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## ABSTRACT

Theoretical issues in baryon spectroscopy are discussed within the context of the constituent quark model. We argue that real progress in this field can come only by considering the spectrum, strong decays, and electromagnetic couplings of the states within a unified framework.

**1. Introduction***1.1 Current theoretical issues*

The broad features of the baryon spectrum have long been attributed to the exchange of a single gluon between two quarks at small separations; recent work [1] has questioned this, and attributed some or all of the contact interaction to pseudoscalar-boson exchange. We will discuss the evidence for the one-gluon exchange mechanism coming from the tensor interaction, and how this relates to the apparent lack of spin-orbit interactions in baryons. The theoretical case for the ‘missing’ baryons will be presented, as will a discussion of the role played by relativity in the calculation of the spectrum and electromagnetic transitions between baryon states. Problematic states such as the Roper resonance  $N_{\frac{1}{2}}^{+}(1440)$ ,  $\Delta_{\frac{3}{2}}^{+}(1600)$ , and  $\Lambda_{\frac{1}{2}}^{-}(1405)$  will be examined. A case will be made for strong effects on the spectrum of the states and their strong and electromagnetic decays of decay channel couplings and higher Fock-space components in the wavefunctions.

*1.2 Baryon spectroscopy and the constituent quark model*

There are around 27 well established nonstrange baryon states discovered primarily in  $\pi N$  elastic and inelastic scattering, and many one and two-star candidates [2]. The corresponding number for strangeness  $-1$  states is around 17, seen in more difficult  $\bar{K}N$  elastic and inelastic scattering experiments and in production. Although it *is not* QCD, the constituent quark model (CQM) adequately describes the systematics of the spectrum of these states, as well as their electromagnetic and strong transitions. Promising *ab initio* approaches to QCD such as lattice gauge theory are likely to have problems with quite well understood states like the  $N_{\frac{1}{2}}^{+}(1710)$ , the third state with nucleon quantum numbers, for some time to come. The CQM is therefore useful to have in our bag of tricks, but can calculations within this scheme be systematically improved? We will argue that this is the case.

Much of the success of the CQM can be attributed to the choice of degrees of freedom, which is three constituent quarks. These are dressed valence quarks with finite spatial extent and masses of around 200–300 MeV for the light quarks, and 150

MeV heavier for the strange quark. The gluon fields are taken to be in their adiabatic ground state, which generates a potential in which the quarks move. This is effectively linear at large separation and can be written as the sum  $V_{\text{string}} = \sum_i b l_i + C$  of the energies of strings connecting the quarks to a string junction point, where  $b$  is the meson string tension. The short-distance spin-dependent and Coulomb potentials are conventionally ascribed to one-gluon exchange between quarks at small separations.

A model such as this is obviously only valid for ‘soft’ physics, and where gluonic excitation is unlikely. It also ignores possibly large mass shifts from coupling to higher components of Fock space such as  $qqq(\bar{q}q)$ . It is not, however, necessarily nonrelativistic. The relativized model [3, 4] treats mesons and baryons on the same footing using a Hamiltonian made up of the relativistic kinetic energy of the quarks and a potential. The latter tends in the nonrelativistic limit  $p_i/m_i \rightarrow 0$  to a confining potential which is the sum of  $V_{\text{string}}$  and associated spin-orbit potentials from Thomas precession, and a one-gluon-exchange potential which contains a Coulomb interaction, hyperfine contact and tensor interactions, and the associated spin-orbit potential. Away from the  $p_i/m_i \rightarrow 0$  limit the potentials are allowed to depend on the momentum of the quarks in a simple way, and the potentials undergo smearing of the interquark coordinate due to relativistic effects and the finite size of the effective quarks in a cut-off field theory. The strong coupling constant also runs, saturating to approximately 0.6 near  $Q^2 = 0$ .

Various calculations within similar models have gone beyond the questionable approximation of first-order wavefunction perturbation theory; see, for example, Refs. [5] and [6]. In Refs. [4] and [5] the Hamiltonian is calculated in a large coupled harmonic-oscillator basis and diagonalized to find the energies and wavefunctions.

### 1.3 Decay models

Models which describe the spectrum in the absence of a strong decay model are of limited usefulness, as the predicted spectrum must be compared to the results of an experiment which forms excited baryons in a given input and exit channel. Poor or absent evidence for predicted baryon states with weak couplings to given formation and decay channels must result from experiments which form excited baryons using these channels.

