BNL PREPRINT

BNL-QGS-99-402

Interference of Resonances, Resonances and Background in Coupled-Channel Formalism

S.U.Chung

Physics Department, Brookhaven National Laboratory, Upton, NY 11973, USA

V.L.Korotkikh

Scobeltsin Institute of Nuclear Physics, Moscow States University, 119899 Moscow, Russia

April 20, 1999

Abstract

An Unitarized Breit-Wigner form for a resonance production with taking into account of the background is suggested in K-matrix approach. The numerical examples demonstrate the strong interference effects.

Introduction

The K-matrix approach allows to consider the interference effects as between overlapping resonances so between the resonances and background.

In the note [1] we put a question of the problem of physical background in the mass dependence of produced meson systems. There is a possibility to take the background into account in K-matrix approach [2]. The K-matrix approach allows us to write a Lorentz-invariant amplitude which satisfies the unitarity condition. The important amplitude properties for two overlapping resonances in a single, two and three channels were studied in work [3].

More than thirty years ago the problem of interference with background was considered in other field of physics. U.Fano [4,5] received a formulae for describing the profiles of absorption lined in the ionization continuum of atomic spectra. He used a nonrelativistic approach in the frame of conventional quantum mechanics.

Now we generalize Fano's formula to the relativistic unitarised amplitude of resonance production with physical background using Aitchison's [6] idea of the production amplitude incorporation with T-matrix. This approach maintains unitarity and allows to study the most general form of background contribution including the complex phase itself. The interference of resonance and background can change not only the resonance width but also a mass dependence of wave phase.

Let's see below the simple examples.

One resonance and background in a single channel

Let's start from our parameterization of K-matrix and P-vector of production in work [7]. For simplicity we ignore the Lorenz-invariant presentation of K-matrix and the mass dependence of widths $\Gamma_{\alpha i}(m)$, barrier factors $\overline{B}_L^{(\alpha)}(q_i)$ and phase-space factors $\rho_i(m)$. We also ignore the direct background contribution in K-matrix, $C_{ij}^{(\alpha)}$, but not in P-vector production. The direct background contribution was studied in [8] for analysis of $\pi^0\pi^0$ system. Then formulae (4) and (14) from [7] can rewrite as follows

$$
\begin{aligned}
K_{ij}(m) &= \sum_\alpha m_\alpha \frac{\sqrt{\Gamma_{\alpha i}\cdot\Gamma_{\alpha j}}}{m_\alpha^2 - m^2}, && (1)\\
P_i(m) &= \sum_\alpha m_\alpha \left\{ \frac{\sqrt{\Gamma_\alpha^0 \cdot \Gamma_{\alpha i}}}{m_\alpha^2 - m^2} V_\alpha + \frac{\sqrt{\Gamma_\alpha^0\cdot\Gamma_{Bi}}}{D^{(B)}} V_i^{(B)} \right\}. && (2)
\end{aligned}
$$

Let's see only single channel $i = 1$ and one resonance $\alpha = a$. In that case

$$
\begin{aligned}
\Gamma_{a1} &= \Gamma_a^0, && (3)\\
\frac{\sqrt{\Gamma_\alpha^0\cdot\Gamma_{Bi}}}{D^{(B)}} &= \Gamma_a^0 \cdot \frac{\gamma_{bi}}{D^{(B)}} = B. && (4)
\end{aligned}
$$

Then the equation (1) and (2) are rewrited as

$$
\begin{aligned}
K_{11}(m) &= \frac{m_a\cdot\Gamma_a^0}{m_a^2 - m^2},\\
P_1(m) &= \frac{m_a\cdot\Gamma_a^0}{m_a^2 - m^2} V_a + B\cdot V_1^{(B)}, && (5)
\end{aligned}
$$

where $B\cdot V_1^{(B)}$ is an amplitude of background in the first channel.

The amplitude $F = (I - iK)^{-1}P$ will be equal to

$$
F_1(m) = \frac{1}{1 - i\frac{m_a\cdot\Gamma_a^0}{m_a^2 - m^2}} \left(\frac{m_a\cdot\Gamma_a^0}{m_a^2 - m^2} V_a + B\cdot V_1^{(B)} \right). \qquad (6)
$$

Here the values V_a and $V_1^{(B)}$ are complex.

Let's denote

$$
\begin{aligned}
\varepsilon(m) &= \frac{m_a^2 - m^2}{m_a\cdot\Gamma_a^0} = \frac{1 - \left(\frac{m}{m_a}\right)^2}{\left(\frac{\Gamma_a^0}{m_a}\right)^2}, && (7)\\
d &= B\frac{V_1^{(B)}}{V_a}. && (8)
\end{aligned}
$$

Then the equation (6) has a simple form

$$
F_1^{(a)}(m) = V_a \frac{1 + d\cdot\varepsilon(m)}{\varepsilon(m) - i} \qquad (9)
$$

and the cross-section is equal to

$$|F_1^{(a)}(m)|^2 = |V_a|^2 \frac{|1 + d \cdot \varepsilon(m)|^2}{1 + \varepsilon^2(m)}. \qquad (10)$$

With $q = 1/d$ we get a relativistic generalization of famous Fano's formula [4]

$$|F_1^{(a)}(m)|^2 = |V_a \cdot d|^2 \frac{|q + \varepsilon(m)|^2}{1 + \varepsilon^2(m)}. \qquad (11)$$

For nonrelativistic case he used [4]

$$\varepsilon_{nr}(m) = \frac{m - m_a}{\Gamma_a^0/2}. \qquad (12)$$

A parameter q (Fano's q-index) generates the influence of background contribution.

In our case we take the amplitude (9) and note that a parameter d (8) may be a complex magnitude:

$$d = d_0 e^{i\delta_0} \qquad (13)$$

If the phase $\delta_0 = 0$, then d is real and a denominator of amplitude (9) gives us a clear Breit-Wigner phase $\delta_{BW}(m)$,

$$F_{BW}(m) = \frac{1}{\varepsilon(m) - i} = \frac{i}{1 + i\varepsilon(m)} = \frac{e^{i\delta_{BW}(m)}}{\sqrt{1 + \varepsilon^2(m)}}, \qquad (14)$$

where

$$\delta_{BW}(m) = \frac{\pi}{2} - \delta_a(m), \qquad (15)$$

$$\tan \delta_a(m) = \varepsilon(m). \qquad (16)$$

So,

$$\delta_{BW}(m) = \begin{cases} 0, & m \ll m_a \text{ and } m_a \gg \Gamma_a^0 \\ \frac{\pi}{2}, & m = m_a \\ \pi, & m \gg m_a \text{ and } m \gg \Gamma_a^0 \end{cases} \qquad (17)$$

But if the background is a complex value, $\delta_B \neq 0$, then

$$\begin{aligned} F_1^{(a)}(m) &= V_a \cdot F_{BW}(m) \cdot (1 + d \cdot \varepsilon(m)), \qquad (18) \\ &= |V_a| \cdot |F_{BW}(m)| \cdot |1 + d \cdot \varepsilon(m)| e^{i(\delta_{pr} + \delta_{BW}(m) + \delta_B(m))}. \end{aligned}$$

Here δ_{pr} is an amplitude production phase and δ_B is a phase of the factor $(1 + d\varepsilon(m))$ which takes into account the background:

$$\tan \delta_B(m) = \frac{d_0 \cdot \sin \delta_0 \cdot \varepsilon(m)}{1 + d_0 \cdot \cos \delta_0 \cdot \varepsilon(m)}. \qquad (19)$$

We see that even if the background amplitude $B \cdot V_1^{(B)}$ in (5) has no mass dependence then the total phase of amplitude (18)

$$\delta_{tot}^{(1)}(m) = \delta_{pr} + \delta_{BW}(m) + \delta_B(m) \tag{20}$$

has the additional mass dependence caused by the phase $\delta_B(m)$.

