model with the $\frac{3}{2}^+$ states in $\underline{18}$ is that the decay $Y_1^*(1660) \to Y_1^*(1385) + \pi$ is expected to be enhanced relative to the other Y_1^* decays. This coupling is nonvanishing in the W_3 limit, in which the other isobar decay couplings vanish. There is some evidence for such a strong $Y_1^*(1660) \to Y_1^*(1385) + \pi$ decay mode.[2]

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[1]M. Taher-Zadeh et al., Phys. Rev. Letters 11, 470 (1963).

[2]L. W. Alvarez et al., Phys. Rev. Letters 10, 184 (1963); P. L. Bastien and J. P. Berge, Phys. Rev. Letters 10, 188 (1963).

[3]S. L. Glashow and A. H. Rosenfeld, Phys. Rev. Letters 10, 192 (1963); A. W. Martin, to be published.

[4]J. J. Sakurai has suggested [Phys. Letters 10, 132 (1964)] that $Y_1^*(1660)$ belongs to the representation $\underline{10}^*$ along with $N^*(1512)$ and a kaon-nucleon bound state (with hypercharge +2). This bound state has yet to be discovered.

[5]The latest phase-shift analyses [L. David Roper, Phys. Rev. Letters 12, 340 (1964); Auvil and Lovelace, to be published] use only angular distributions in elastic scattering to determine the phase shifts in the second resonance region. The parity ambiguity is removed by a continuity argument, which is somewhat unjustified in view of the large gaps in the data in the energy region between the first and second resonances. On the other hand, these analyses demonstrate convincingly that more than one phase shift is large and rapidly changing in the second resonance region. We know of no analysis of photoproduction in this region which takes more than one final state in addition to the first resonance into account, and therefore we doubt that the parity ambiguity is yet resolved.

[6]This assignment has also been suggested recently by Sakurai (reference 4) and no doubt by many others.

[7]We are using the notation of A. W. Martin and K. C. Wali, Nuovo Cimento 31, 1324 (1964).

[8]Very rough partial widths from reference 2 are

$$\Gamma_{\Sigma\pi} = 13 \text{ MeV}, \quad \Gamma_{\Lambda\pi} = 11 \text{ MeV}, \quad \Gamma_{\bar{K}N} \leq 3 \text{ MeV}.$$

See Glashow and Rosenfeld, reference 3. It is the smallness of the $\bar{K}N$ decay mode that constrains f' to the region near $\frac{1}{2}$.

[9]A. W. Martin and K. C. Wali, Phys. Rev. 130, 2455 (1963).

[10]We have used Eq. (6) of reference 7 to relate coupling constants to partial widths for the isobar decays.

[11]J. Schwinger, Phys. Rev. Letters 12, 237 (1964).

[12]R. E. Cutkosky, Phys. Rev. Letters 12, 530 (1964).

[13]S. L. Glashow and D. J. Kleitman, to be published.

[14]J. J. de Swart, Rev. Mod. Phys. 35, 916 (1963).

[15]It seems clear, if W_3 symmetry should have some approximate validity, that one must still have an intermediate stage of SU(3) symmetry. This was the viewpoint from which mass relations for the representation $\underline{18}$ were derived in reference 16. In commenting on this work I. S. Gerstein and K. T. Mahanthappa [Phys. Rev. Letters 12, 570 (1964)] give an argument to the effect that an SU(3) stage of approximation is impossible. Their argument is entirely incorrect.

[16]E. Johnson and R. Sawyer, Phys. Letters 9, 212 (1964).

[17]A fact clearly pointed out in reference 16. There are two reasons for not changing the assignments in Schwinger's scheme in order to make $\underline{18}$ (rather than $\underline{18}^*$) coupled to the baryon-pseudoscalar channel, as was done by Gerstein and Mahanthappa. In the first place the pion nucleon coupling would then vanish in the symmetry limit. In the second place, as pointed out by several authors (references 12 and 13) the single baryon-exchange mechanism would be repulsive in the representation $\underline{18}$.

[18]The argument that single baryon exchange must drive some $\frac{3}{2}^+$ resonances is unimpressive; there are too many simplifying assumptions in the calculation and too many other forces at work.

