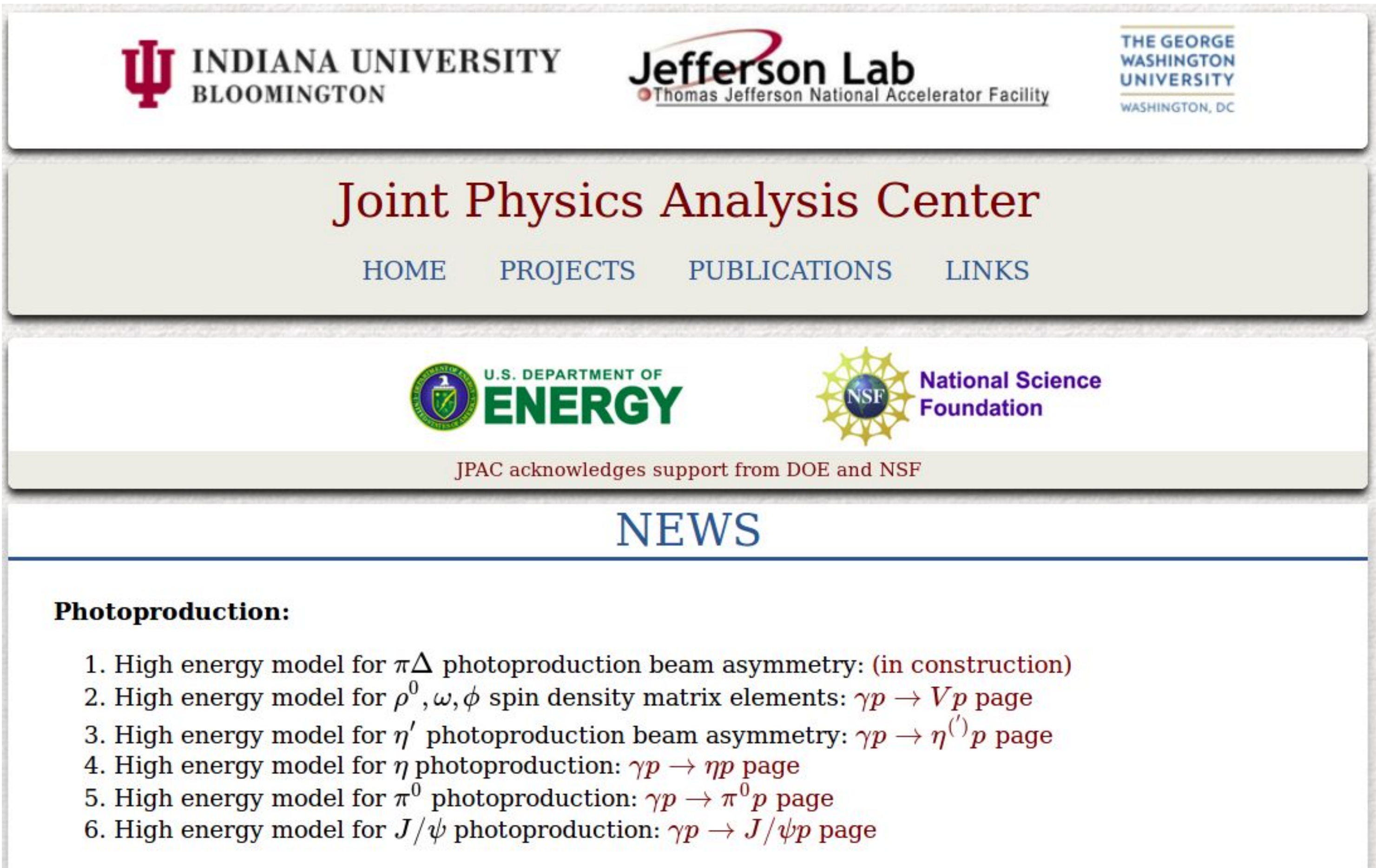
[V.Mathieu, J.N. *et al.*, (2018) arXiv:1802.09403]

Summary

Theory support for GlueX and CLAS with JPAC

- *Various photoproduction reactions analyzed*
  - πN, $\pi\Delta$, ηN, $\eta' N$ + many more
  - Comparison to first GlueX data
    - Unnatural exchanges negligible
    - Natural exchanges dominate
  - Importance of analytic constraints (FESR)
  - Connection between baryon spectroscopy and high-energy data
  - SDME predictions for neutral meson prediction (Pomeron dominated)

http://www.indiana.edu/~jpac/



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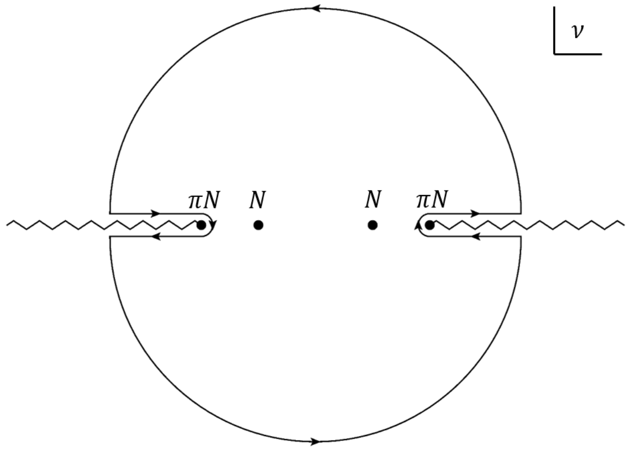
JPAC acknowledges support from DOE and NSF

