Light mesons from JPAC+COMPASS analyses

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Contents

Introduction

Joint Physics Analysis Center

[collage by Vincent Mathieu]

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Madrid U.

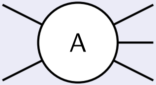
Viktor Mokeev
JLab

Miguel Albaladejo
JLab

JPAC effort

Collaboration of theoreticians and experimentalists

Amplitude analysis



- Writting amplitudes using general QFT constraints
- Analysis of experimental data
- Analytic continuation, pole search

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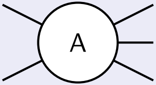

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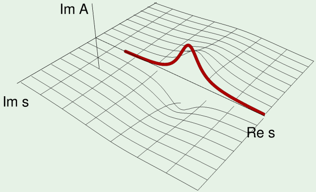
General properties of the scattering amplitude



Analyticity + Unitary + Crossing symmetry

- Scattering amplitude is an analytic function in $s = E^2$ complex plane,
- The Real axis $\rightarrow$ physical world,
- Resonances = poles of the unphysical sheet.

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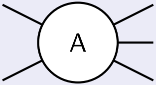

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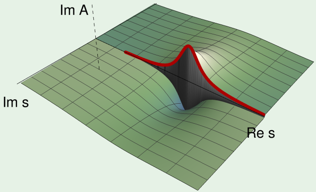

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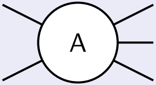

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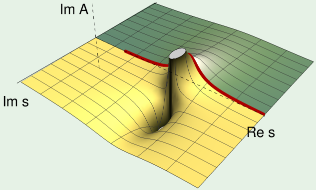

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- Scattering amplitude is an analytic function in $s = E^2$ complex plane,
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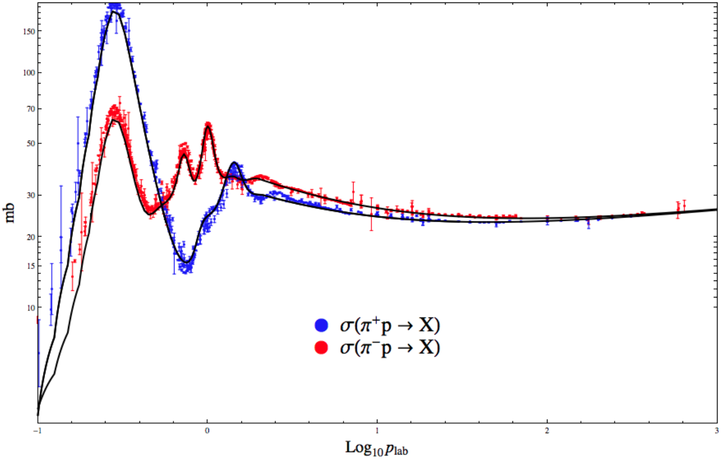
Two regimes of scattering

Hadronic duality [V.Mathieu,et al.,PRD92 (2015), 074004]



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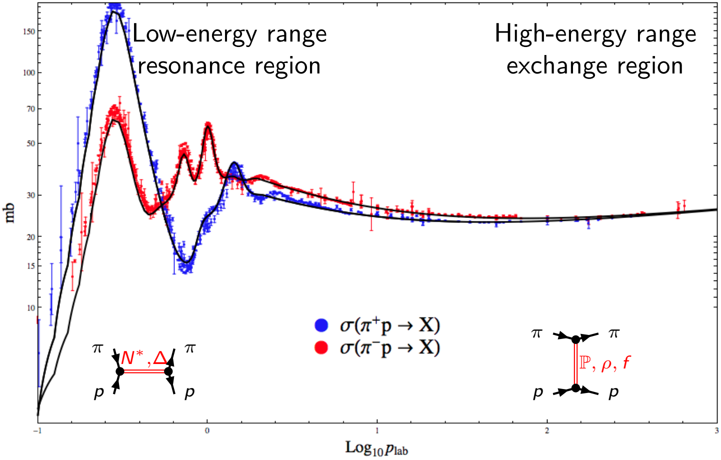
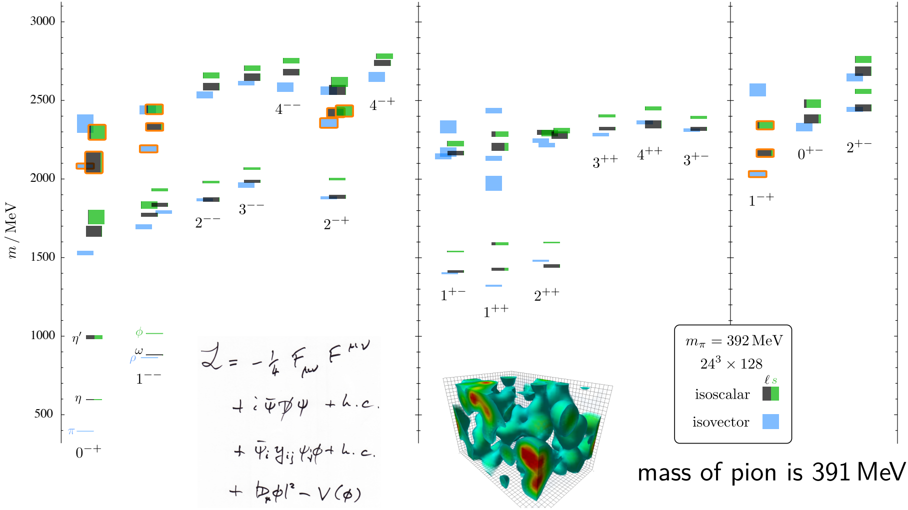


Hadronic excitations

Results of lattice QCD

[Dudek et al., PRD 88, 094505 (2013)]