COMMENTS ON HIGHER RESONANCE MODELS

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Some time ago, it was observed[1] that in the elastic scattering of an unstable particle ("isobar") on one of its decay products ("meson"), the exchange diagram, Fig. 1(a), can be an energy-conserving process in the physical region [see Fig. 1(b)]. As a consequence, the diagram of Fig. 1(a) has a singularity near the physical region, in fact, in the physical region in the limit of vanishing isobar width (we shall call such singularities P singularities). This singularity contributes a peak to the isobar-meson scattering cross section.[2] But since all physically observed interactions are initiated with stable particles (weak interactions ignored),

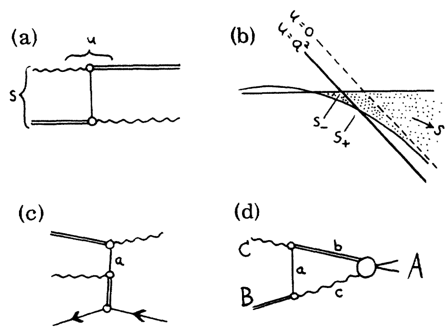

FIG. 1. (a) An elastic isobar-meson scattering diagram which yields P singularities. The mass of the exchanged particle is a. (b) A portion of the Mandelstam plane (s, t, u are the usual center-of-mass energy squares) showing the physical region (dotted) for elastic isobar-meson scattering, and the location of the pole of the diagram of Fig. 1(a), at $u = a^2$. In each partial-wave amplitude, this pole implies logarithmic branch points where it enters and leaves the physical region at $s = s_-$ and s_+; these branch points can be connected by a cut. (c) A production process diagram in which the isobar-meson scattering diagram of Fig. 1(a) occurs peripherally. (d) A production process diagram in which the isobar-meson scattering diagram of Fig. 1(a) occurs as a final-state rescattering. The letters denote the masses of the lines.

isobar-meson scattering is not observed directly. The purpose of this note is to point out that for at least two simple ways that the isobar-meson scattering enters as a subprocess in a physical process [see Figs. 1(c) and 1(d)], these peaks are absent.[3]

We shall also briefly discuss the possibility that a P singularity may induce nearby singularities, and hence be indirectly responsible for peaks: We come to the conclusion that such induced peaks will not lie very close to the energy of the P singularity.

Two simple diagrams in which the diagram of Fig. 1(a) enters as a subdiagram are shown in Figs. 1(c) and 1(d). In the peripheral diagram, Fig. 1(c), it is obvious that the singularity of the process of Fig. 1(a) is not in the physical region, since the four-momentum of the exchanged line a is spacelike in the physical region of the production process.

In the rescattering process, Fig. 1(d), a more elaborate analysis is needed to show that the P singularity is not near the physical region. Note how this result differs from the case of a resonance pole of a rescattering amplitude, which does reoccur as a pole of a production amplitude. The P singularity, it turns out, is not near the physical region because it is on the other side of a cut due to the isobar-meson intermediate state of Fig. 1(d). This was first pointed out, in this connection, in an earlier work[4] and subsequently verified by several authors.[5-7]

The most direct demonstration is to recognize that Fig. 1(d) is a triangle diagram, with well-known singularities.[8] The location of the triangle singularity (Landau curve) is given by

$$\alpha^2 + \beta^2 + \gamma^2 - 2\alpha\beta + 2\beta\gamma + 2\gamma\alpha + \frac{\alpha\beta\gamma}{abc} = 0, \qquad (1)$$

where $\alpha = a[A^2 - (b+c)^2]$, $\beta = b[B^2 - (c+a)^2]$, and $\gamma = c[C^2 - (a-b)^2]$.

We are interested in the "elastic" case, $C = c$ and $B = b$. We are also particularly interested in the transition between stability and instability of the isobar; when $b \approx a + c$, Eq. (1) yields for the location of the triangle singularities in the A plane

$$A_\pm \approx b + c + [b - (a+c)]N_\pm, \qquad (2)$$

where

$$N_\pm = \frac{1}{2(b+c)}\left\{\left[\frac{b}{a}(c+b+a)\right]^{1/2} \pm \left[\frac{c}{a}(c+b-a)\right]^{1/2}\right\}^2.$$

Note that if a is stable (which is assumed), $N_\pm$ are positive; in the static limit, $a, b \to \infty$, we have $N_\pm \to 1$ and the logarithmic singularities $A_\pm$ degenerate into a pole.