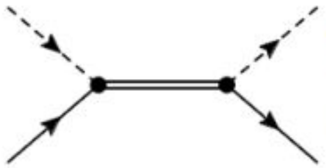
NEWS

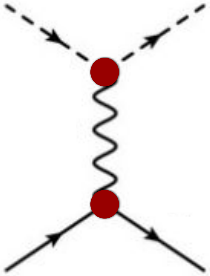
Photoproduction:

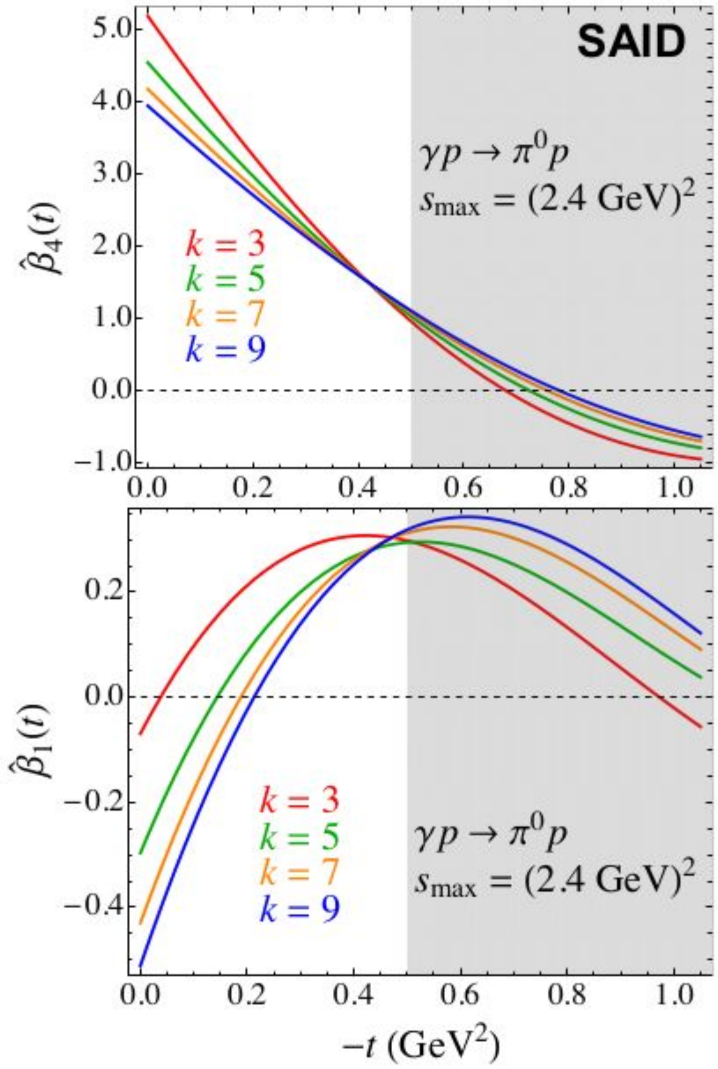
1. High energy model for $\pi\Delta$ photoproduction beam asymmetry: (in construction)
2. High energy model for ρ^0, ω, ϕ spin density matrix elements: $\gamma p \to V p$ page
3. High energy model for η' photoproduction beam asymmetry: $\gamma p \to \eta^{(')} p$ page
4. High energy model for η photoproduction: $\gamma p \to \eta p$ page
5. High energy model for π^0 photoproduction: $\gamma p \to \pi^0 p$ page
6. High energy model for J/ψ photoproduction: $\gamma p \to J/\psi p$ page

Finite-Energy Sum Rules









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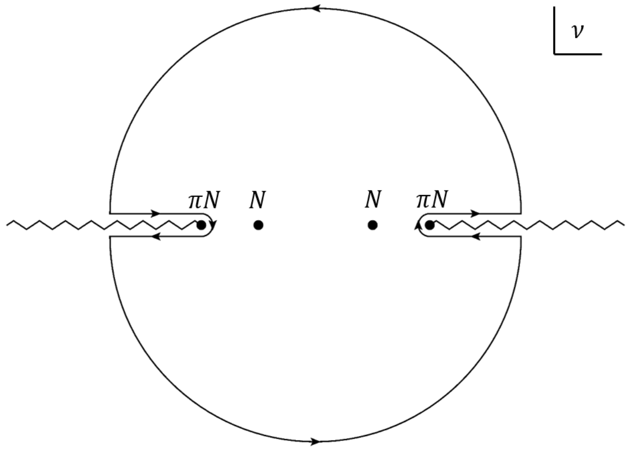
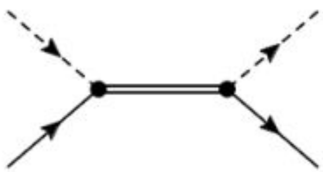
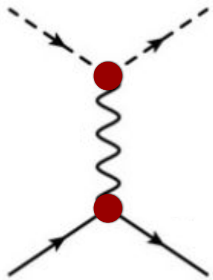
Finite-Energy Sum Rules







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