Baryon spectroscopy in the Quark Model

Collaborators: Danielle Morel (FSU), Philip Page (LANL), Winston Roberts (Jlab/ODU), Sameer Walawalkar (FSU), Mike Pichowsky (Kent State)

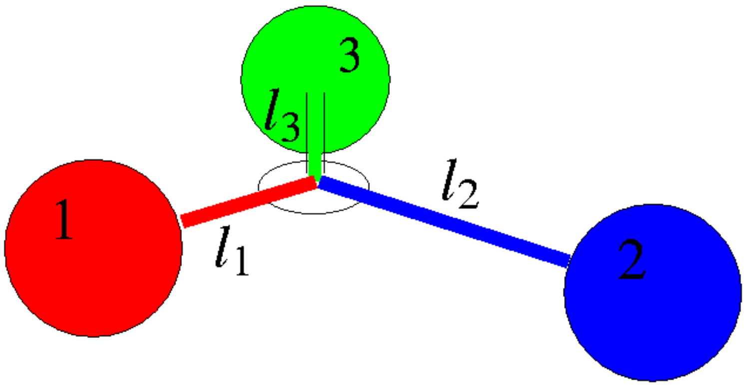
- What is a quark model?
  - Effective degrees of freedom, properties
- How should we treat confinement?
  - Recent guidance from lattice QCD
- What are the q-q "residual" interactions
  - One-gluon, one-boson, instanton-induced?
- What should we do about qqq($q\bar{q}$) states?
  - Self energies, mixings
- Conclusions

The Cork Model

Up Charm Top

Down Strange Bottom

Effective degrees of freedom



- Constituent quarks?
  - Have effective masses
  - Not point-like (have EM and strong form factors)
- Di-quark cluster + quark?
  - Two degrees of freedom at low energy ➔ fewer excited states
- Collective excitations of string-like model?
  - Algebra-based models: radial excitations from rotations and vibrations of strings ➔ more excited states
- Light quarks in a bag, pions coupled to surface?
  - Difficult to describe highly excited states

⇨ **Constituent quarks + flux tubes**

What are their properties?

- Dynamically generated constituent masses, which can run with Q^2
  - Evidence from lattice that at low Q^2, $m \approx \Lambda_{QCD}$
  - Similar results from Dyson-Schwinger Bethe-Salpeter studies of hadrons, quark propagators
- Models with $K=(p^2+m^2)^{\frac{1}{2}}$ could use $0<m_{ud}<250$ MeV
  - isospin-violating mass splittings ⇨ $K_d-K_u \approx$ 5-10 MeV
  - From current algebra $\Delta m = m_d-m_u \approx$ 5-10 MeV
  - $K_d-K_u = (p^2+[m_u+\Delta m]^2)^{\frac{1}{2}} - (p^2+m_u^2)^{\frac{1}{2}} \approx (m_u/K_u)\Delta m$ if $\Delta m << K_u$
- So $m_u \approx K_u \approx \Lambda_{QCD} \approx 200$ MeV !
  - Strange quark has $m_s \approx m_u + 150 - 200$ MeV

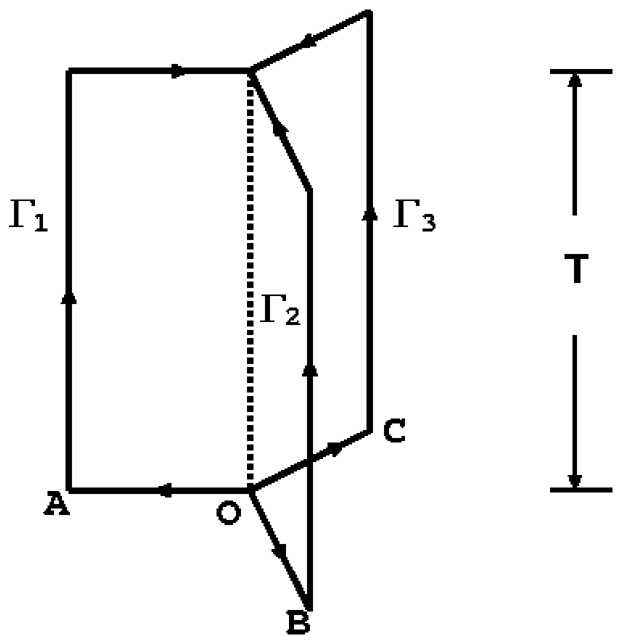
…What are their properties?

- Effective sizes, form factors
  - Strong form factors: Gaussian (convenience), monopole
    - Make finite contact interactions $\propto \delta^3(\mathbf{r}_i - \mathbf{r}_j)$
    - Heavier quarks more point-like
  - EM form factors required to fit nucleon G_E, G_M
    - Even in relativistic (light-cone) calculations
    - E.g. $F_1^i(Q^2)=e_i/(1+Q^2/\Lambda_1^2)$, $F_2^i(Q^2)=\kappa_i/(1+Q^2/\Lambda_2^2)^2$
    - The κ_i should be environment sensitive
      - Lattice (Leinweber and Voloshyn), BM loops

      ⇨ See talk by P. Gonzalez, Tues.@3:00 !
  - Strong and EM sizes should be similar

How should we treat confinement?

- Quenched lattice measurement of QQQ potential
- Takahashi, Matsufuru, Nemoto and Suganuma, PRL**86** (2001) 18.
- Measure potential with 3Q-Wilson loop (static quarks) for 0<t<T
- Also fit $Q\bar{Q}$ potential to compare σ and Coulomb terms



… How should we treat confinement?