In the stable case, $b < a + c$, we have $A_\pm < b + c$, i.e., the singularities lie below the normal bc threshold; it is known that they lie in the second sheet, reached through the bc cut. We can continue to the unstable case, passing the critical point $b = a + c$ by using the Feynman rule: The internal mass a should have a small negative imaginary part. Thus the singularities $A_\pm$ have a small positive imaginary part as they pass the normal threshold, and so are not close to the physical region; see Fig. 2(a).

For completeness we discuss briefly the treatment of b as an unstable particle; this treated at length in, for instance, reference 7. An unstable particle is, of course, not a physical state, and in diagrams such as Figs. 1(d) and 2(b), taking them either as Feynman diagrams or "unitarity" diagrams, the line b stands for a sum over physical states, such as the two-body state consisting of particles C and a. In the analogous diagram, Fig. 2(b), it is well known how a resonant pole in the C-a scattering amplitude occurs as a near-

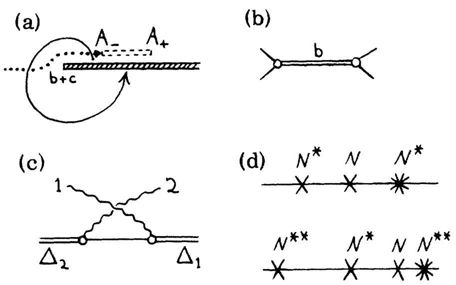

FIG. 2. (a) The location of the singularities of the diagram of Fig. 1(d) for the "elastic" ($C=c$ and $B=b$) and unstable ($b>a+c$) case. The branch point at $b+c$ is the normal threshold, the branch points $A_\pm$ are the triangle singularities. The solid arrow shows how $A_\pm$ are to be reached from the physical region. The dotted arrow shows the motion of $A_\pm$ as one continues from the stable case $b<a+c$. (b) A diagram in which b occurs as an intermediate state. (c) An isobar-meson scattering diagram of the type of Fig. 1(a) in the static model. (d) The poles in the ω plane (ω = meson energy) of the meson-N (upper diagram) and meson-N^* (lower diagram) scattering amplitudes in the static model. Those which lie at energies $\omega>m$ (m = meson mass) are really displaced off the real axis. The ×'s denote poles due to the exchange of the labelled isobars in the crossed channel; the *'s denote the pole due to the labelled isobar in the direct channel.

by pole in the amplitude of Fig. 2(b), and so in this sense the resonance b is here equivalent to a particle. In just the same way, the resonance b is equivalent to a particle in diagrams such as Fig. 1(d).

We would now like to discuss the possible effect of the P singularity as a "driving force" in dynamical models, that is, the possibility that it may be responsible for an isobar-meson ("three-body") resonance. Hwa,[5] followed by Gyuk and Tuan,[9] have argued that under suitable circumstances a three-body pole can occur close to the energy of a P singularity, but they have made no actual calculations. Peierls[10] has argued similarly that a P singularity in an isobar-meson scattering amplitude should generate a three-body resonance pole at a somewhat higher energy, the resonance being closer to the P singularity the stronger the coupling. We shall see below that there is some truth to this, although it seems that if the coupling is made strong, so as to make the three-body resonance pole approach the P singularity, the isobar becomes more tightly bound, so that the P singularity itself receeds to lower energy. Calculations involving the process of Fig. 1(a) have been done by Harrington,[11] Mandelstam, Paton, Peierls, and Sarker,[12] and Srivastava,[6] all with the result that no nearby pole was generated.

A very special model, which should be mentioned, is that of Peierls and Tarski.[13] This consists of a static scatterer on which two mesons scatter independently.[14] Thus if meson 1 (2) has a scattering resonance (or bound state) with excitation energy $\Delta_{1(2)}$ (i.e., energy above the ground state of the scatterer), then there is a three-body isobar with excitation energy $\Delta=\Delta_1+\Delta_2$. This occurs formally in the following way: Any production amplitude into the three-body state consisting of the scatterer, meson 1, and meson 2 has a factor[13]

$$1/D_1(\omega_1)D_2(\omega_2), \tag{3}$$

where $D_j(\omega_j)$ is the denominator, or Jost, function of the scattering of meson j; if the scattering of meson j has a resonance at energy Δ_j, D_j has a zero at $\omega_j=\Delta_j$. The factor, Eq. (3), exhibits no dependence on the total energy ω ($\omega=\omega_1+\omega_2$), but this dependence appears if we form the production cross section, which will be proportional to