The properties of this phase are the next:

$$\delta_B(m) = \begin{cases} \delta_0, & m \gg m_a \text{ and } m \gg \Gamma_a^0 \\ 0, & m = m_a \\ \pm\frac{\pi}{2}, & m = m_1^{(B)} \\ \delta_0, & m \ll m_a \text{ and } m_a \gg \Gamma_a^0, \end{cases} \tag{21}$$

where $m_1^{(B)}$ is determined by the equation

$$1 + d_0 \cdot \varepsilon(m_1^{(B)}) = 0. \tag{22}$$

Two Resonance and Background in a Single Channel

In this case we get from (1) and (2)

$$\begin{aligned} K_{11}(m) &= \frac{m_a \cdot \Gamma_a}{m_a^2 - m^2} + \frac{m_b \cdot \Gamma_b}{m_b^2 - m^2}, & (23)\\ P_1(m) &= \frac{m_a \cdot \Gamma_a}{m_a^2 - m^2} V_a + \frac{m_b \cdot \Gamma_b}{m_b^2 - m^2} V_b + B \cdot V_1^{(B)}. & (24) \end{aligned}$$

Let's use (7) and denote

$$\mu_{ab}(m) = \frac{\varepsilon_a(m) \cdot \varepsilon_b(m)}{\varepsilon_a(m) + \varepsilon_b(m)} = \frac{1}{1/\varepsilon_a(m) + 1/\varepsilon_b(m)}. \tag{25}$$

Then the amplitude is equal to

$$F_1^{(ab)}(m) = \frac{1}{\mu_{ab}(m) - i} \left(\frac{\varepsilon_b(m)}{\varepsilon_a(m) + \varepsilon_b(m)} V_a + \frac{\varepsilon_a(m)}{\varepsilon_a(m) + \varepsilon_b(m)} V_b + \mu_{ab}(m) C \right), \tag{26}$$

where

$$C = B V_1^{(B)} = C_0 e^{i\varphi_0}, \qquad C_0 = |C|. \tag{27}$$

Let's see some special cases.

If two resonance are isolated and $m \approx m_\alpha$, $\alpha = a, b$, then

$$F_1^{(\alpha)}(m) \approx V_\alpha \frac{1 + d\varepsilon_\alpha(m)}{\varepsilon_\alpha(m) - i}; \qquad d = B \frac{V_1^{(B)}}{V_\alpha,} \tag{28}$$

which coincides with (9).

If two resonances have the same peak mass $m_a = m_b = m_\alpha$, but different widths, $\Gamma_a \neq \Gamma_b$, then

$$F_1^{(ab)}(m) = \frac{1}{\varepsilon_\alpha(m, \Gamma_{tot}) - i}\left(\frac{\Gamma_b V_a + \Gamma_a V_b}{\Gamma_{tot}} + \varepsilon_\alpha(m, \Gamma_{tot}) \cdot B V_1^{(B)}\right), \tag{29}$$

where $\varepsilon_\alpha(m, \Gamma_{tot})$ is the equation (7) with

$$\Gamma_a^0 = \Gamma_{tot} = \Gamma_a + \Gamma_b. \tag{30}$$

And if we have two different resonances which are excited by the same intensities, $V_a = V_b = V_0$, then from (26)

$$F_1^{(ab)}(m) = \frac{V_0}{\mu_{ab}(m) - i}(1 + \mu_{ab}(m) \cdot d); \qquad d = B\frac{V_1^{(B)}}{V_0}. \tag{31}$$

We see in all cases that the background contribution of the amplitude $F_1^{(a)}(m)$ has mass dependence. And if the parameter d is a complex value, then $F_1^{(a)}(m)$ will have the additional mass dependent phase $\delta_B(m)$ as we see it in (18).

Two Resonance and Background in Two Coupled Channels

We again ignore the barrier factors $\overline{B}_L^{(\alpha)}(q_i)$.We also ignore the background contribution in K-matrix directly. We study the background contribution in P-vector production. Let's start with formulae (16) and (17) from [7]:

$$K_{ij}(m) = \sum_\alpha m_\alpha \Gamma_\alpha^0 \frac{\gamma_{\alpha i}\gamma_{\alpha j}}{m_\alpha^2 - m^2}, \tag{32}$$

$$P_i(m) = \sum_\alpha m_\alpha \Gamma_\alpha^0 \frac{\gamma_{\alpha i}}{m_\alpha^2 - m^2} V_\alpha + C_i e^{i\varphi_i}. \tag{33}$$

We get for two channels $i, j = 1,\ 2$ and for $\alpha = a,\ b$:

$$\Gamma_a = \Gamma_a^0(\gamma_{a1}^2 + \gamma_{a2}^2) = \Gamma_a^0, \qquad \gamma_{a1}^2 + \gamma_{a2}^2 = 1, \tag{34}$$

$$\Gamma_b = \Gamma_b^0(\gamma_{b1}^2 + \gamma_{b2}^2) = \Gamma_b^0, \qquad \gamma_{b1}^2 + \gamma_{b2}^2 = 1, \tag{35}$$

$$K_{11}(m) = \frac{m_a\Gamma_a}{m_a^2 - m^2}\gamma_{a1}^2 + \frac{m_b\Gamma_b}{m_b^2 - m^2}\gamma_{b1}^2, \tag{36}$$

$$K_{22}(m) = \frac{m_a\Gamma_a}{m_a^2 - m^2}\gamma_{a2}^2 + \frac{m_b\Gamma_b}{m_b^2 - m^2}\gamma_{b2}^2, \tag{37}$$

$$K_{12}(m) = \frac{m_a\Gamma_a}{m_a^2 - m^2}\gamma_{a1}\gamma_{a2} + \frac{m_b\Gamma_b}{m_b^2 - m^2}\gamma_{b1}\gamma_{b2}, \tag{38}$$

$$P_1(m) = \frac{m_a\Gamma_a}{m_a^2 - m^2}\gamma_{a1}V_a + \frac{m_b\Gamma_b}{m_b^2 - m^2}\gamma_{b1}V_b + C_1 e^{i\varphi_1}, \tag{39}$$

$$P_2(m) = \frac{m_a\Gamma_a}{m_a^2 - m^2}\gamma_{a2}V_a + \frac{m_b\Gamma_b}{m_b^2 - m^2}\gamma_{b2}V_b + C_2 e^{i\varphi_1}. \tag{40}$$

The amplitudes of K-matrix approach are equal to

$$G_i^{(ab)}(m) = \sum_{j=1}^{2} (1 - iK)^{-1}_{ij} P_j(m), \qquad i = 1,\ 2. \tag{41}$$

For two channels and two poles we have the next list of parameters:

$$V_a, \quad m_a, \quad \Gamma_a, \quad V_b, \quad m_b, \quad \Gamma_b, \quad \gamma_{a1}, \quad \gamma_{b1}, \quad c_1, \quad \varphi_1, \quad c_2, \quad \varphi_2. \tag{42}$$

Numerical examples

The results for one resonance in a single channel with and without background are presented in fig. 1 and 2. We take not very large contribution of background ($d_0 = 0.2$) and its phase is close to 180° or 0°. The background moves the position of resonance and changes the phase on the sides of resonance.

The case of two isolated resonances in K-matrix approach is demonstrated on fig. 3b and 3d. All fig. a and c of fig. 3–11 show the Breit-Wigner resonances with our parameters which we use in K-matrix calculations.

Fig. 4 shows two coinciding resonances with different widths. The interference of two resonances in K-matrix approach is seen in fig. 5–7 when we change the position of resonance (fig. 5), the width of one resonance (fig. 6) and the phase of background (fig. 7).

You can see in all the figures 5d)-10d) a jump of phase from π to zero. It is only an effect of drawing. Consider it as a smooth phase transfer from π to $(\phi + \pi)$, because the phase is determined with accuracy of π.

The calculation with background contribution is demonstrated by the dotted lines. The background in our calculations is arbitrary, of course, but not small (see fig.1-7,9 $C_0/V_a = 0.5$, $C_0/V_b = 0.33$) in order to see its influence. In fig.8,10,11 the background is rather small.