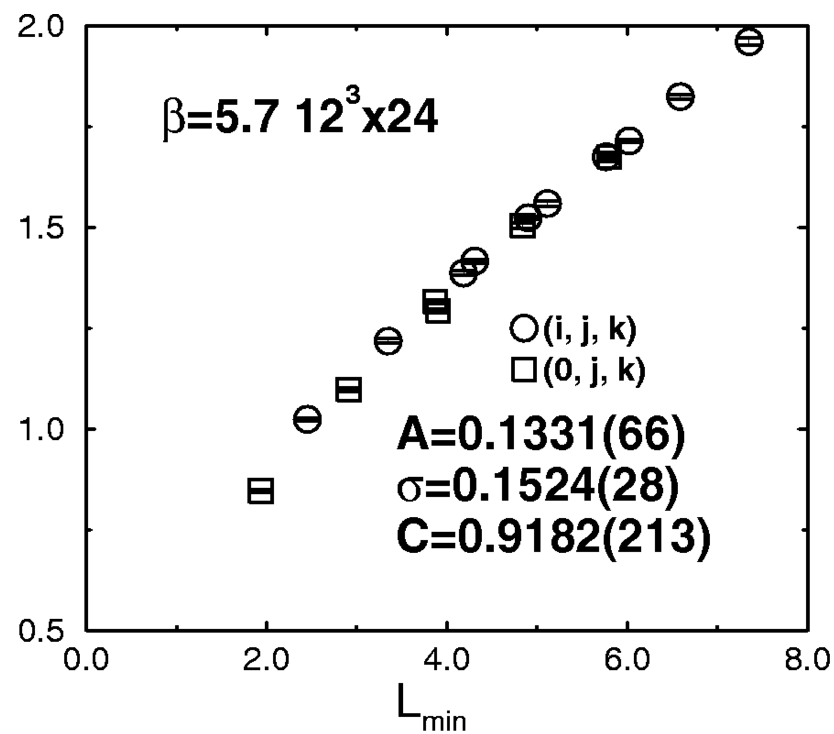
- Fit 16 QQQ configurations to

$$V_{3Q} = -A_{3Q}\sum_{i<j}\frac{1}{|\mathbf{r}_i - \mathbf{r}_j|} + \sigma_{3Q}L_{\min} + C_{3Q}$$

- Lmin = min. length Y-shaped string _
- 3Q, QQ string tensions similar
- Coulomb terms in OGE ratio ½

TABLE I. The coefficients in Eq. (6) for the $3Q$ potential and those in Eq. (5) for the Q-$\bar{Q}$ potential in the lattice unit.

	σ	A	C
$3Q$	0.1524(28)	0.1331(66)	0.9182(213)
Q-$\bar{Q}$	0.1629(47)	0.2793(116)	0.6203(161)



- σ is in lattice units a^{-2}
- Meson string tension 0.89 GeV/fm (a=0.19 fm)

… How should we treat confinement?

- Also tried fit to function

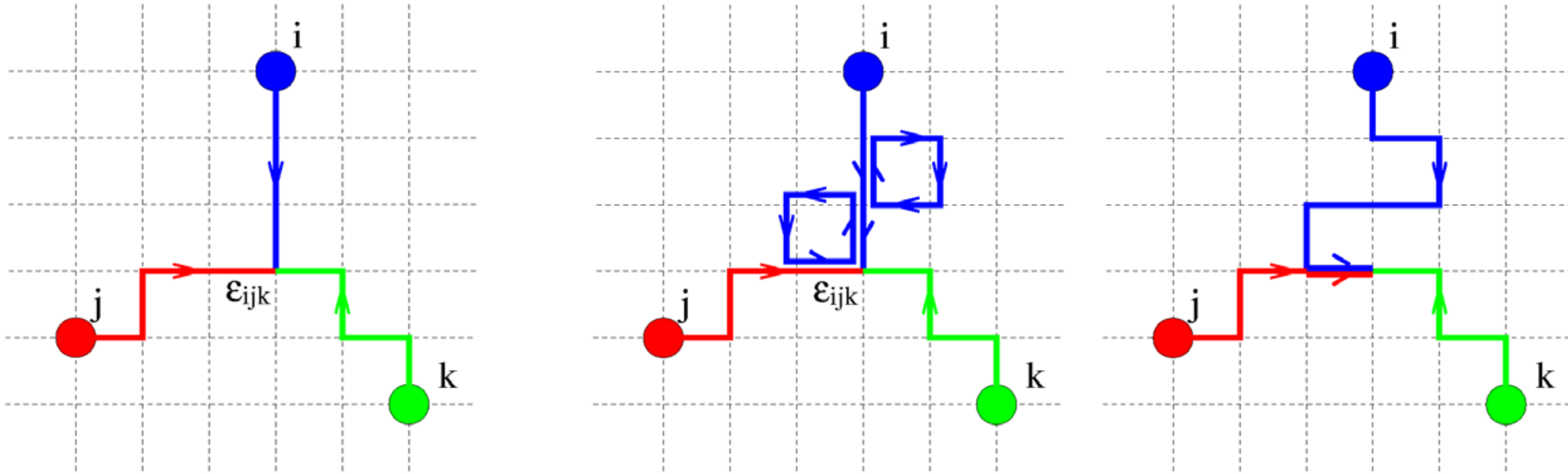
$$V_{3Q} = -A_\Delta \sum_{i<j} \frac{1}{|\mathbf{r}_i - \mathbf{r}_j|} + \sigma_\Delta \sum_{i<j} |\mathbf{r}_i - \mathbf{r}_j| + C_\Delta$$

- Fit worse: χ^2 per d.f. 3.8 ⇨10.1
- Result is a reduced string tension $\sigma_\Delta = 0.53\ \sigma$
  - Simply a geometrical factor
  - Perimeter P satisfies $1/2 < L_{min}/P < 1/(3)^{1/2} = 0.58$
  - Accidentally close to ratio $\langle \Lambda_i . \Lambda_j \rangle_{baryons}$ / $\langle \Lambda_i^* . \Lambda_j \rangle_{mesons}$ but confinement is **not** (colored) vector exchange!

⇨string-like potential good for QQQ baryons!