$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + i \bar{\psi} \not{D} \psi + h.c. + \bar{\psi}_i y_{ij} \psi_j \phi + h.c. + |D_\mu \phi|^2 - V(\phi)$$

mass of pion is 391 MeV

Hadronic excitations

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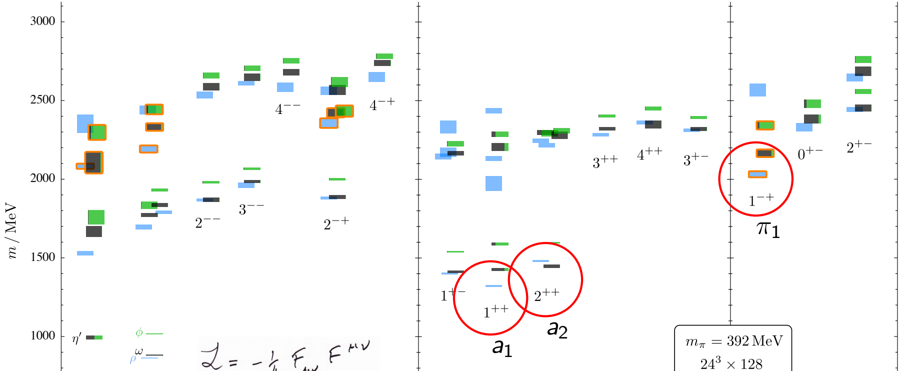
[Dudek et al., PRD 88, 094505 (2013)]



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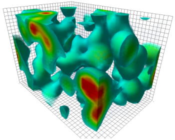
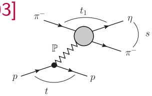


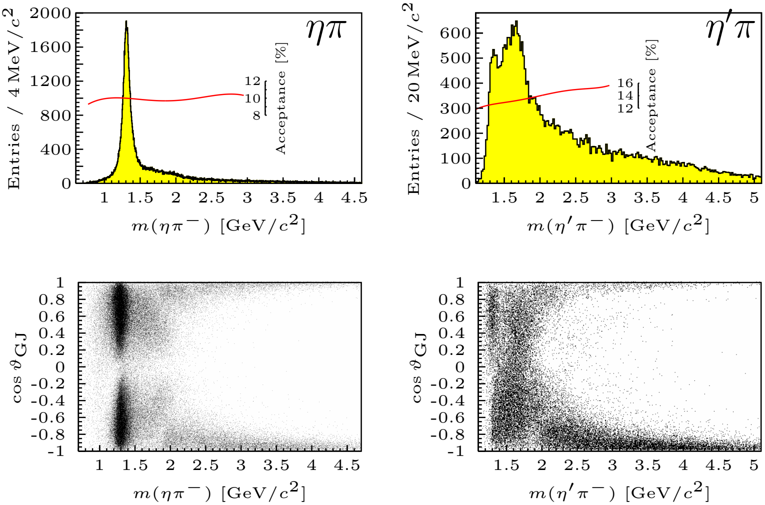
mass of pion is 391 MeV

$\eta^{(\prime)}\pi$ analyses

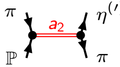
$\eta\pi$ vs $\eta'\pi$ at COMPASS

[(COMPASS) PLB 740 (2015) 303]

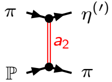




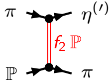
Resonance production



η-forward production



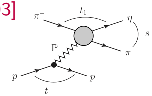
π-forward production

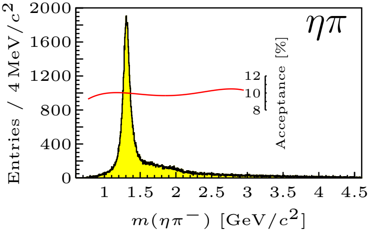


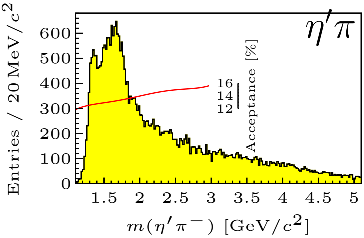
$\eta^{(\prime)}(0^-)\pi(0^-)$, $J^{PC} = L^{P+} \Rightarrow \cos\theta_{\rm GJ}$ asymmetry $\Rightarrow$ exotic waves!

$\eta\pi$ vs $\eta'\pi$ at COMPASS

[(COMPASS) PLB 740 (2015) 303]







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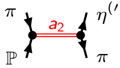
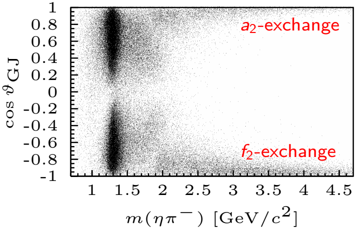
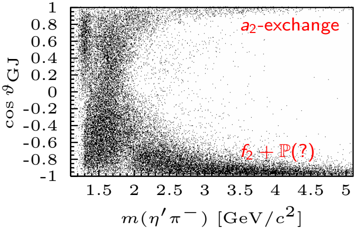
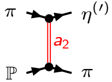






η-forward production



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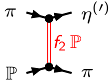
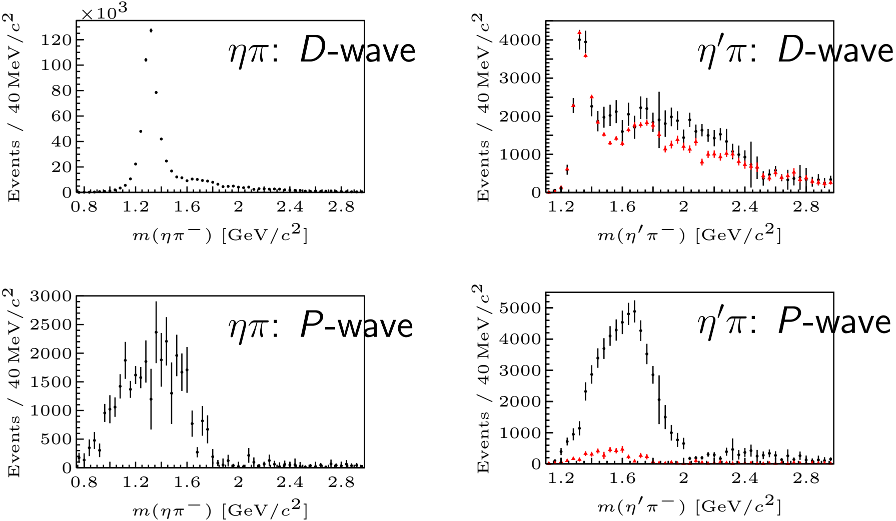


$\eta^{(\prime)}(0^-)\pi(0^-)$, $J^{PC} = L^{P+}$ $\Rightarrow$ $\cos\theta_{GJ}$ asymmetry $\Rightarrow$ exotic waves!