Fig. 8, 9 and 10 show the results for two coupled channels. We include very weak coupling with parameters:

$$\gamma_{a1} = 0.99, \quad \gamma_{a2} = 0.141,$$
$$\gamma_{b1} = 1.00, \quad \gamma_{b2} = 0.00$$

and a background only in the first channel:

$$c_1 = 3.0, \quad \varphi_1 = 170°;$$
$$c_2 = 0.0.$$

In fig. 8 we take the same parameters of poles as in fig. 5, but include the coupling of channels. Then we force the background contribution in fig. 9.

In fig. 10 we exchange the coupling constants of channels comparing with fig. 8:

$$\gamma_{a1} = 1.00, \quad \gamma_{a2} = 0.00,$$
$$\gamma_{b1} = 0.99, \quad \gamma_{b2} = 0.141.$$

Fig. 9b and 10b demonstrate the strong interference of two resonances which gives us an artifact of additional resonance between two true resonances.

In fig. 11 we decrease strongly the partial width γ_{b1} which couples the second resonance b with the first channel comparing with fig. 8:

$$\gamma_{a1} = 1.00, \quad \gamma_{a2} = 0.00,$$
$$\gamma_{b1} = 0.04, \quad \gamma_{b2} = 0.999.$$

We see that, though the resonance b has a very little probability to decay in the first channel, he gives a peak, but with small width about 30 MeV instead of the input width 300 MeV.

Resume

Our study shows the strong interference effects as between overlapping resonances so between the resonances and background. These effects have to be taken into account in the interpretation of mass dependent fit of experimental data.

This research was supported in part by the U.S. Department of Energy and the Russian State Committee for Science and Technology.

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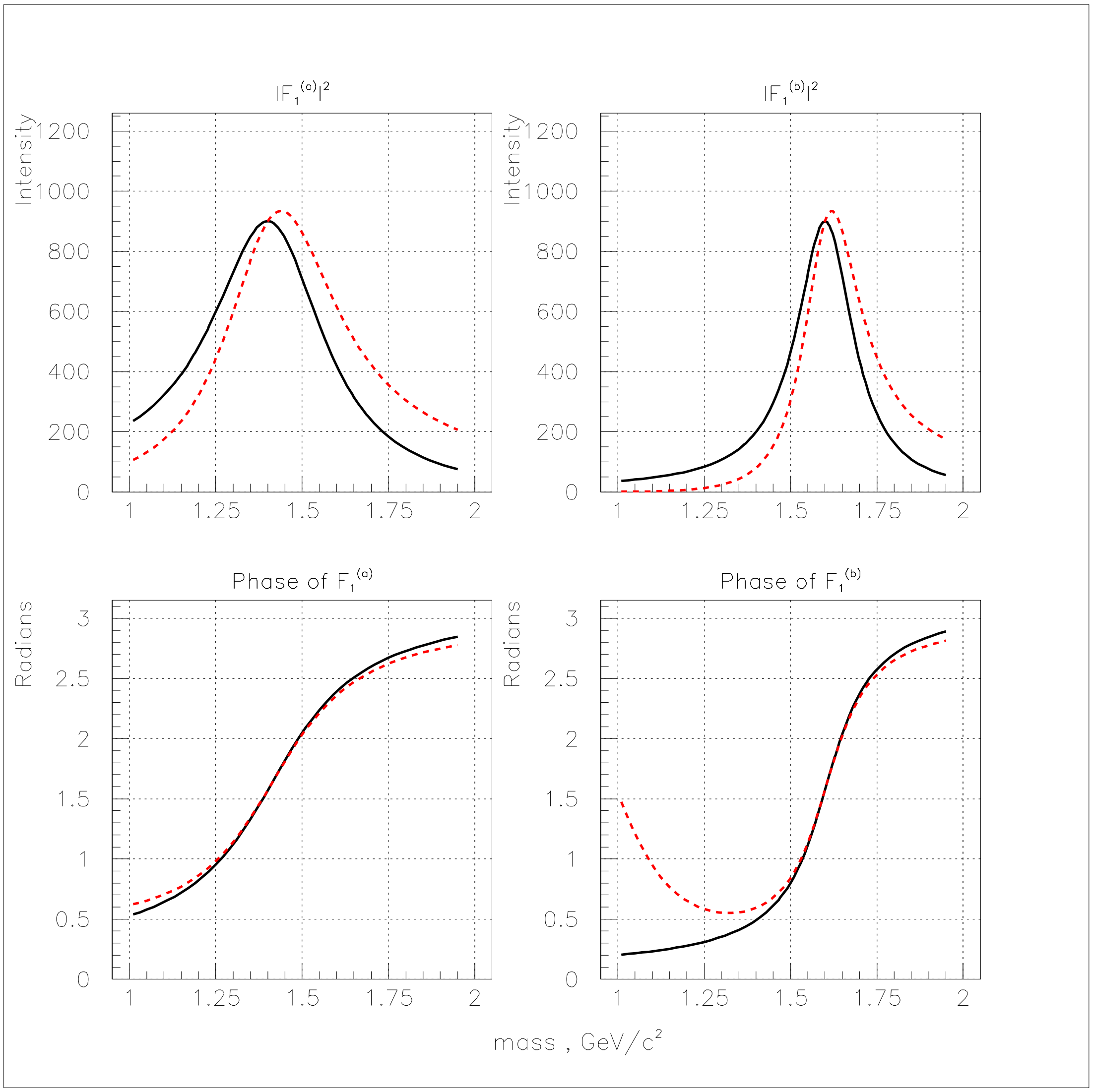

Fig. 1. Intensities and phases for one resonance and background in a single channel (9). a) and c) for $m_a = 1.4$ GeV, $\Gamma_a = 0.4$ GeV, b) and d) for $m_b = 1.6$ GeV, $\Gamma_b = 0.2$ GeV. Black lines are without, dotted line with background ($d_0 = 0.2$, $\delta_0 = 170°$).