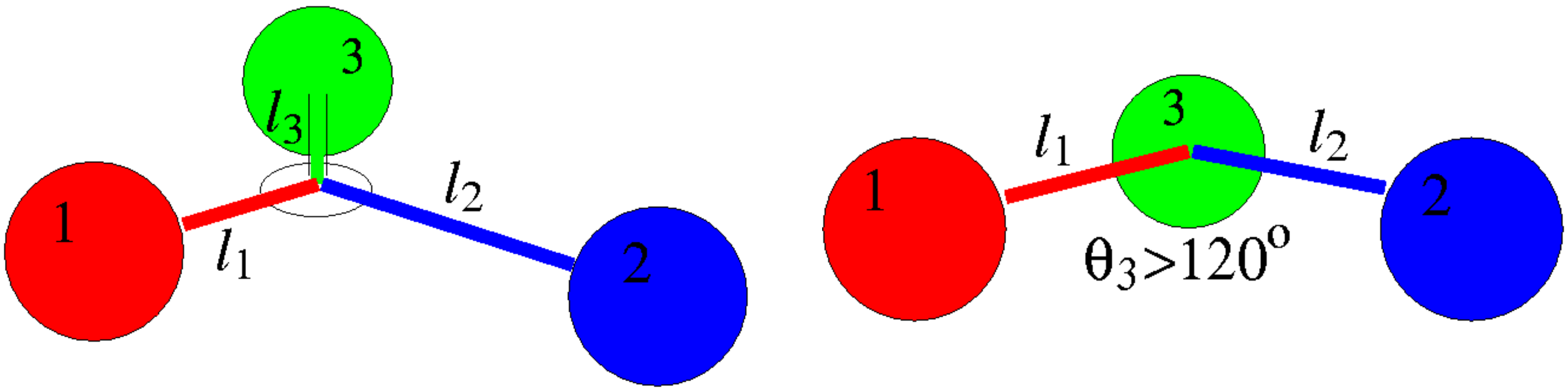
⇨Assume it is good for qqq baryons

Flux-tube model



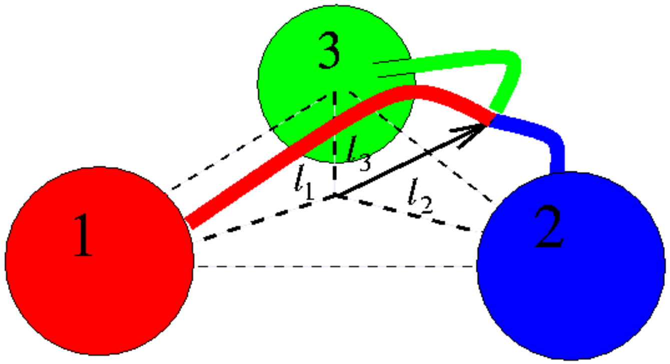
- Based on strong-coupling lattice QCD
  - –Color fields confined to narrow tubes, energy ∝ length
  - –Junction, to maintain global color gauge invariance
  - –Plaquette operator from lattice action:
    - –Moves tubes transverse to their original direction
    - –Moves junction, but leaves string excited

Model confining interaction



- Flux tubes, combined with adiabatic approx.
  - confining interaction: minimum length string
  - $V_B(\mathbf{r}_1,\mathbf{r}_2,\mathbf{r}_3)=\sigma(l_1+l_2+l_3)=\sigma L_{min}$
    - note σ is meson string tension
    - linear at large q-junction separations
  - Strings should be allowed to be dynamical
    - See talk by Philip Page, Thus.@11:15 !

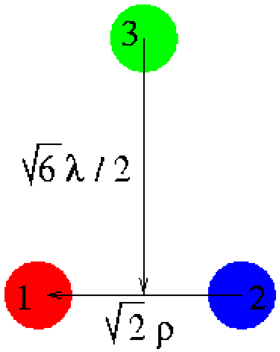
…Model confining interaction

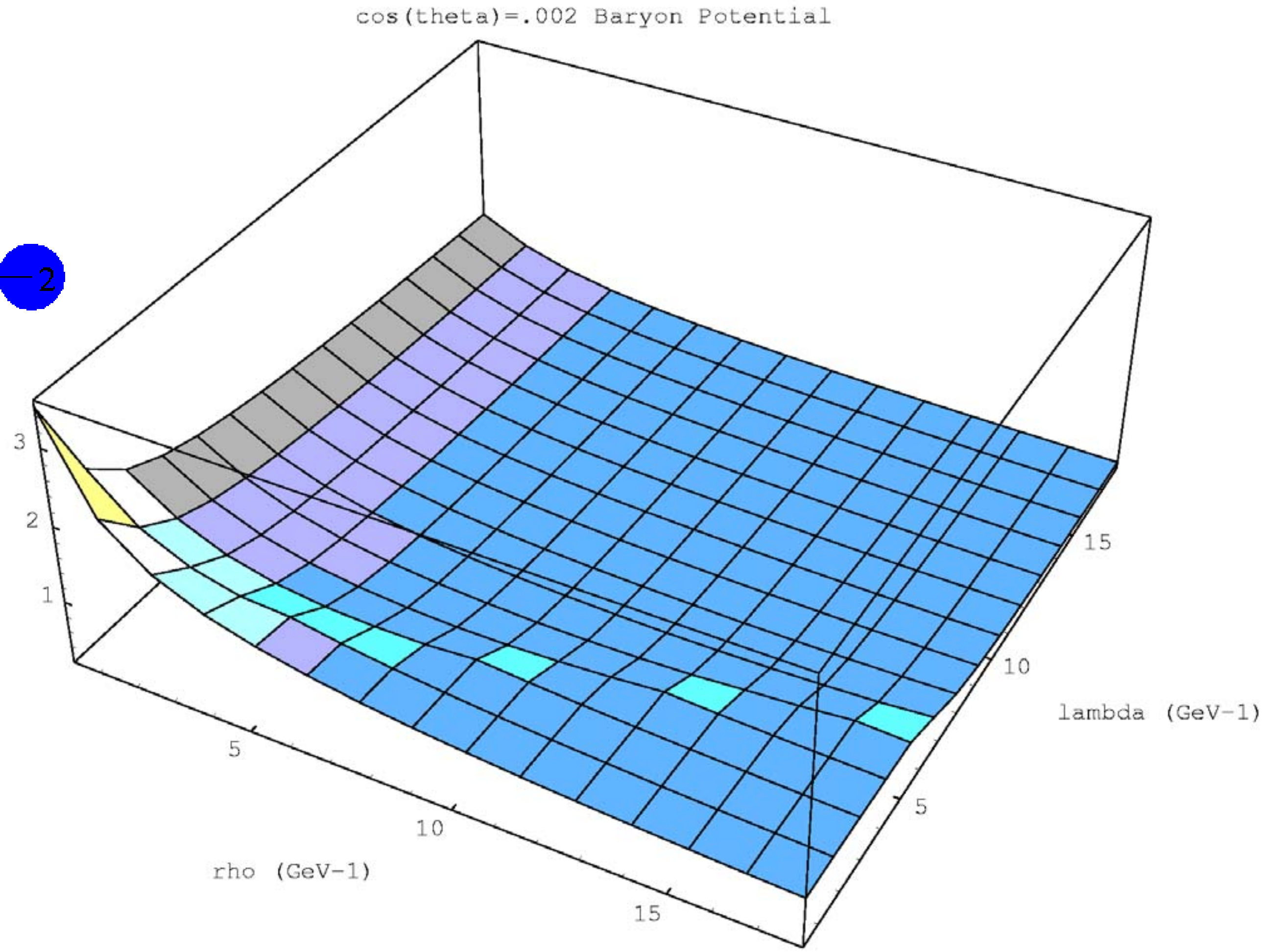


- Fix quark positions $\mathbf{r}_i$, allow flux tubes to move
  - Junction moves relative to its equilibrium position
  - Strings move transverse to their equilibrium directions
- Ground state of string defines adiabatic potential
  - $V_B(\mathbf{r}_1,\mathbf{r}_2,\mathbf{r}_3)=\sigma(l_1+l_2+l_3)=\sigma L_{min}$, plus zero point motion

…Adiabatic potentials

- plot of $V_B - b\sum_i l_i$ as a function of ρ, λ, with $cos(\theta_{\rho\lambda}) = 0$:





What are their residual interactions?