A systematic analysis of the coupling to strong decay channels was performed with configuration-mixed nonrelativistic wavefunctions in the elementary-meson emission model, where point-like mesons are allowed to couple to the quarks, by Koniuk and Isgur [7]. The pair-creation model [8] of Lyouanc *et al.* quite successfully describes the strong decays as proceeding through the creation of a color-singlet  $q\bar{q}$  pair with vacuum ( ${}^3P_0$ ) quantum numbers, which necessitates that their spins are coupled to  $S = 1$  and that they are in a relative  $P$ -wave. The resulting decay operator is of the form

$$T = -3\gamma \sum_{i,j} \int d\mathbf{p}_i d\mathbf{p}_j \delta(\mathbf{p}_i + \mathbf{p}_j) C_{ij} F_{ij}$$

$$\sum_m C(1, 1, m, -m; 0, 0) \chi_{1m}^{ij} \mathcal{Y}_{1-m}(\mathbf{p}_i - \mathbf{p}_j) b_i^\dagger(\mathbf{p}_i) d_j^\dagger(\mathbf{p}_j), \quad (1)$$

where  $C_{ij}$  is a color-singlet wavefunction,  $F_{ij}$  is a flavor-singlet wavefunction,  $\chi$  is a triplet spin wavefunction, and  $\mathcal{Y}$  is a solid harmonic. This model has been applied to baryon decays with nonrelativistic-model wavefunctions [8, 9, 10] and to meson decays [11] and baryon decays [12, 13] using relativized-model wavefunctions. The result is an adequate description of the strong decays with essentially a single parameter, the strength constant  $\gamma$ .

## 2. Baryon Spectrum and Decays

As an example of how models of this type describe the masses and properties of baryons, we consider the low-lying negative-parity nonstrange excited states. The nonrelativistic model works well here, and all of the states predicted by the model are seen. The predictions of the relativized model of Ref. [4] are shown in Figure 1, along with the Particle Data Group quoted range of masses for resonances of this type. The coarse features of the spectrum are due to the contact splitting, which adds to the energy of a group of five states, and subtracts from that of the two others.

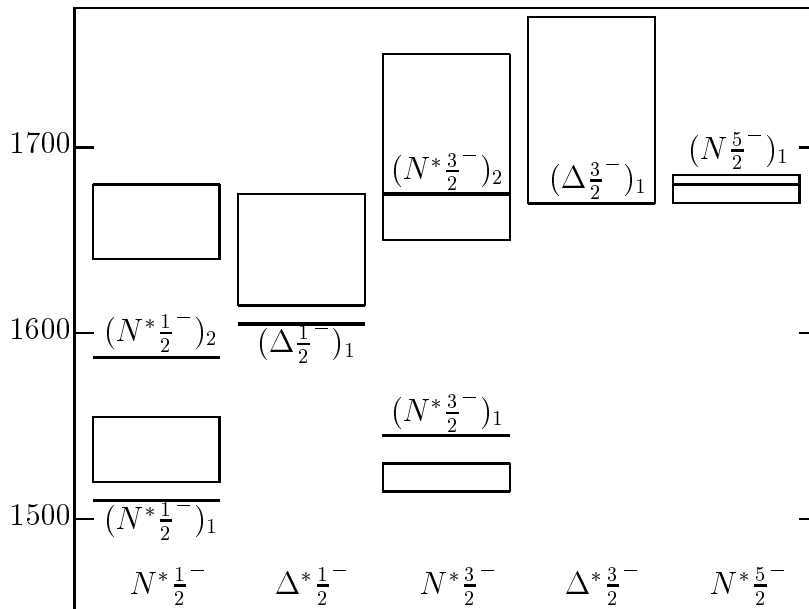


Figure 1: low-lying negative-parity excited nonstrange baryons. Bars are the model predictions shifted by +50 MeV, boxes give the Particle Data Group range of masses.

These groups are further split by the tensor and spin-orbit interactions [for example the  $(\Delta_{\frac{3}{2}}^{-})_1$  and  $(\Delta_{\frac{1}{2}}^{-})_1$  are split by the spin-orbit interaction]. Our predictions are shown shifted by +50 MeV to roughly reproduce the band center of mass, which we predict (this is fit in the nonrelativistic model). In the relativized model these states

are predominantly in the  $N = 1$  band of the harmonic-oscillator basis, but are not restricted to this band since the potential is linear plus Coulomb.