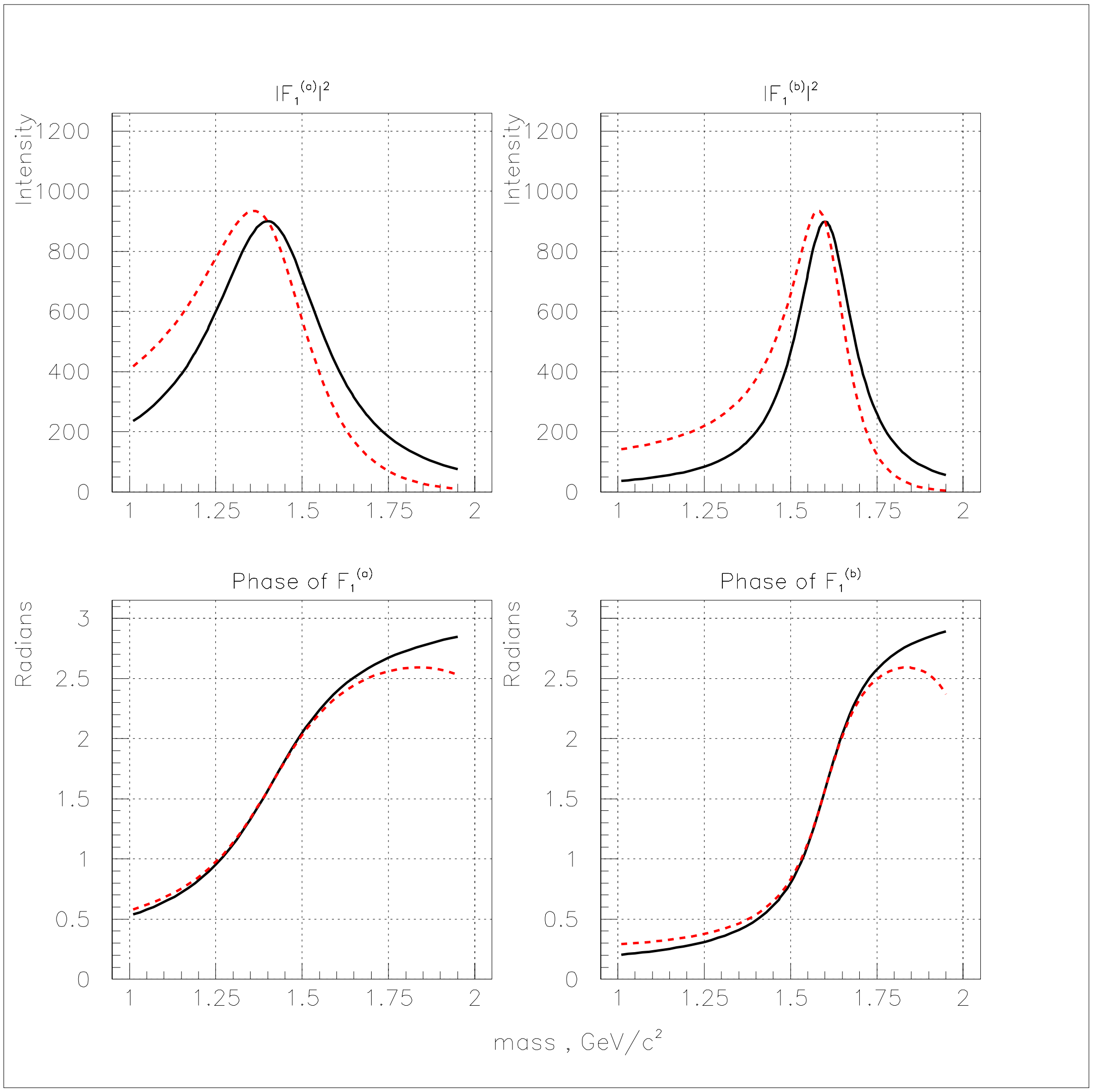

Fig. 2. The same as fig. 1 but with $\delta_0 = 10°$.

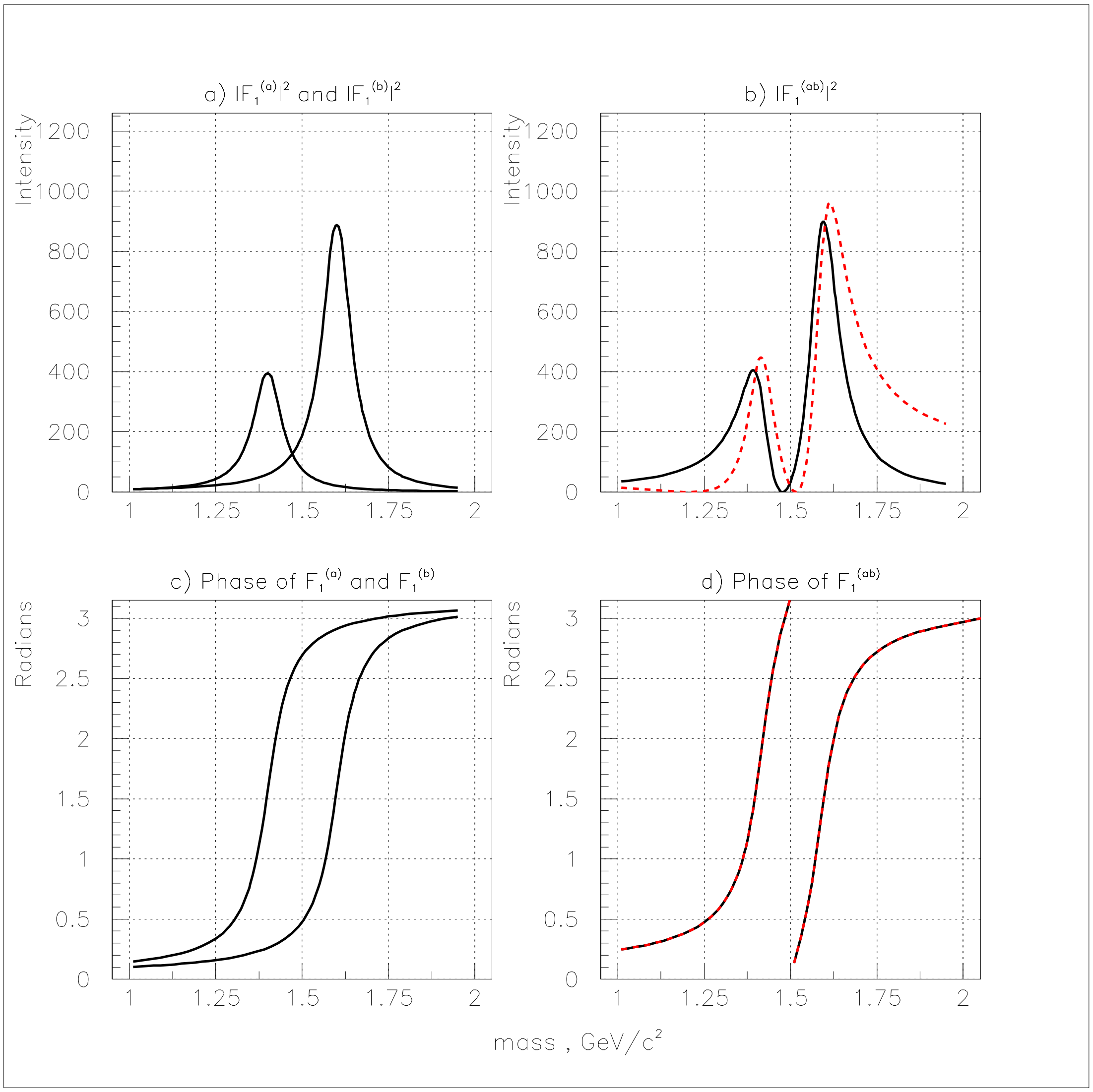

Fig. 3. Two narrow resonances in a single channel. a) and c) for Breit-Wigner amplitudes ($V_\alpha F_{BW}(m)$), b) and d) for K-matrix amplitude (26). Parameters:

$V_a = 20$, $m_a = 1.4$ GeV, $\Gamma_a = 0.1$ Gev, $V_b = 30$, $m_b = 1.6$ GeV, $\Gamma_b = 0.1$ GeV, $C_0 = 10$, $\varphi_0 = 180°$.

Black lines are without, dotted line with background.