- Ground-state spectrum suggests flavor-dependent short-range (contact) interactions
- One-gluon exchange: good fit to ground states with (color-magnetic dipole-dipole), e.g. Σ-Λ ⇨DeRujula,Georgi,Glashow

$$M = \sum_{i=1}^{3} m_i + \frac{2\alpha_s}{3}\frac{8\pi}{3}\langle\delta^3(\mathbf{r})\rangle \sum_{i<j=1}^{3} \frac{\mathbf{S}_i \cdot \mathbf{S}_j}{m_i m_j}$$

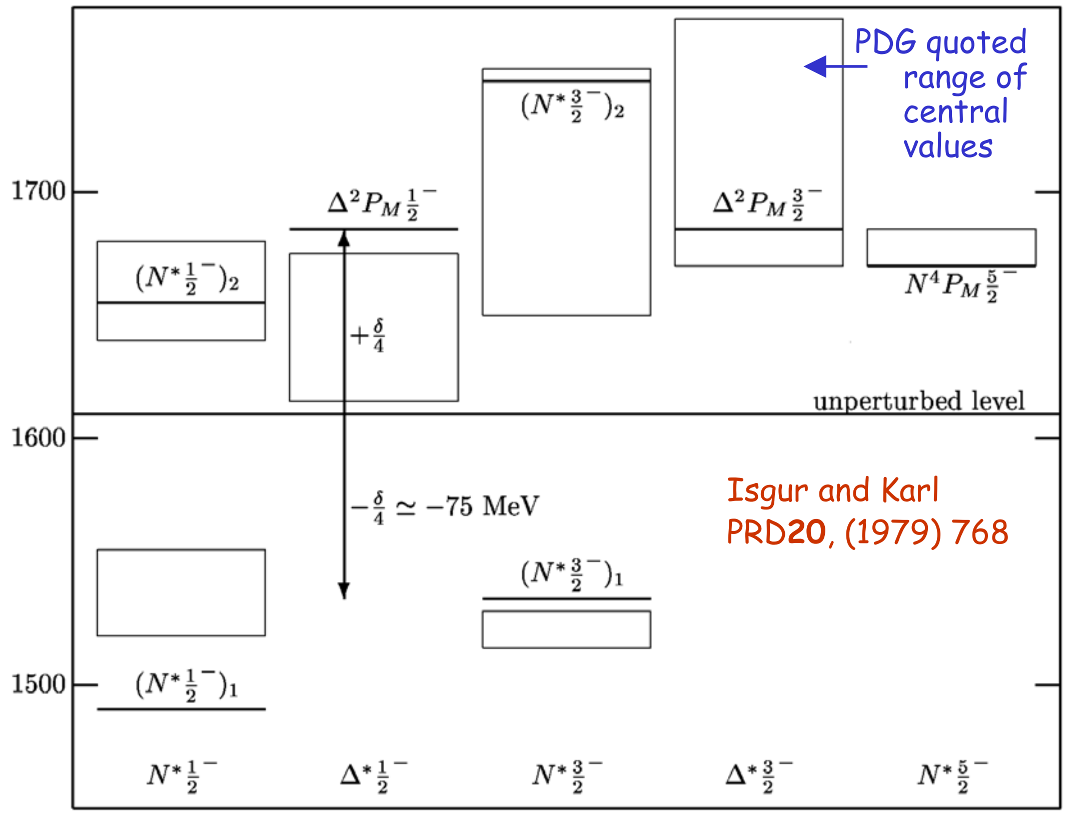
  - Gives, e.g. $(m_\Sigma - m_\Lambda)/(m_\Delta - m_N) = 2/3\ (1 - m_{u,d}/m_s) \approx 2/3\ (0.6) = 0.27$, expt.⇨(1193-1116)/(1232-939)=0.26
- Explains regularities in meson spectrum (e.g. evolution of vector-pseudoscalar splitting with quark mass)
  - Unclear why this should work for light quarks…
- Taken at face value predicts tensor interaction

$$H^{ij}_{\rm hyp} = \frac{2\alpha_s}{3m_i m_j}\left\{\frac{8\pi}{3}\mathbf{S}_i \cdot \mathbf{S}_j \delta^3(\mathbf{r}_{ij}) + \frac{1}{r_{ij}^3}\left[\frac{3(\mathbf{S}_i \cdot \mathbf{r}_{ij})(\mathbf{S}_j \cdot \mathbf{r}_{ij})}{r_{ij}^2} - \mathbf{S}_i \cdot \mathbf{S}_j\right]\right\}$$

- And spin-orbit interactions, at a level not present in analyses

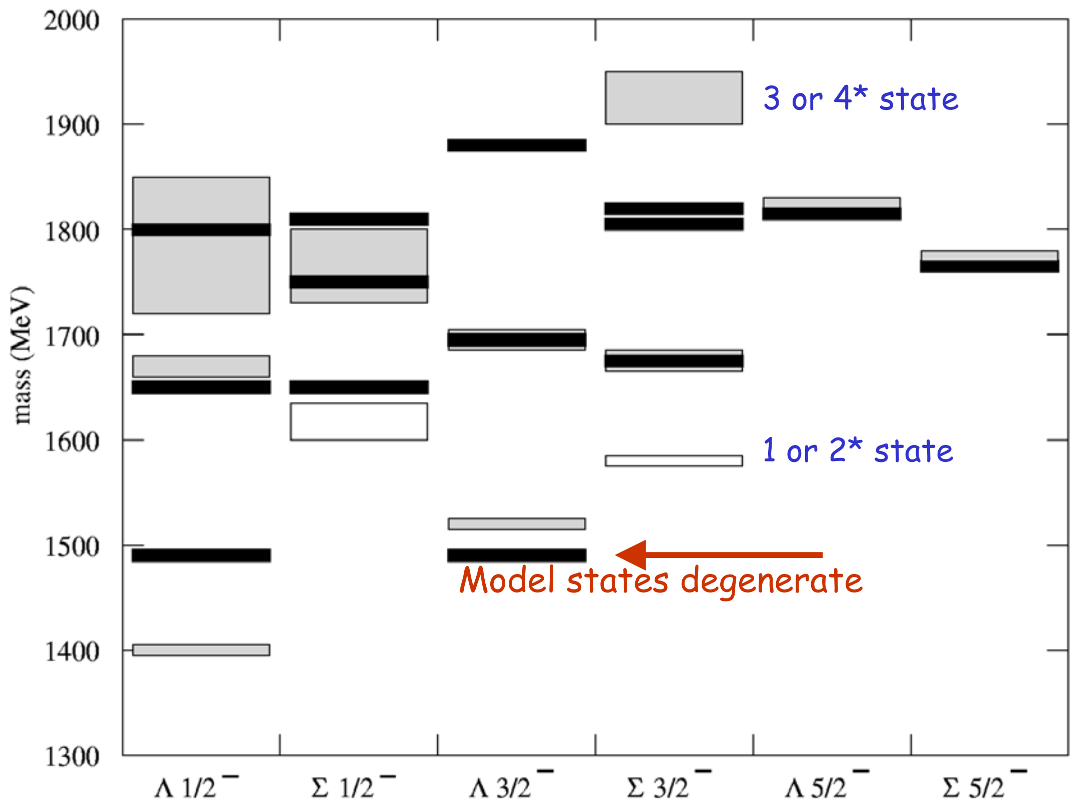
Residual interactions…

- Contact splitting active in L=1 excited states
- Characteristic splitting is $(m_\Delta - m_N)/2$
- Add consistent tensor interaction
- No strong evidence for tensor from spectrum
- Best evidence from decays, $S_{11}(1535) \rightarrow N\eta$