$\eta^{(\prime)}\pi$ partial wave analysis

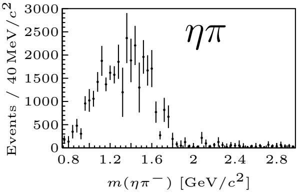
[(COMPASS) PLB 740 (2015) 303]

Two-main contribution: *P*- and *D*-waves



PDG status: exotic π_1 states

Two candidates



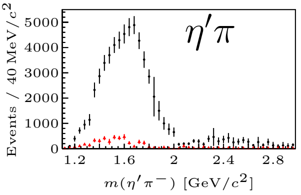
$\pi_1(1400)$ $I^G(J^{PC}) = 1^-(1^{-+})$

See also the mini-review under non-$q\bar{q}$ candidates in PDG 2006 , Journal of Physics G33 1 (2006).

$\pi_1(1400)$ MASS	1354 ± 25 MeV (S = 1.8)
$\pi_1(1400)$ WIDTH	330 ± 35 MeV

Decay Modes

Mode		Fraction (Γ_i/Γ)	Scale Factor/ Conf. Level	P (MeV/c)
Γ_1	$\eta\pi^0$	seen		557
Γ_2	$\eta\pi^-$	seen		556
Γ_3	$\eta'\pi$			318



$\pi_1(1600)$ $I^G(J^{PC}) = 1^-(1^{-+})$

$\pi_1(1600)$ MASS	1662^{+8}_{-9} MeV
$\pi_1(1600)$ WIDTH	241 ± 40 MeV (S = 1.4)

Decay Modes

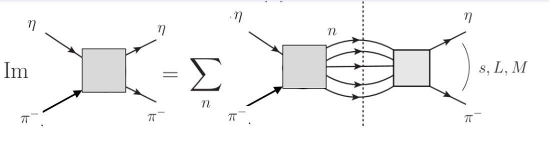
Mode		Fraction (Γ_i/Γ)	Scale Factor/ Conf. Level	P (MeV/c)
Γ_1	$\pi\pi\pi$	seen		803
Γ_2	$\rho^0\pi^-$	seen		641
Γ_3	$f_2(1270)\pi^-$	not seen		318
Γ_4	$b_1(1235)\pi$	seen		357
Γ_5	$\eta'(958)\pi^-$	seen		543
Γ_6	$f_1(1285)\pi$	seen		314

Amplitude for $\eta\pi$ production [A.Jackura,MM,A.Pilloni,et al. (JPAC-COMPASS), PLB779, 464-472]

N-over-*D* method

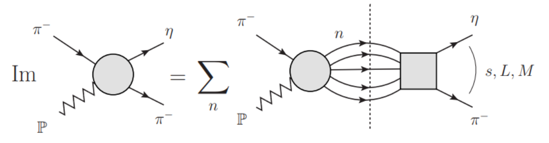
Scattering amplitude: $\eta\pi \to \eta\pi$, D-wave

$$T = \frac{N(s)}{D(s)}$$



Production amplitude: $\pi\mathbb{P} \to \eta\pi$, D-wave

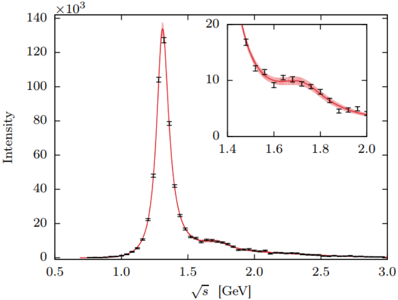
$$a = \frac{n(s)}{D(s)}$$



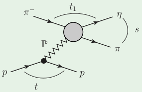
- $D(s)$ is universal, has only the right-hand cut.
- $N(s)$ and $n(s)$ have the left-hand cut only (exchanges)

Tensor mesons ($J^{PC} = 2^{++}$)

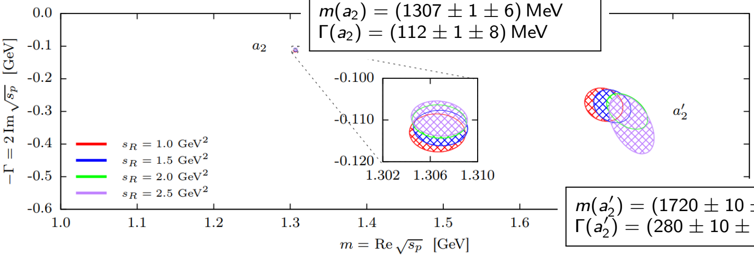
Advanced $\eta\pi$ analysis [A.Jackura,MM,A.Pilloni,et al. (JPAC-COMPASS), PLB779, 464-472]



Single channel: $\eta\pi$ D-wave



- Elastic unitarity
- Two CDD-poles



$m(a_2) = (1307 \pm 1 \pm 6)$ MeV
$\Gamma(a_2) = (112 \pm 1 \pm 8)$ MeV

$m(a_2') = (1720 \pm 10 \pm 60)$ MeV
$\Gamma(a_2') = (280 \pm 10 \pm 70)$ MeV

Coupled-channel amplitude [A.Rodas,A.Pilloni,MM,et al. (JPAC), PRL122 (2019)]

Scattering amplitude: $\eta^{(\prime)}\pi \to \eta^{(\prime)}\pi$, P/D-waves

$$T = \frac{N(s)}{D(s)}$$

$$\rho N^J_{ki}(s') = \delta_{ki} \frac{(p_{\eta^{(\prime)}\pi}\sqrt{s}/2)^{2J+1}}{(s'+s_L)^{2J+1+\alpha}},$$

- 2 K-matrix pole for D-wave
- 1 K-matrix pole for P-wave

$$D^J_{ki}(s) = \left[K^J(s)^{-1}\right]_{ki} - \frac{s}{\pi}\int_{s_k}^{\infty} ds' \frac{\rho N^J_{ki}(s')}{s'(s'-s-i\epsilon)}.$$