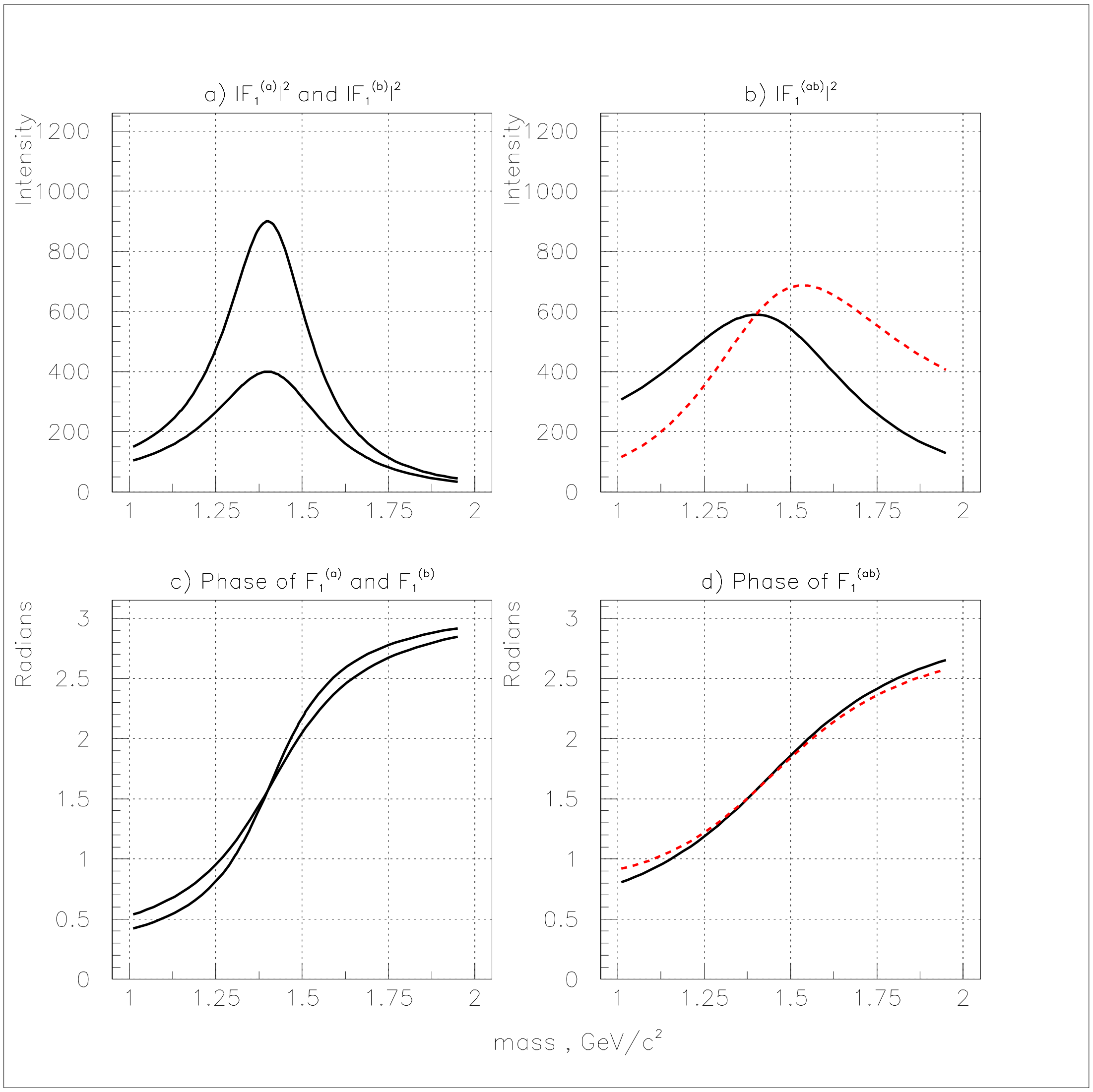

Fig. 4. Two resonances with the same peak mass, but with different widths in a single channel. The same as fig. 3, but with parameters:

$V_a = 20$, $m_a = 1.4$ GeV, $\Gamma_a = 0.4$ GeV, $V_b = 30$, $m_b = 1.4$ GeV, $\Gamma_b = 0.3$ GeV, $c_0 = 10$, $\varphi_0 = 170°$.

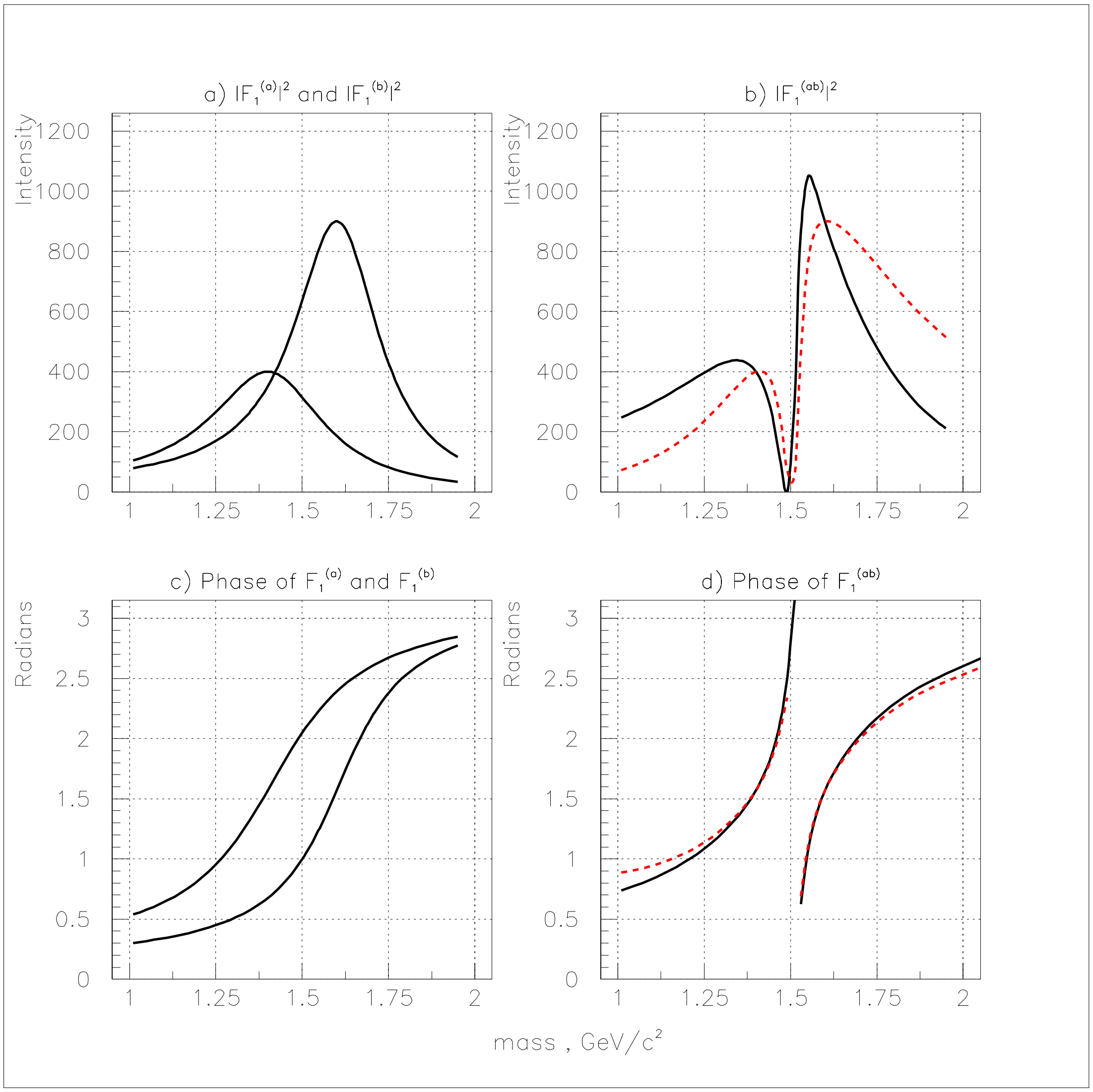

Fig. 5. Two resonances with the different peak mass and widths in a single channel. The parameters:

$V_a = 20$, $m_a = 1.4$ GeV, $\Gamma_a = 0.4$ GeV, $V_b = 30$, $m_b = 1.6$ GeV, $\Gamma_b = 0.3$ GeV, $c_0 = 10$, $\varphi_0 = 170°$.