Residual interactions…

- Also applied to L=1 strange baryons
- Degenerate lightest $\Lambda 1/2^-$ and $\Lambda 3/2^-$
- data sparse, analyses even less certain
- Consistent (OGE plus confinement) spin-orbit cannot explain $\Lambda 3/2^-(1520)$-$\Lambda 1/2^-(1405)$



Residual interactions…

- Model has been applied to **all** baryons (Isgur, Karl,…)
- Variational calculation in large HO basis (SC, N. Isgur)
  - String confinement, plus associated spin-orbit
  - Include OGE Coulomb, contact, tensor, spin-orbit
  - Relativistic KE, relativistic corrections in potentials, e.g.

$$\left(\frac{m_i m_j}{E_i E_j}\right)^{\frac{1}{2}+\epsilon_{\rm cont}} \frac{8\pi}{3}\alpha_s(r_{ij})\frac{2}{3}\frac{\mathbf{S}_i\cdot\mathbf{S}_j}{m_i m_j}\left[\frac{\sigma_{ij}^3}{\pi^{\frac{3}{2}}}e^{-\sigma_{ij}^2 r_{ij}^2}\right]\left(\frac{m_i m_j}{E_i E_j}\right)^{\frac{1}{2}+\epsilon_{\rm cont}}$$

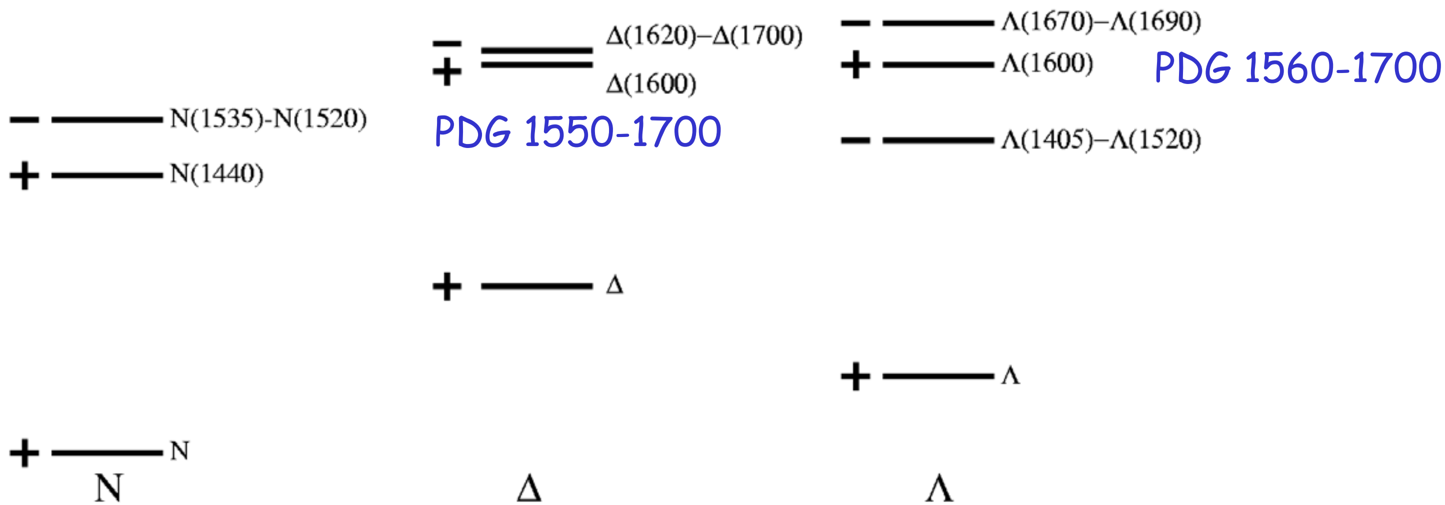
  - Photocouplings calculated with H_{int} expanded to $O(p^2/m^2)$
  - Strong decays calculated in pair creation ($^3P_0$) model (with W. Roberts)
- Reasonable agreement; allows prediction of favorable channels to find 'missing' baryons
- Puzzles: Roper mass; $\Lambda 3/2^-(1520)$-$\Lambda 1/2^-(1405)$; L=1 too light by 50 MeV, positive parity too massive by 50 MeV,…

Residual interactions…

- Another possibility: should light quarks exchange pions?
  Robson; Buchmann, Faessler,…
- Gluons not active in light-quark hadrons: flavor dependence through exchange of octet of pseudoscalars (GBE)
- Contact interaction:

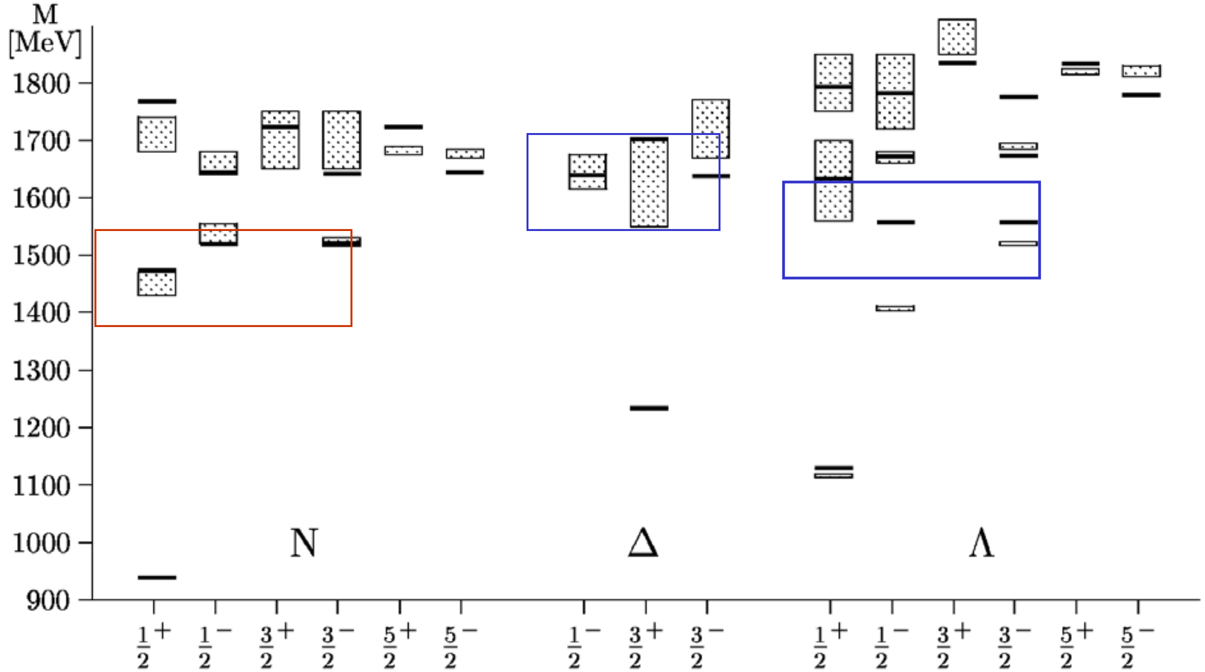
$$H_\chi \sim -\sum_{i<j} \frac{V(\mathbf{r}_{ij})}{m_i m_j} \boldsymbol{\lambda}_i^{\mathrm{F}} \cdot \boldsymbol{\lambda}_j^{\mathrm{F}} \, \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j$$