Production amplitude: $\pi\mathbb{P} \to \eta^{(\prime)}\pi$, P/D-waves

$$a = \frac{n(s)}{D(s)}$$

$$a^J_i(s) = q^{J-1} p^J_i \sum_k n^J_k(s) \left[D^J(s)^{-1}\right]_{ki}$$

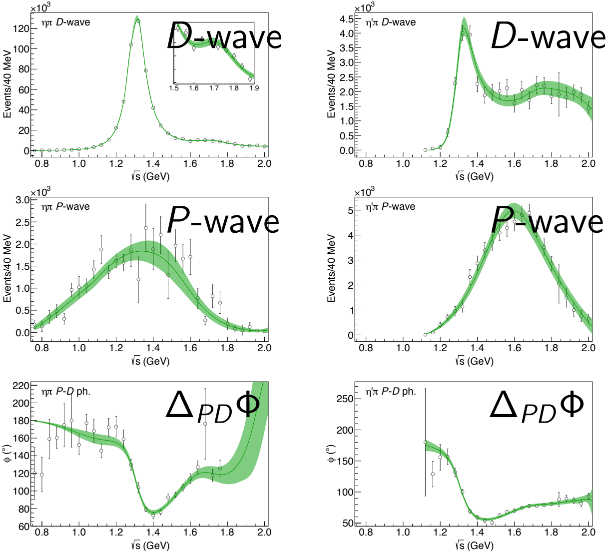
- left poles to model unknown production function $n(s)$

- $D(s)$ has only the right-hand cut.
- $N(s)$ and $n(s)$ have the left-hand cut only (exchanges)

Fit to the data

[A.Rodas,A.Pilloni,MM,et al. (JPAC), PRL122 (2019)]

$\chi^2/\mathrm{ndf} = 162/122$, the band - 2σ bootstrap error



D-wave difference

- Kinematics ($m_{\eta'} > m_\eta$)

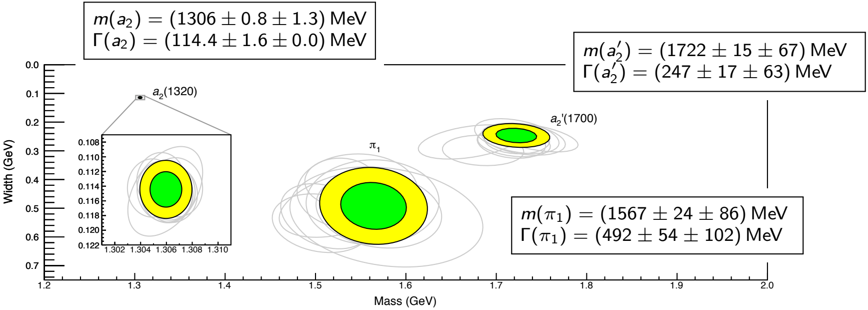
$\Rightarrow$ Same amplitude.

P-wave difference

- production mechanism
- + kinematics.

Results: pole positions

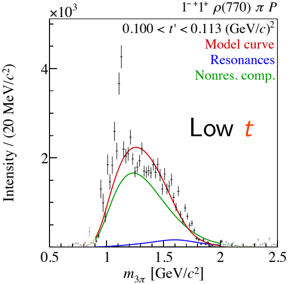
[A.Rodas,A.Pilloni,MM,et al. (JPAC), PRL122 (2019)]

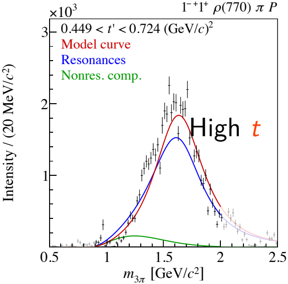


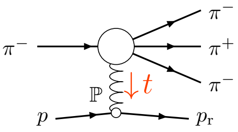
- Change parametrization of the denominator $\rho N^J_{ki}(s') = \delta_{ki}\,\frac{(p_{\eta^{(\prime)}\pi}\sqrt{s}/2)^{2J+1}}{(s'+s_L)^{2J+1+\alpha}}$,
  - $s_R = 1\,\text{GeV} \to 0.8, 1.8\,\text{GeV}$.
  - $\alpha = 2 \to 1\text{GeV}$.
  - Different function, $\rho N^J_{ki}(s') = \delta_{ki}\, Q_J(z_{s'})\, s'^{-\alpha}\lambda^{-1/2}(s', m^2_{\eta^{(\prime)}}, m^2_\pi)$
- Change of parameters in the numerator $n(s)$
  - Effective transferred momentum $t_{\text{eff}} = -0.1\,\text{GeV}^2 \to -0.5\,\text{GeV}^2$.
  - Order of the polynomial 3rd-order $\to$ 4th-order.

Same π_1 as in 3π?

[See a talk of B.Ketzer, afternoon]

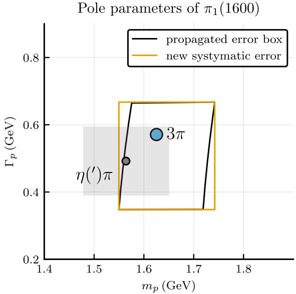






The COMPASS fit:

- Signal by BW amplitude
- Flexible background



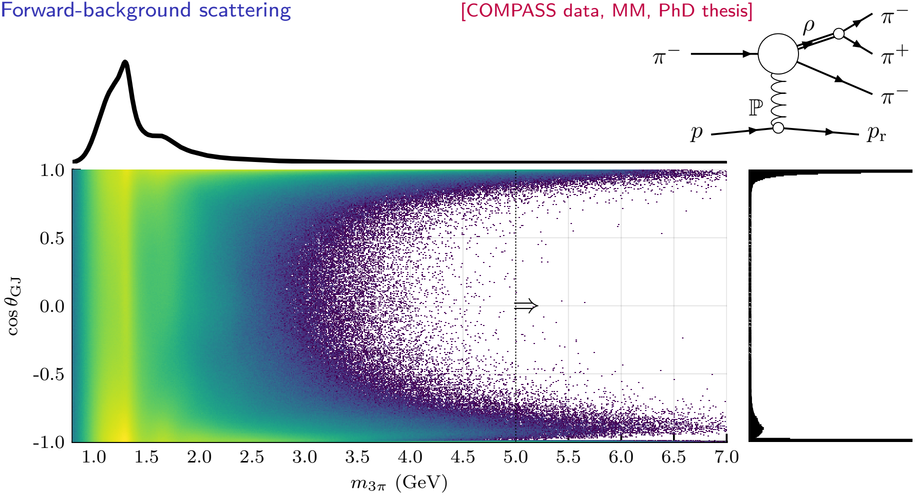
Consistent results on $\pi_1(1600)$ pole:

- $\rho\pi$ Breit-Wigner parameters
  $\Rightarrow$ pole position
- $\eta^{(\prime)}\pi$ systematic margins

Three-pions physics

Diffractive production of 3π off proton target

Forward-background scattering [COMPASS data, MM, PhD thesis]

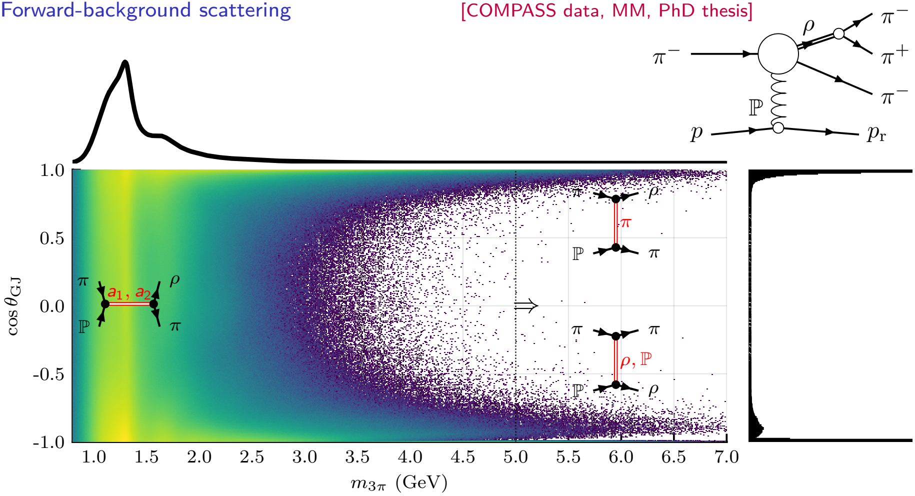


The high-energy exchange processes penetrate to the low energy and make resonance characterization difficult

Diffractive production of 3π off proton target

Forward-background scattering

[COMPASS data, MM, PhD thesis]

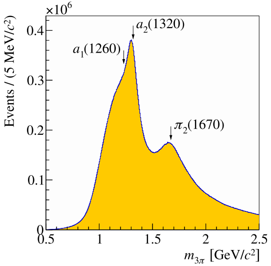


The high-energy exchange processes penetrate to the low energy and make resonance characterization difficult

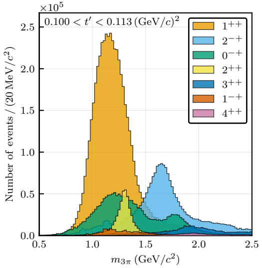
Three-pion resonances

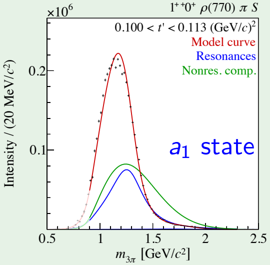
[see talk by D.Ryabchikov, afternoon]

COMPASS PWA [PRD95 (2017) 032004]



Partial Wave Analysis ⟶





COMPASS Fit:

- Signal by BW amplitude $\sim 50\%$
- Background $\sim 50\%$

$m_{a_1}^{(\mathrm{BW})} = (1299^{+12}_{-28})$ MeV,

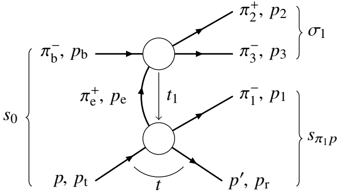
$\Gamma_{a_1}^{(\mathrm{BW})} = (380 \pm 80)$ MeV.

Large uncertainty due to unknonwn background.

Model for the forward scattering

[MM, A.Jackura (JPAC) in preparation]

Deck effect



$$= (T_{\pi_1 p})_{\lambda\lambda'} \frac{1}{m_\pi^2 - t_1} T_{\pi_2\pi_3} \quad (1)$$

- Two diagrams (π^- symmetrization)

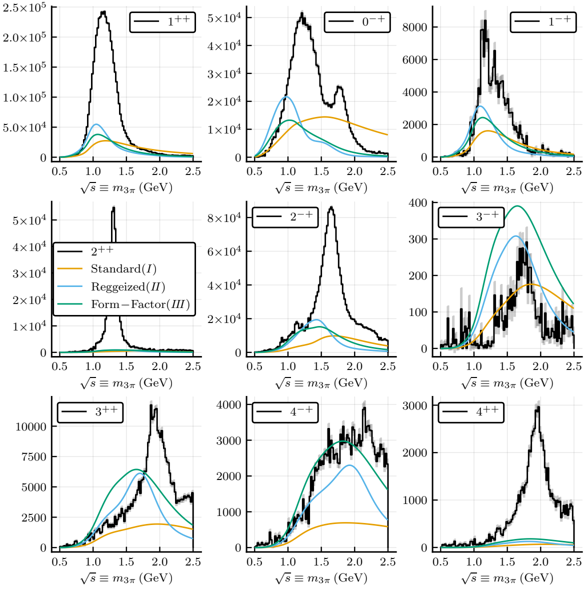
$$\mathcal{B}_{\lambda\lambda'} = \mathcal{B}^{(1)}_{\lambda\lambda'} + \mathcal{B}^{(3)}_{\lambda\lambda'}$$

- High energy $p\pi$ scattering
- $\pi\pi$ scattering dominated by resonances in lower partial waves
  - Relative strength of S, P, D-waves is controlled by unitarity

$$\mathcal{B}^{(1)} = s_{\pi p} F(t) \frac{\mathrm{FF}(t_1)}{m_\pi^2 - t_1}\left[\frac{2}{3}\, t^{(\sigma_1, f_0)}(\sigma_1) + 3\, t^{(\rho)}(\sigma_1, t_1) P_1(\cos\theta_{\pi\pi}) + \frac{10}{3}\, t^{(f_2)}(\sigma_1, t_1) P_2(\cos\theta_{\pi\pi})\right].$$

Comparison with the COMPASS data

[MM PhD thesis]