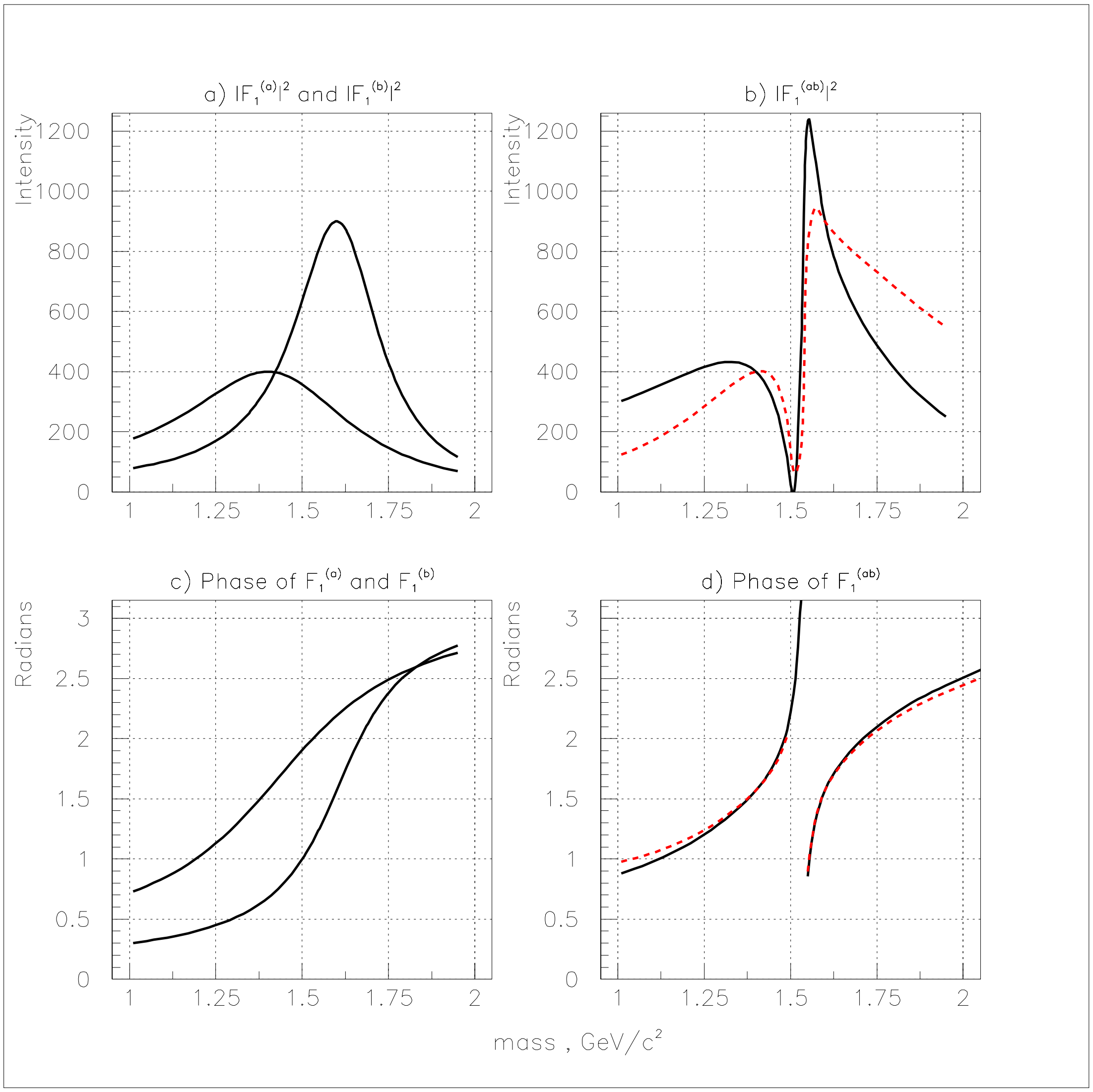

Fig. 6. Two resonances with the different peak mass and widths in a single channel and large width of the first resonance. The same as fig. 5 but with parameters:

$V_a = 20$, $m_a = 1.4$ GeV, $\Gamma_a = 0.6$ GeV, $V_b = 30$, $m_b = 1.6$ GeV, $\Gamma_b = 0.3$ GeV, $c_0 = 10$, $\varphi_0 = 170°$.

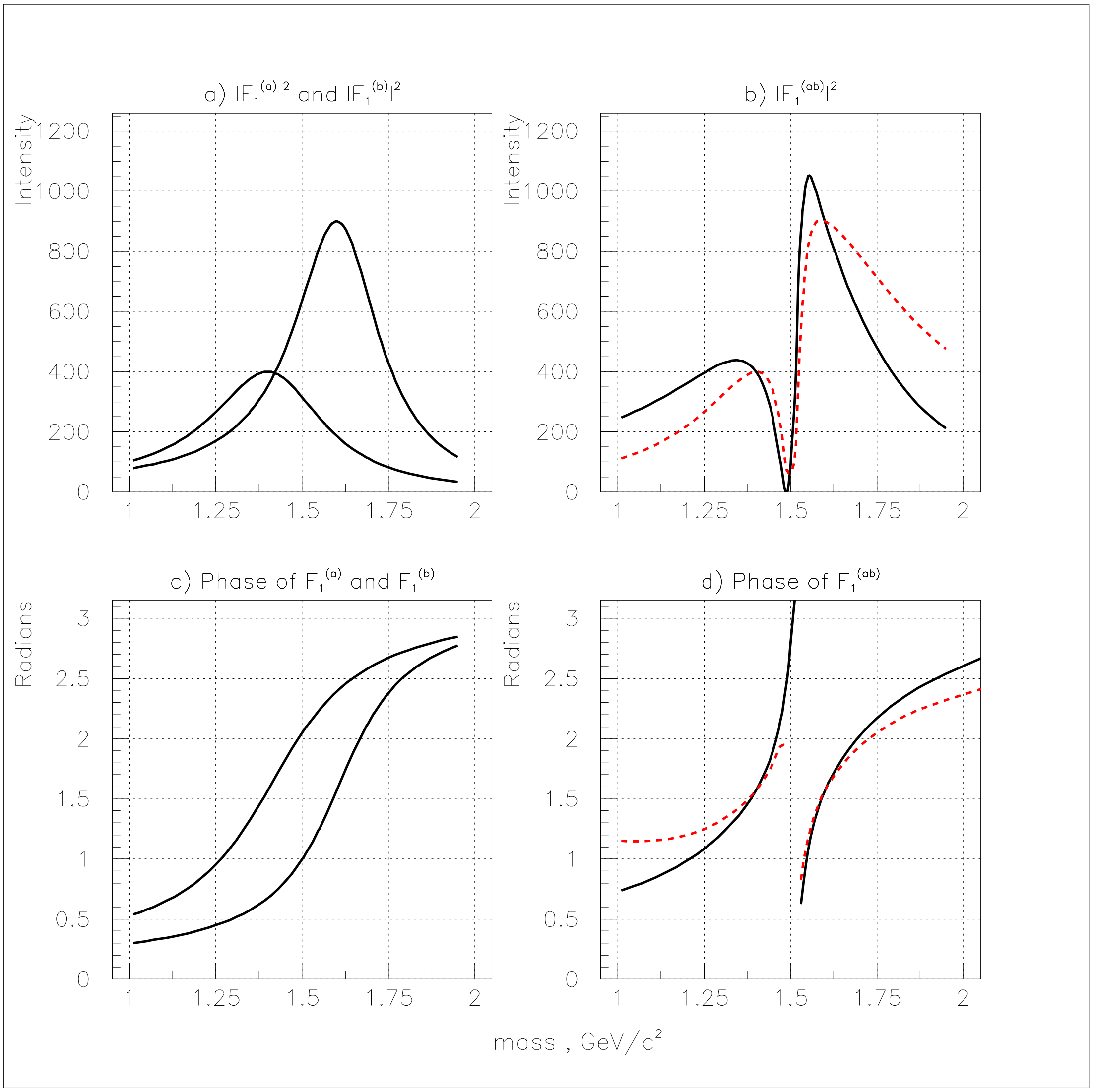

Fig. 7. Two resonances with the different peak mass and widths in a single channel and phase of background, which is strongly differ from 180° and 0°. The same as fig. 5 but with parameters:

$V_a = 20$, $m_a = 1.4$ GeV, $\Gamma_a = 0.4$ GeV, $V_b = 30$, $m_b = 1.6$ GeV, $\Gamma_b = 0.3$ GeV, $c_0 = 10$, $\varphi_0 = 145°$.