- Order of states inverted? Natural with GBE
  - ⇨ Glozman & Riska (GR)



Residual interactions…

- GR fit radial matrix elements of $V(r_{ij})$ to spectrum
- **Calculated** in variational H.O. basis with consistent tensor Glozman, Plessas, Theussl, Wagenbrunn, & Varga
- add nonets of exchange vector mesons and scalars
  - Relativistic K.E., string confinement; calculate decays



Residual interactions…

- Another flavor-dependent possibility: instanton-induced interactions ⇨ see talks by: Ulrich Löring, Tues.@5:30; D. Diakonov, Wed.@8:30

$$\langle q^2; S, L, T | W | q^2; S, L, T \rangle = -4g\, \delta_{S,0}\, \delta_{L,0}\, \delta_{T,0} \mathcal{W}$$

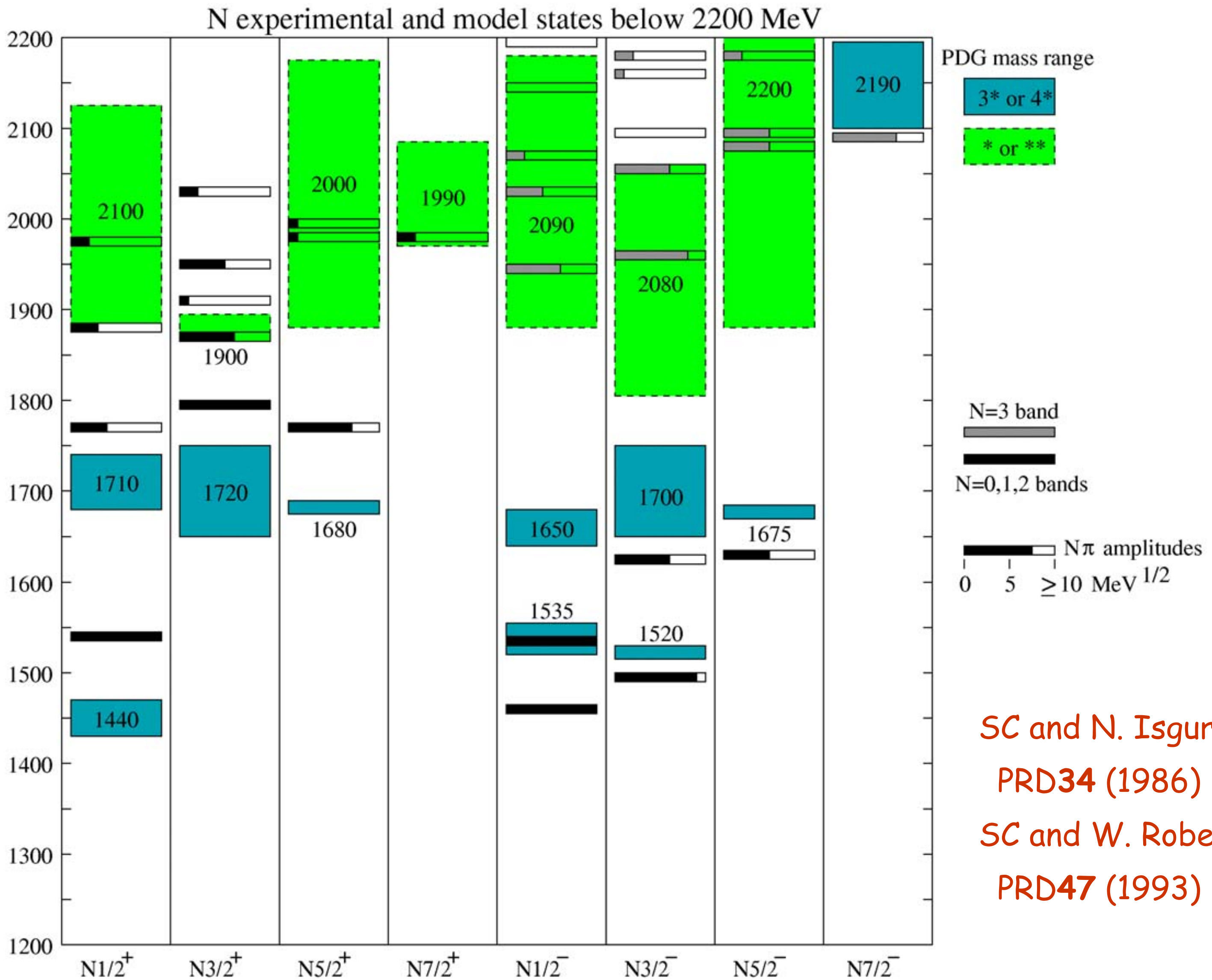
- Present if qq in S-wave, I=0, S=0 state
- W is a contact interaction
- Applied to excited states Blask, Bohn, Huber, Metsch & Petry
  - Efficient model—few parameters
  - Small splittings in P-wave Σ, positive-parity states generally too heavy, by 250 MeV
  - Works reasonably in ground states, P-wave N and Λ

Baryon spectroscopy requires a decay model

- It is not enough to give the masses of your states!
  - Analyses of data generally have fewer states in them than in models (some model states are "missing")
    - But see talk by Javier Vijande, Thus.@2:15 !
  - You must also predict which of your states are likely to be seen in analyses of scattering data
  - $^3P_0$ is popular phenomenological decay model
  - See talks by P. Gonzalez & B.Desplanques, Tues@3:00, 3:15
    - Has advantage that emitted mesons have structure