Pion propagator:

- Standard Deck

$$\frac{1}{m_\pi^2 - t_1},$$

- Regge Deck

$$\frac{e^{-i\pi\alpha(t_1)/2}}{m_\pi^2 - t_1}\left(\frac{s' - u'}{2s_{\mathrm{sc}}}\right)^{\alpha(t_1)}$$

- Form-factored Deck

$$\frac{e^{bt_1}}{m_\pi^2 - t_1},$$

$a_1(1260)$ state – isospin parter of ρ

$a_1(1260)$ WIDTH

INSPIRE search

VALUE (MeV)	EVTS	DOCUMENT ID		TECN	COMMENT
250 to 600	**OUR ESTIMATE**				
389 ± 29	**OUR AVERAGE** Error includes scale factor of 1.3.				
430 ±24 ±31		DARGENT	2017	RVUE	$D^0 \to \pi^-\pi^+\pi^-\pi^+$
367 ±9 $^{+28}_{-25}$	420k	ALEKSEEV	2010	COMP	190 $\pi^- \to \pi^-\pi^-\pi^+ Pb'$
• • • We do not use the following data for averages, fits, limits, etc. • • •					
410 ±31 ±30		1 AUBERT	2007AU	BABR	10.6 $e^+ e^- \to \rho^0\rho^\pm\pi^\mp\gamma$
520 - 680	6360	2 LINK	2007A	FOCS	$D^0 \to \pi^-\pi^+\pi^-\pi^+$
480 ±20		3 GOMEZ-DUMM	2004	RVUE	$\tau^+ \to \pi^+\pi^+\pi^-\nu_\tau$
580 ±41	90k	SALVINI	2004	OBLX	$\overline{p}\, p \to 2\pi^+ 2\pi^-$
460 ±85	205	4 DRUTSKOY	2002	BELL	$B^{(*)} K^- K^{*0}$
814 ±36 ±13	37k	5 ASNER	2000	CLE2	10.6 $e^+ e^- \to \tau^+\tau^-$, $\tau^- \to \pi^-\pi^0\pi^0\nu_\tau$

$a_1(1260)$ state – isospin parter of ρ

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• • • We do not use the following data for averages, fits, limits, etc. • • •						
410 ±31 ±30		1	AUBERT	2007AU	BABR	10.6 $e^+ e^- \to \rho^0\rho^\pm\pi^\mp\gamma$
520 - 680	6360	2	LINK	2007A	FOCS	$D^0 \to \pi^-\pi^+\pi^-\pi^+$
480 ±20		3	GOMEZ-DUMM	2004	RVUE	$\tau^+ \to \pi^+\pi^+\pi^-\nu_\tau$
580 ±41	90k		SALVINI	2004	OBLX	$\bar{p}\, p \to 2\pi^+ 2\pi^-$
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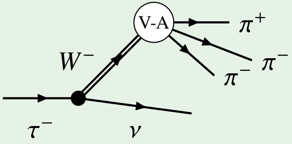
$\tau^- \to \pi^-\pi^+\pi^-\nu$

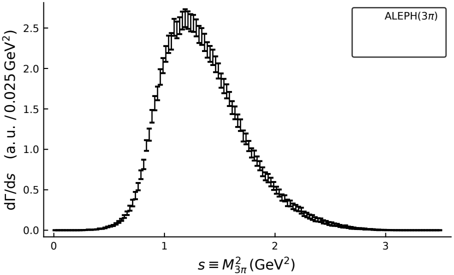


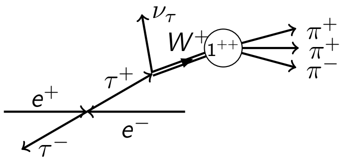
- V-A: Vector (1^{--}) or Axial (1^{++})
- Isospin 1 due to the charge
- Negative G-parity $\Rightarrow$ positive C-parity

$\Rightarrow J^{PC} = 1^{++}$

Fit to ALEPH data

[data from ALEPH, Phys.Rept.421 (2005)]





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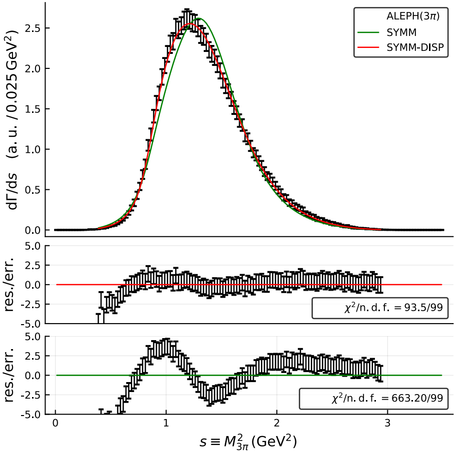


Dispersive model vs Non-dispersive model

- Difference: LH singularities
- The dispersive model fits significantly better

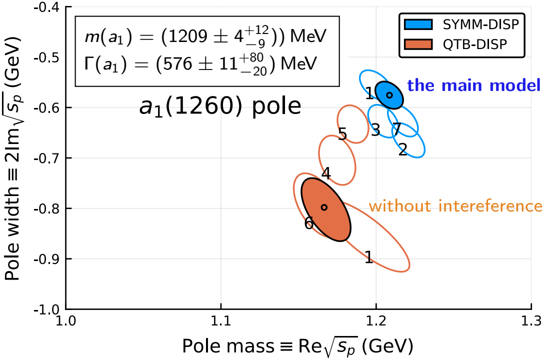
Fit function

$$\chi^2(c, m, g) = (\vec{D} - \vec{M}(c, m, g))^T C_{\text{stat}}^{-1} (\vec{D} - \vec{M}(c, m, g)),$$

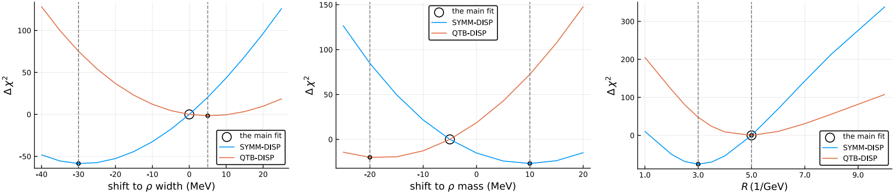
- Stat. cov. matrix is used in the fit
- Syst. cov. matrix – in the bootstrap