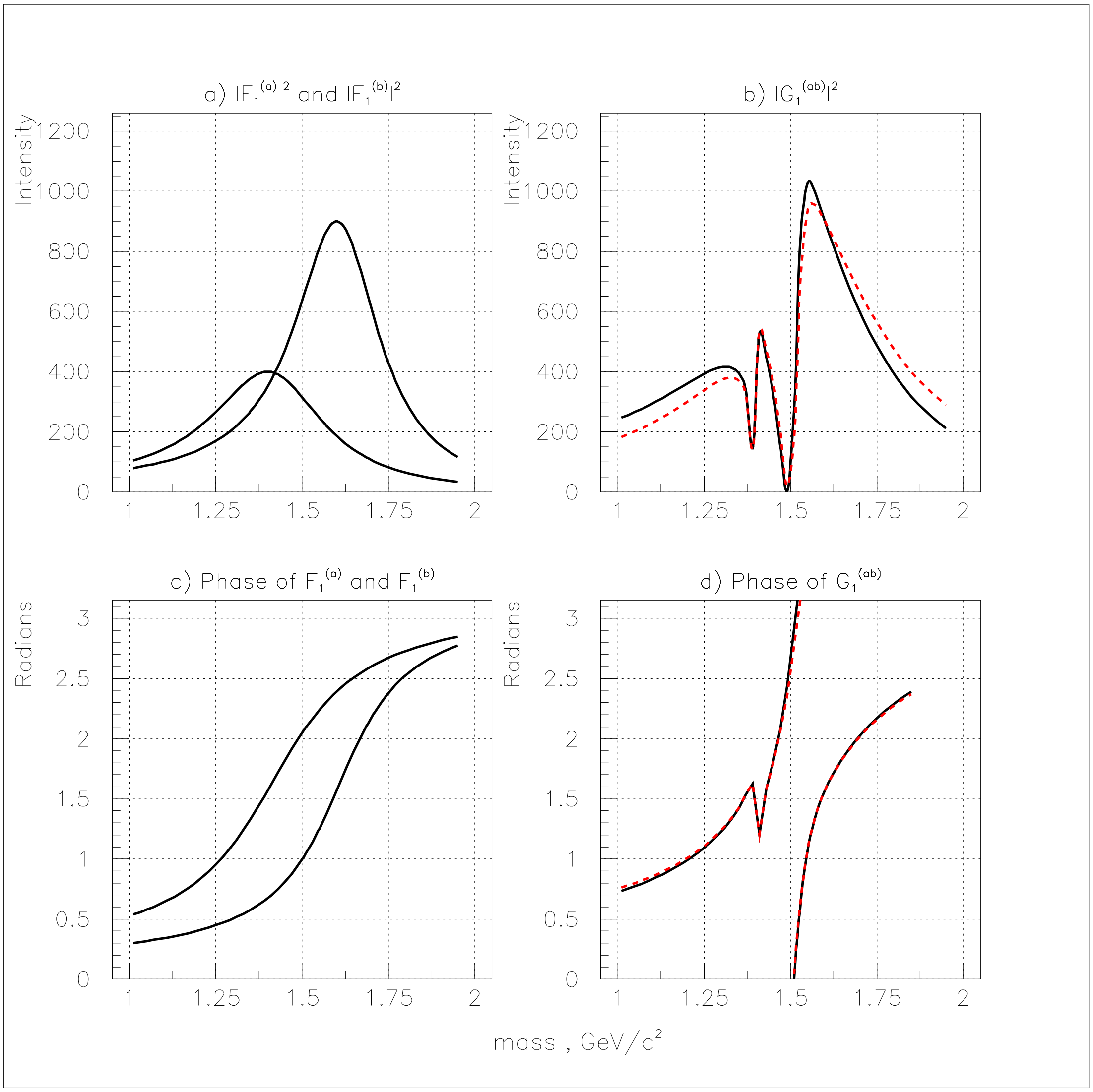

Fig. 8. The intensities and phases for two resonances in two coupled channels in K-matrix approach. a) and c) for Breit-Wigner amplitudes, b) and d) amplitude $G_1^{(ab)}(m)$ for the first channel (41). The parameters are:

$V_a = 20$, $m_a = 1.4$ GeV, $\Gamma_a = 0.4$ GeV, $V_b = 30$, $m_b = 1.6$ GeV, $\Gamma_b = 0.3$ GeV, $\gamma_{a1} = 0.99$, $\gamma_{b1} = 1.00$, $c_1 = 3$, $\varphi_1 = 170°$, $c_2 = 0$.

Black lines are without, dotted lines with background.

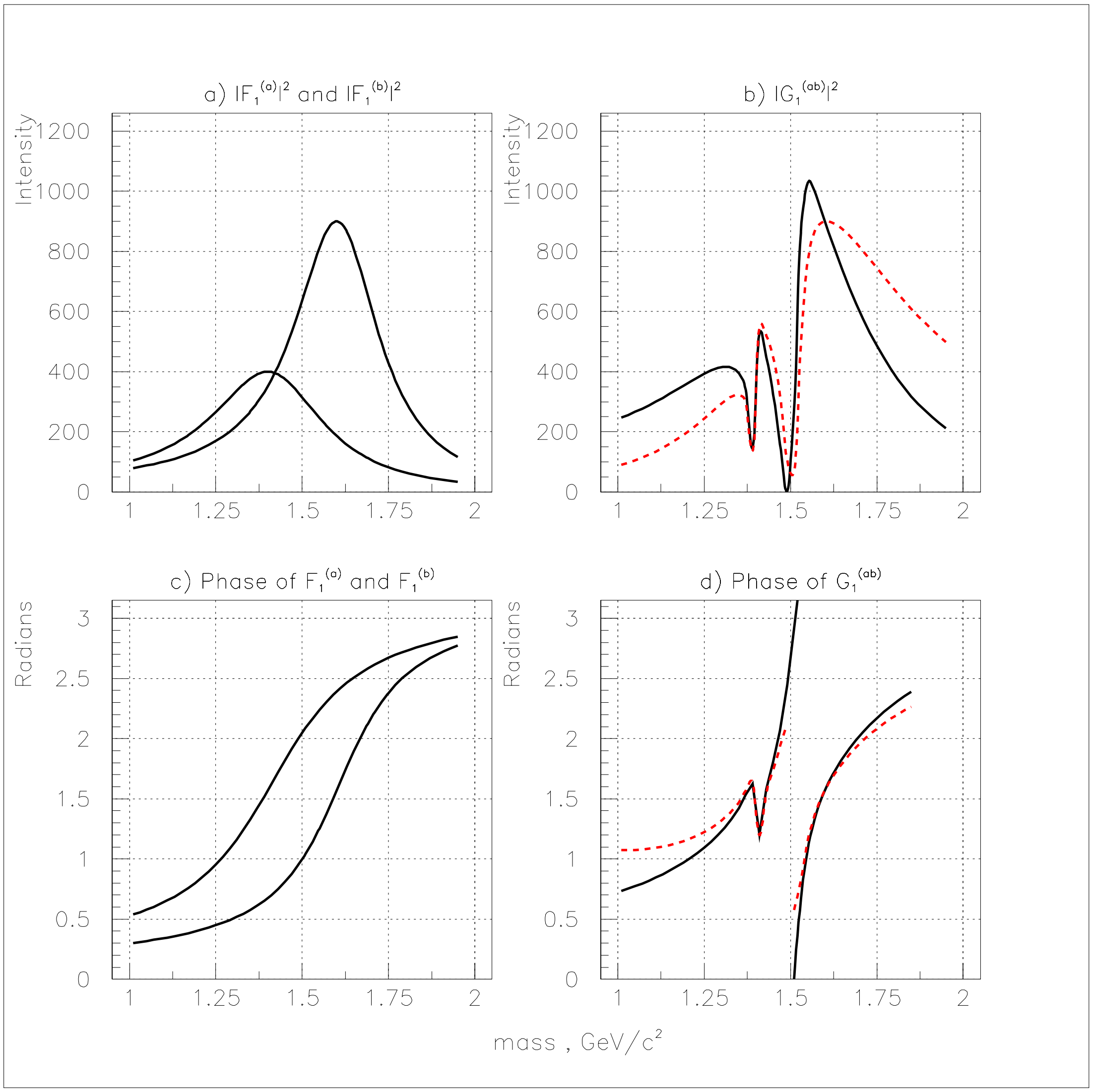

Fig. 9. Two resonances in two coupled channels. The same as fig. 8, but with more large background:
$c_1 = 10$, $\varphi_1 = 155°$.