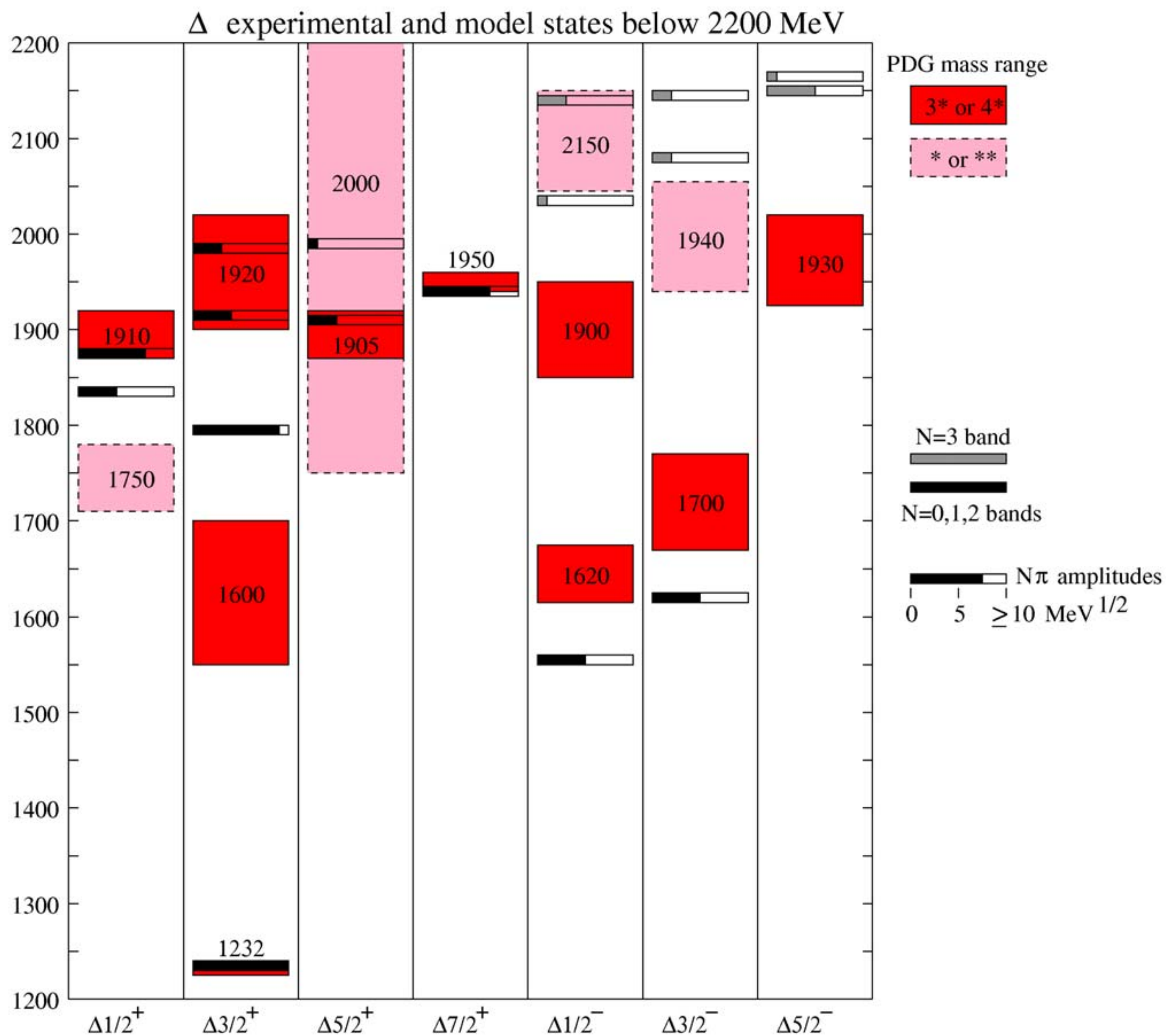
    ⇨Can correlate many decays with very few parameters

Nucleon model states and Nπ couplings



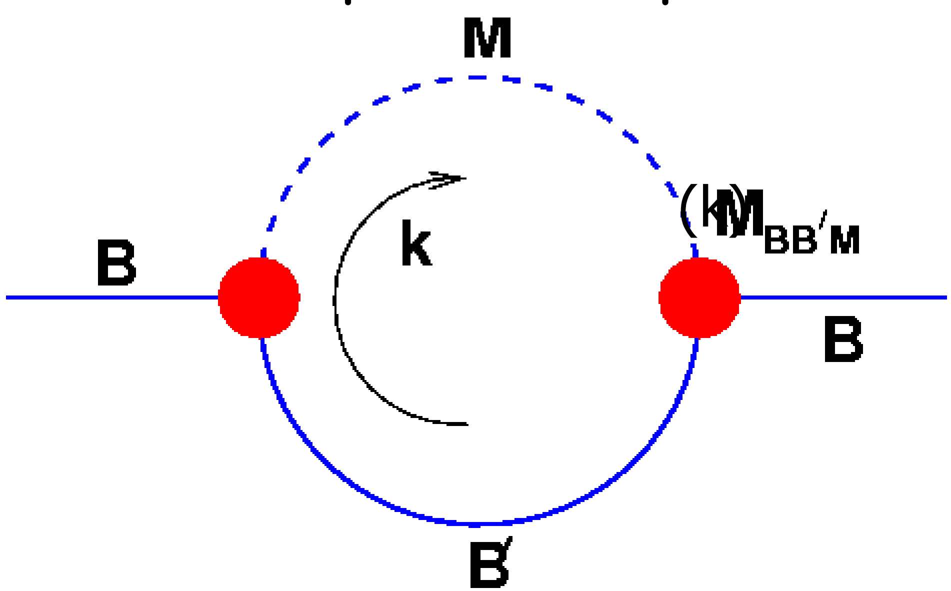
SC and N. Isgur, PRD**34** (1986) 2809;
SC and W. Roberts, PRD**47** (1993) 2004

Δ model states and Nπ couplings



Unquenching the quark model

- In QCD $qqq(q\bar{q})$ configurations possible in baryons: effect on CQM?
  - Model with baryon-meson intermediate states, loops ⇨ self energies
  - High-momentum part of loops contains OBE



Unquenching the quark model…

To calculate self energies and mixing:

- Need model of B ⇨ B′ M vertices and their momentum dependence
- Need model of spectrum (including states not seen in experiment) ⇨ wave functions ⇨ vertices

– Solve $E+\Sigma_B(E)=M_B$ self-consistently to find E (bare mass) at which sum is physical mass M_B

– Find effects on spectrum by examining splittings in bare masses

Unquenching the quark model…

- Baryon self energy due to individual B′M loops comparable to widths - convergence?
- Best calculations applied to N, Δ, Λ, Σ, ground and L=1 states
- Intermediate states BM
  - Ground state mesons $M \in \{\pi, K, \eta, \eta', \rho, \omega, K^*\}$
  - Ground states baryons $B \in \{N, \Delta, \Lambda, \Sigma, \Sigma^*\}$
  - π, K, η : Blask, Huber & Metsch ZPA**326** (1987) 413
  - All ps and vector M: Zenczykowski Ann.Phys.(NY)**123** (1986) 453; Silvestre-Brac and Gignoux PRD**43** (1991) 3699; Fujiwara, Prog. Theor. Phys. 90 (1993) 105.