First measurement of the $a_1(1260)$ pole position

The result and systematic studies [MM (JPAC), PRD98 (2018), 096021]



#	Fit studies
1	$s < 2\ \mathrm{GeV}^2$
2	$R' = 3\ \mathrm{GeV}^{-1}$
3	$m'_\rho = m_\rho + 10$ MeV
4	$m'_\rho = m_\rho - 10$ MeV
5	$m'_\rho = m_\rho - 20$ MeV
6	$\Gamma'_\rho = \Gamma_\rho + 5$ MeV
7	$\Gamma'_\rho = \Gamma_\rho - 30$ MeV



Summary

Meson spectroscopy

- Using hadronic scattering as QCD excitation laboratory.
- Mapping gluonic degrees of freedom to structures of excited states is an essential test of QCD.
- Non-perturmative methods are required

Resent impact of JPAC to light meson spectroscopy

Extensive analyses and extraction of resonance poles:

- Tensor states: $a_2(1320)$ and $a_2(1700)$
- Establishing single exotic $\pi_1(1600)$
- Ground axial state $a_1(1260)$

JPAC effort

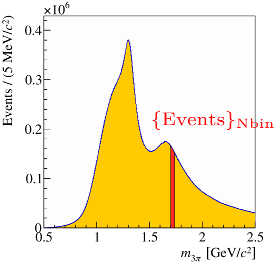
- >50 research papers in PRD, PLB, PRL, EJPC (>10 in 2018)
- >100 invited talks and seminars
- Collaboration with GlueX, CLAS12, COMPASS, MAMI, BaBar, LHCb,...
- Summer Schools on Reaction Theory (2015, 2017)

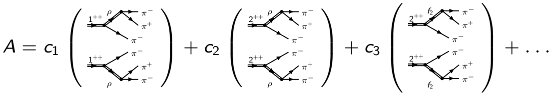
Thank you

Three-particles PW technique

COMPASS@PWA

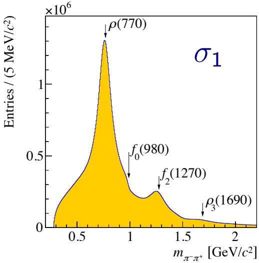
Model: a sum of 88* partial waves

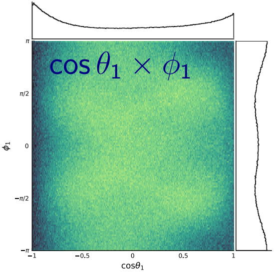


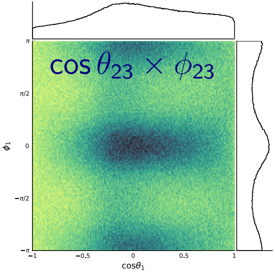


Likelihood Fit: a product of probability per event

$$L = \prod_{e=1}^{\text{Nevents}} \frac{A(\tau_e)}{\int \mathrm{d}\tau_e A(\tau_e)}, \quad \tau = (\sigma_1, \cos\theta_1, \phi_1, \cos\theta_{23}, \phi_{23})$$







Tour to the complex plane

[MM (JPAC), PRD98 (2018), 096021]

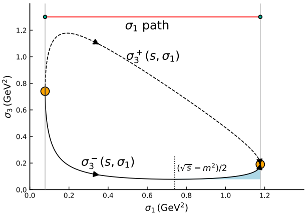
Analytical continuation

$$|t_{I}^{-1}(s)| = \left| \frac{m^2 - s}{g^2} - i\left(\frac{\tilde{\rho}(s)}{2} + \rho(s)\right)\right|.$$

- Analytical continuation of $\rho(s)$: integral over the Dalitz plot for the complex energy

$$\rho(s) = \sum_{\lambda} \int \mathrm{d}\Phi_3 \left| f_\rho(\sigma_1) d_{\lambda 0}(\theta_{23}) - f_\rho(\sigma_3) d_{\lambda 0}(\hat{\theta}_3 + \theta_{12}) \right|^2$$

- Analytic contuation of ρ-meson decay amplitude f_ρ
  - Breit-Wigner amplitude with the dynamic width
  - P-wave Blatt-Weisskopf factors



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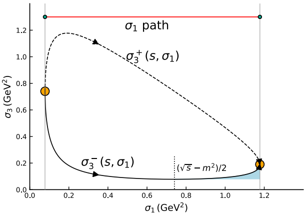

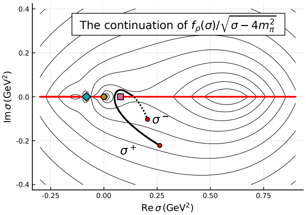

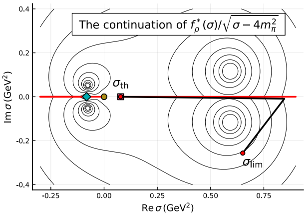

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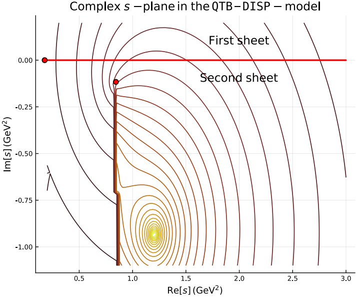

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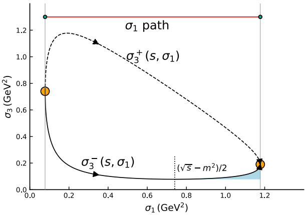

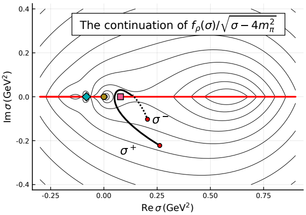

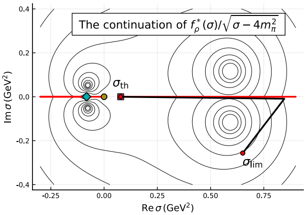

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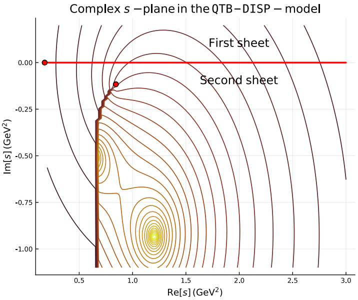