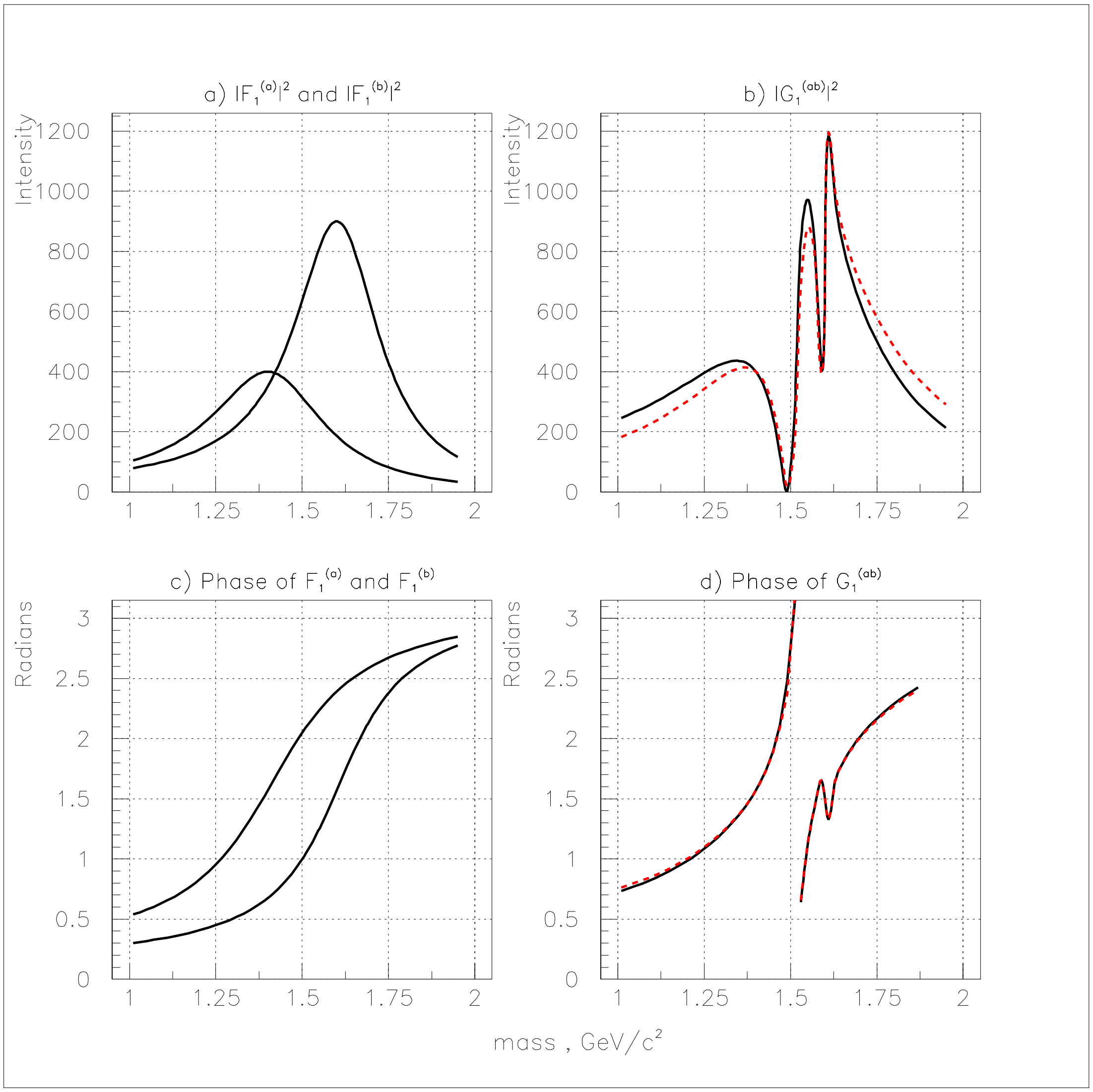

Fig. 10. Two resonances in two coupled channels. The same as fig. 8, but we exchange the values of γ_{a1} and γ_{b1} and put its the next:

$\gamma_{a1} = 1.00$, $\gamma_{b1} = 0.99$.

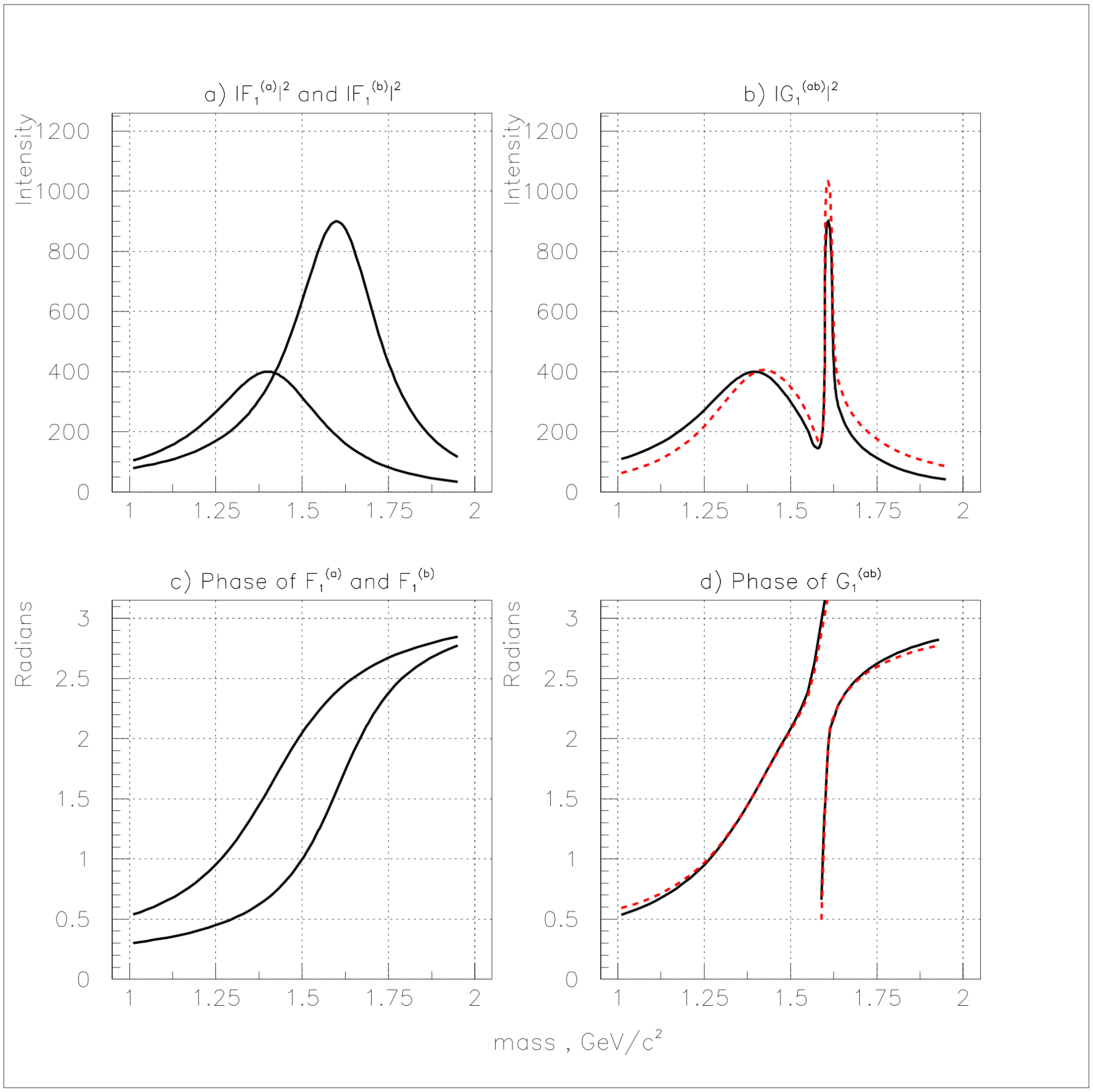

Fig. 11. Two resonances in two coupled channels. The same as fig. 8, but we strongly decrease the value of γ_{b1} and take:

$\gamma_{a1} = 1.00$, $\gamma_{b1} = 0.04$.