Unquenching the quark model…

- Couple mesons to baryons
  - Point-like elementary particles
  - Using pair creation ($^3P_0$) or similar model ✓
- Either time-ordered perturbation theory

$$\Sigma_R(E) = \sum_B \int d^3p \frac{V^\dagger_{\pi RN}(p) V_{\pi RN}(p)}{E - E_R(p) - E_\pi(p)}$$

- Or dispersion relation

$$m_A^2 - (m_A^0)^2 = \sum_i w_i^A \int_{S^i_{\rm thr}} \frac{\rho(s, m_B, m_M)}{m_A^2 - s} ds$$

⇨Zenczykoswki calculates ρ using $^3P_0$

Unquenching the quark model…

- Effects on spectrum are substantial (≈ 50-100 MeV):
  - Zenczykowski finds many mass splittings close to analyses **without** qqq residual interactions
  - Other calculations show splittings in bare masses which resemble spin-orbit effects
    - Solution to spin-orbit problem in baryons?
- Lack self-consistent treatment of external and intermediate states—converged?
    - Such convergence slower in $^3P_0$ NRQM: mesons, Geiger and Isgur
    - Faster in covariant model based on Schwinger-Dyson Bethe-Salpeter approach: SC, Pichowsky, Walawalkar

Unquenching the quark model…

- Essential problem: there are lots of B′M thresholds nearby in energy
  - Nρ, Δρ similar thresholds to N(1535)π, Λ(1116)K, etc.
  - Cannot study spectrum by restricting M to π (or even all pseudoscalars) or B′ to N, Δ (or even all octet and decuplet ground states)

Unquenching the quark model…

- Zenczykowski:
  - assume $SU(3)_f \otimes SU(2)_{spin}$, only ground state B and M exist
  - all octet and decuplet ground state baryons have mass M_B and same wvfn.
  - all pseudoscalar and vector ground state mesons have mass M_M and same wvfn.
  - All loop integrals now the same, apart from SU(6) factor at vertices

⇨ Sum of loops for N and Δ same *only* if include all B′M combinations consistent with quantum numbers!

Unquenching the quark model…

- Convergence examined using unmixed oscillator wave functions for intermediate baryons
  - Brack and Bhaduri, PRD35 8451 ('87)

$$\Sigma_\pi(i) = -\frac{1}{4\pi^2}\sum_B P\int_0^\infty \frac{k^2\overline{|\mathcal{M}_{Bi}|^2} F_\pi^2(k)dk}{\omega_k[\omega_k-(E_B-E_i)]}$$

  - $B\pi$, with $B \in$ {N=0,1,2 & 3 band N, Δ states}
  - Find splitting Δ-N converges by N=3, but those of negative-parity states do not

Self energies in relativized $^3P_0$ model

- PhD thesis of Danielle Morel (FSU)
  - ⇨ See her talk, Tues@6:00 !
- Calculate vertices as a function of loop momentum using $^3P_0$ model (analytic, Maple)
  - Use mixed relativized-model wavefunctions (expanded up to N=7 band)
  - Include intermediate states BM with
    - Mesons $M \in \{\pi, K, \eta, \eta', \rho, \omega, K^*\}$
    - Baryons $B \in \{N, \Delta, \Lambda, \Sigma, \Sigma^*\}$, including all excitations up to N=3 band

Self energies in relativized $^3P_0$ model…

- Integrate over loop momentum, over pole if above threshold

$$\Sigma_{N*}(E) = \sum_{BM} \int d^3k \frac{\left|M_{N*\to BM}(k)\right|^2}{E - \sqrt{m_M{}^2 + k^2} - \sqrt{M_B{}^2 + k^2}}$$

- Usual $^3P_0$ model gives vertices that are too hard, loops get large contributions from high momenta
  - Geiger and Isgur give pair-creation operator a form factor ~ $\exp(-f^2[p_q - p_{\bar{q}}]^2)$, with $f^2=3.0$ GeV^{-2}
  - Gives vertex a spatial size of ~0.35 fm
- Silvestre-Brac and Gignoux use similar form factor ⇨ reasonable splittings of –ve parity states

Self energies and mixing

- Self energies are not diagonal!
  - States with same quantum numbers mix through BM loops
  - Need to **diagonalize** self energy matrix
  - E.g. N and Roper, 2X2 mixing through all Bπ intermediate states:
    - Need N⇨Bπ⇨N, R⇨Bπ⇨R, N⇨Bπ⇨R, R⇨Bπ⇨N
    - Energy dependent 2 x 2 matrix $\Sigma(E)$
  - How can we find the self energies of the un-mixed states? (Thanks to Mike Pichowsky…)

Self energies and mixing…

- Poles in Green's function are at zeros of $G_0^{-1}-\Sigma(E)$
- quadratic equation for counter terms

$$\left\| \begin{pmatrix} E-M_N+i\varepsilon & 0 \\ 0 & E-M_R+i\varepsilon \end{pmatrix} - \begin{pmatrix} \Sigma_{NN}(E) & \Sigma_{NR}(E) \\ \Sigma_{RN}(E) & \Sigma_{RR}(E) \end{pmatrix} - \begin{pmatrix} \delta M_N & 0 \\ 0 & \delta M_R \end{pmatrix} \right\| = 0$$

- Solve at $E=M_N$ and $E=M_R$: ⇨ two equations in two unknowns, find δM's

Conclusions

- Quark models of baryons undergoing renaissance!
- Lattice QCD, Schwinger-Dyson approaches:
  - Can help identify degrees of freedom and nature of confinement
  - Able to calculate ground and some first excited states
  - Can work together with quark models (extrapolation to light quark masses)
- The next Fock-space component is likely more important than differences among qqq models
  - calculating its effects requires:
    - Use of full SU(6)-related set of intermediate states, spatially-excited intermediate baryons
    - Realistic model of off-shell vertex amplitudes
    - Careful treatment of mixing effects

Tallahassee may be a small city…

…but people there have heard about baryons