### 2.1 The tensor interaction

The  ${}^3P_0$  model gives the absolute  $N\pi$  coupling strengths of these states with reasonable accuracy given its simplicity, and fits well the relative strengths within a given partial wave. As an example of another strong coupling, consider the  $N\eta$  decays of the two  $S_{11}$  states  $N\frac{1}{2}^-$  (1535) and  $N\frac{1}{2}^-$  (1650). The lighter state has little phase space to decay to  $N\eta$  but has a 30-55% branching ratio to this channel, whereas the upper state has considerable phase space but seldom decays this way. Tensor mixing between the two  $P$ -wave basis states with total quark spin of  $\frac{1}{2}$  and  $\frac{3}{2}$  causes a cancellation of the  $N\eta$  amplitude in the  $N\frac{1}{2}^-$  (1650) and a significant enhancement in the  $N\frac{1}{2}^-$  (1535). This mechanism holds in the  $SU(6)_W$  decay model, as well as in the elementary-meson emission model [7] and the  ${}^3P_0$  model [9, 13]. Any model which attempts to explain the splittings in the spectrum must also explain these tensor mixings (and mass splittings).

The evidence for tensor splittings in the spectrum is inconclusive, and we would argue that a better place to look for evidence of a tensor force is in the effects of tensor mixings on the strong decays (as above) or in the electromagnetic couplings. As an example of the latter,  $D$ -wave mixing into the predominantly  $S$ -wave nucleon and  $\Delta(1232)$  wavefunctions due to the tensor interaction causes an E2 amplitude in the predominantly M1 (spin-flip)  $\gamma N \rightarrow \Delta$  transition. In principle a measurement of the ratio E2/M1 at the photon point can indirectly measure this tensor mixing; in practice the extraction of this small quantity (for example the nonrelativistic model [14] predicts a ratio of -0.4%) from the photoproduction data is complicated. Modern analyses [15] of the  $\Delta$ -region data give -1.0 to -3.0%. CEBAF experiments will examine the  $Q^2$  dependence of this ratio and the longitudinal multipole ratio C2/M1 in electroproduction, and re-examine the photon point.

### 2.2 Spin-orbit interactions

Are spin-orbit interactions present in baryons, with a strength commensurate with the vector-exchange contact interaction and the confining interaction? These interactions are conventionally [16] described as too large and left out. A partial cancellation of the vector and scalar spin-orbit interactions occurs, but not for the three-body spin-orbit interactions. As above, there are some spin-orbit splittings in the spectrum, so leaving these interactions out is unsatisfactory.

In the relativized model the contact interaction is evaluated nonperturbatively, and the usual  $\delta^3(\vec{r}_{ij})$  form is smeared out by relativistic effects and the finite size of the constituent quarks. The perturbative evaluation of the  $\delta^3(\vec{r}_{ij})$  interaction in the nonrelativistic model underestimates its strength; in the relativized model for the same contact splitting we require a value of  $\alpha_s = 0.6$ , about three times smaller than that required in the nonrelativistic model. The result is a smaller associated spin-orbit interaction. We have also used some of our freedom to fit the momentum dependence of the potentials to further suppress the spin-orbit interactions relative to the contact interaction. These effects, along with a partial cancellation of the vector and scalar

spin-orbit terms, adequately reduce the size of the spin-orbit interactions.

In these models the splitting between  $\Lambda_{\frac{3}{2}}^{-}(1520)$  and  $\Lambda_{\frac{1}{2}}^{-}(1405)$  can arise only from a spin-orbit interaction, *if* we assume no mass shifts arising from decay-channel couplings [or  $qqq(\bar{q}q)$  configurations]. In the relativized model there is very little splitting of these two states from the spin-orbit interaction. The presence of the nearby threshold for  $N\bar{K}$  decay is expected to strongly affect the mass of  $\Lambda_{\frac{1}{2}}^{-}(1405)$  [17]. Obviously *any* model which ignores the effects of decay-channel couplings will not be able to explain this state's mass.