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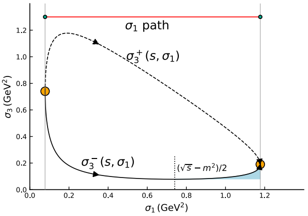

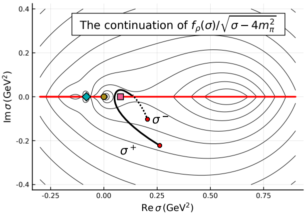

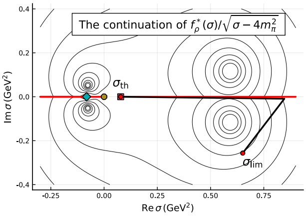

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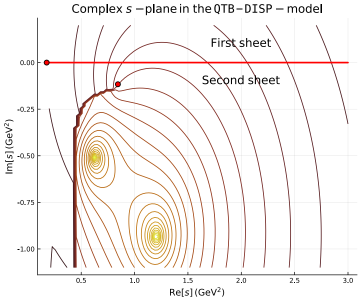

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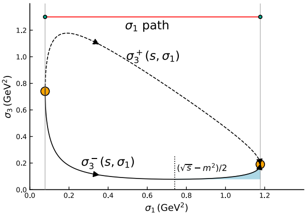

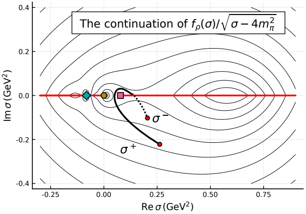

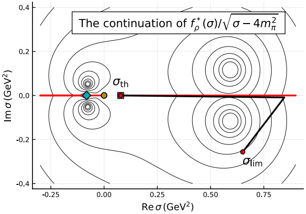

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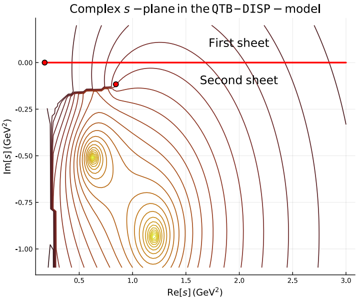

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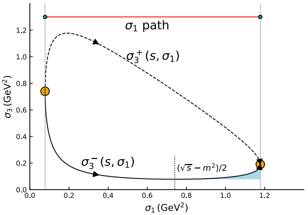
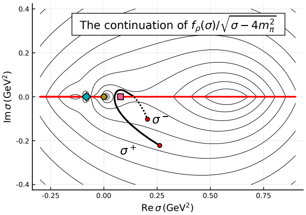
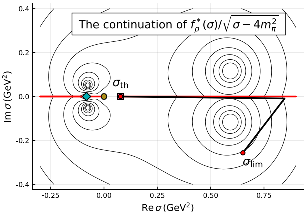




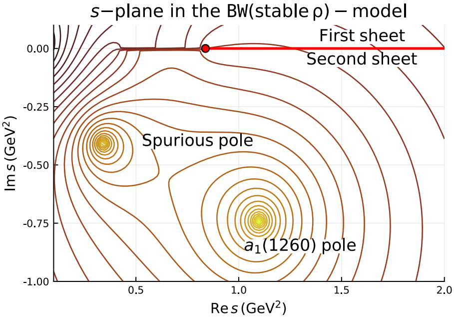


The spurious pole in the Breit-Wigner model

Energy dependent width, stable particles

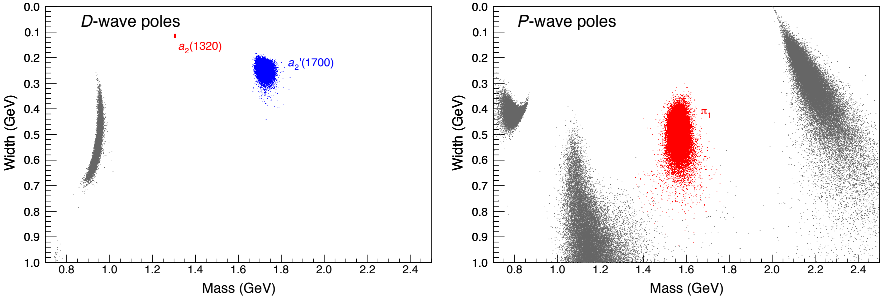
$$t(s) = \frac{1}{m^2 - s - im\Gamma(s)}, \quad \Gamma(s) = \Gamma_0 \frac{p(s)}{p(m^2)} \frac{m}{\sqrt{s}}, \quad p(s) = \frac{\sqrt{(s-(m_1+m_2)^2)(s-(m_1-m_2)^2)}}{2\sqrt{s}}.$$

Example: $m_1 = 140\,\text{MeV}$, $m_2 = 770\,\text{MeV}$, $m = 1.26\,\text{GeV}$, $\Gamma_0 = 0.5\,\text{GeV}$



Live demo

Bootstrap: stability of the poles



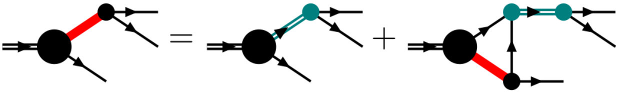
- Statistical bands are obtained by $50k$ bootstrap samples

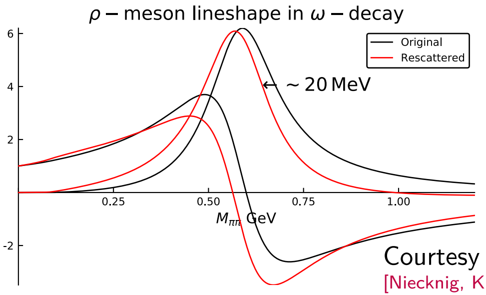
Subchannel dynamics

Khuri-Treiman equations

Consistency equations for the isobar lineshape

- Governed by two-body unitarity
- Model: only RHC for the isobar amplitude
- Uses Analyticity / Cauchy theorem / Omnès trick





KT analysis of $\omega \to 3\pi$

- $\rho \to \rho$ rescattering
- Solved by iteration

Courtesy of Tobias Isken

[Niecknig, Kubis, JHEP 1510 (2015) 142]