### 2.3 Positive-parity states

Figure 2 shows the relativized-model predictions [4] (shifted by  $-40$  MeV) for those low-lying nonstrange positive-parity states which are predicted by our decay model [12] to couple appreciably to the  $N\pi$  formation channel. These states have wavefunctions which are predominantly made up of  $N = 2$  band harmonic-oscillator basis functions. Note that ten states are therefore ‘missing’ [7, 12] from this band, due to smaller  $N\pi$  couplings than those of nearby states in their partial wave. In the CQM the basic structure of the splittings in the spectrum is usually attributed to the contact interaction and anharmonic terms in the confining potential.

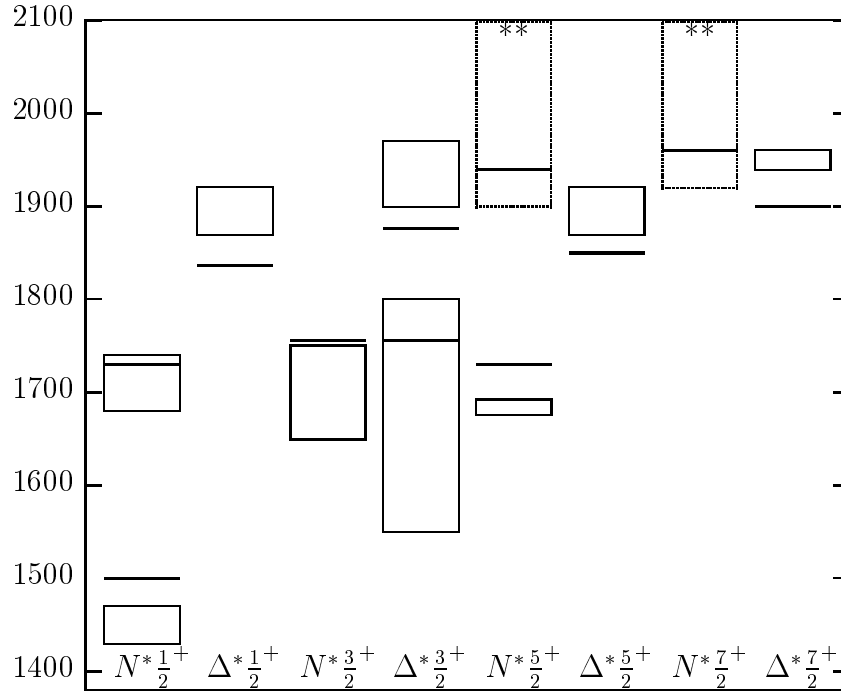


Figure 2: low-lying positive-parity excited nonstrange baryons; caption as in Fig. 1. Only model states which couple appreciably to  $N\pi$  are shown.

The Roper resonance  $N_{\frac{1}{2}}^{+}(1440)$  is 100 MeV lighter than the model predictions. Furthermore [18], within a certain class of models it is impossible to have this state lighter than the  $P$ -wave states of Fig. 1 if we go beyond first order wavefunction perturbation theory. It has been suggested that this state may be a hybrid baryon,

although if this is the case we have to find an additional  $qqq$  state with these quantum numbers in this mass range. The other state in this partial wave,  $N\frac{1}{2}^+(1710)$  fits well into the conventional CQM. Searches have been made for more than one resonance near 1450 MeV, and show a complicated pole structure but only one resonant state with a large  $N\pi$  width in this region, which agrees with our  ${}^3P_0$  decay model [12] predictions. This large width signals that it is possible that the model mass should be shifted downwards by substantial decay channel couplings.

Another problematic state (see Fig. 2) has been the second  $P_{33}$  resonance  $\Delta\frac{3}{2}^+(1600)$ , which like the Roper resonance is too light in comparison to model predictions. The  $N\pi$  elastic scattering analyses yield a mass of [2] 1500-1600 MeV, with a substantial width. However, in the coupled-channel analysis of Manley and Saleski [19], which examines  $N\pi \rightarrow N\pi\pi$  through various quasi-two-body channels, the  $N\pi$  branch for this state was found to be only about 12%, with the bulk of the total width being to the  $p$ -wave  $\Delta\pi$  channel. The mass is found to be  $1706\pm 10$  MeV, within 100 MeV of conventional CQM predictions, once again with a large total width.

#### 2.4 Higher states

Models which expand the states in a large harmonic-oscillator basis have the advantage that they are able to describe states which lie predominantly in bands above the  $N = 2$  oscillator band, such as the more highly-excited negative parity states discovered in  $N\pi$  elastic scattering between roughly 1900 and 2300 MeV. In this region it is crucial that the model also predict which states couple to the formation channel, as the multiplicity of quark model states increases rapidly with energy. There are many broad overlapping states, which must mix through their decay channels, so the narrow resonance approximation implicit in our spectrum and decay calculations is problematic, as is the assumption that the glue remains in its ground state. With these caveats, it is interesting that the  ${}^3P_0$  model consistently [12] predicts that the lightest of the states in each  $N\pi$  partial wave has the largest coupling strength, and that the spectrum of heavier states is roughly fit by a model [4] with a linear potential with essentially the same string tension as in the meson model [3].

### 3. Missing positive-parity nonstrange baryons

Models with fewer than three degrees of freedom, such as the quark-diquark model, have fewer excitations and are able to account for most of the positive-parity states seen in  $N\pi$  scattering. As we have seen, there is a natural explanation for the absence of certain  $qqq$  model states in  $N\pi$  elastic scattering, as their coupling strengths are weaker than other nearby (usually lighter) states in their partial wave. Is it possible to resolve these two explanations?

If these states exist, then they necessarily have large widths to quasi-two-body channels such as  $N\rho$ ,  $\Delta\pi$ , *etc.* In their coupled-channel treatment of nonstrange baryon resonances, Manley and Saleski [19] have analysed  $N\pi \rightarrow N\pi$  partial-wave amplitudes together with isobar-model amplitudes for various  $N\pi \rightarrow N\pi\pi$  quasi-two-body channels. One result is that they find a new resonance  $N\frac{3}{2}^+(1880)$  which may

be ascribed to a ‘missing’ member of the  $[70, 0^+]$  multiplet. This state has an  $N\pi$  branching ratio of about 26%, and poles in this partial-wave amplitude at this energy were in fact noticed in the  $N\pi$  elastic scattering analysis of the VPI group [20].

Inelastic  $N\pi$  scattering still involves one small vertex for states which couple weakly to the  $N\pi$  channel. Experiments at CEBAF which will search for these missing states in, for example,  $\gamma p \rightarrow p\pi^+\pi^-$  [21],  $e^-p \rightarrow e^-p\pi\pi$  [22], and  $e^-p \rightarrow e^-p\omega$  [23] will avoid this problem, and so should find many of these states if they really are excitations of three constituent-quark degrees of freedom. The decay amplitudes for the predicted missing states to decay to these quasi-two-body channels have been calculated in the elementary-meson emission model [7, 24] with nonrelativistic wavefunctions, and in the  $^3P_0$  model [9], with relativized-model wavefunctions in Ref. [13]. Their photoproduction amplitudes have also been calculated consistently in the nonrelativistic [7, 25] and relativized models [26]. Such calculations make definite predictions for the channels and partial waves in which the missing states are likely to be found.

#### 4. Relativity in baryon electromagnetic couplings

In order to be internally self-consistent, models of the baryon spectrum must go beyond the nonrelativistic approximation. A model which carries this out has been described above [3, 4]; here the quarks are given relativistic kinetic energies and the momentum dependence of the potentials (which is necessary for a strongly-bound system) is given a simple parameterization. The resulting baryon spectrum is quite similar to that of the nonrelativistic model, although there are substantial relativistic effects in the photocouplings [25, 26].

When calculating electroproduction amplitudes at substantial  $Q^2$ , however, it becomes necessary to work within a relativistic model due to the sizeable boost given to the struck quark. Various groups [27] are working on calculations of these amplitudes using models formulated using light-cone dynamics, and these calculations demonstrate that relativistic effects on these amplitudes are sizeable, even at  $Q^2 = 0$ .

#### 5. Conclusions

It is not enough to explain the mass splittings of some part or all of the baryon spectrum; models must also be applied to the strong and electromagnetic couplings of the states which result from solving for the spectrum. A model cannot be considered successful until it achieves some degree of consistency in all of these areas. Recent calculations have gone a long way towards establishing that constituent quark models are not necessarily nonrelativistic, and have described the spectrum and strong and electromagnetic couplings of low-lying states with a certain degree of consistency. However, none of the models described here takes into account the presumably substantial effects on the spectrum of the states and their strong and electromagnetic decays of decay-channel couplings. It is likely that many of the problems which still exist in the description of the physics of baryons have this as their source. It is also likely that in the next few years results from the experiments which are currently underway and planned at new facilities such as CEBAF will bring about a renaissance

of this field.

## 6. Acknowledgements

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