$$\int_{m_1}^{\omega-m_2} d\omega_1 \frac{k_1k_2}{|D_1(\omega_1)D_2(\omega-\omega_2)|^2},$$

which has poles at $\omega=\Delta_1+\Delta_2$ and $(\Delta_1+\Delta_2)^*$. These poles will then appear in any amplitude in which the three-body state occurs as an intermediate state.[15] This "three-body" pole at $\omega=\Delta_1+\Delta_2$ is just at the P pole of the meson-isobar scattering of Fig. 2(c). The reason for this identity of the P pole and resonance pole is the very special circumstance that in this model the cuts corresponding to isobar-meson intermediate states (branch points at $\omega=\Delta_1+m_2$ and Δ_2+m_1) are lacking. When any departure occurs from the condition of independence of the interaction of mesons 1 and 2, the isobar-meson state cuts will appear, and the pole which is near the physical region, the resonance pole, will move away from the P pole (which, in fact, changes into a short cut as one leaves the static limit).

To get an idea of how far the three-body pole may be from the P pole in more realistic static models, we consider static models in which the basic meson-scatterer interactions are of Yukawa type, i.e., single meson emission and absorption. In these models, the scatterer is not static with respect to its spin or charge coordinates (we ignore the neutral scalar model), and so the scattering of two mesons will not be independent. One such model is the Lee model with Castillejo-Dalitz-Dyson pole added; this was investigated by Srivastava,[6] who found no resonance in the vicinity of the P-pole position.

A large class of models, not solvable exactly, but more physical than the Lee model, are simply the ordinary static models. According to the results of strong-coupling theory,[16] for strong enough coupling higher isobars (resonances or bound states) exist,[17] that is, the scatterer N has the excited states N^*, N^{**}, etc., where N is the ground-state multiplet of the scatterer, N^* ("isobar") is the multiplet of excited states which appears as poles in meson-N scattering, N^{**} ("higher isobar" or "three-body resonance") is the multiplet which appears (in addition to N^* and N^{**}) as poles in meson-N^* scattering, etc. For simplicity, let us speak of models in which the isobar multiplets (as well as the ground state) are each degenerate, writing Δ_1, Δ_2, etc. for the excitation energies N^*, N^{**}, etc. Then, due to the N-exchange diagram, meson-N^* scattering has a P pole at a meson energy of Δ_1, i.e., at a total excitation energy of $2\Delta_1$. The "three-body" resonant state N^{**}, appearing as another pole in meson-N^* scattering, is at $\omega=\Delta_2-\Delta_1$ [see Fig. 2(d)]. We know that $\Delta_2>2\Delta_1$: In the strong-coupling limit the isobars form a rotational band,[16] the energies satisfying $\Delta_n-\Delta_{n-1} =C_n(\Delta_{n+1}-\Delta_n)$, with $C_n<1$. In the strong-coupling limit, $g\rightarrow\infty$, the C_n are independent of g, and the Δ_n go as $1/g^2$; hence although as $g\rightarrow\infty$ the N^{**} resonance energy Δ_2 approaches the P-pole energy $2\Delta_1$ (as suggested in references 5 and 10), it does so only at the same rate that Δ_1 itself approaches 0, i.e., the ratio $\Delta_2/2\Delta_1$ goes to a constant, >1. For instance, in the $\pi N\,p$-wave static model $\Delta_n\propto(2n+1)(2n+3)-3$, thus $\Delta_2/2\Delta_1 =\frac{4}{3}$. The situation can be roughly described by saying that the meson-N^* scattering is rather similar to the meson-N scattering, hence in the former the distance that the resonance pole (N^{**}) lies higher than the crossed pole (the P pole), i.e., $\Delta_2-2\Delta_1$, is of the same order ($\propto 1/g^2$) as in the latter, i.e., Δ_1. A particularly simple situation holds in the charged scalar model[18]: Here the only crossed poles in meson-N and meson-N^* scattering are those due to N exchange, and the residues are equal (to order $1/g^2$); hence in the strong-coupling limit, meson-N and meson-N^* scattering are identical and so $\Delta_2-2\Delta_1=\Delta_1$, i.e., $\Delta_2/2\Delta_1=\frac{3}{2}$.

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[1]R. F. Peierls, Phys. Rev. Letters 6, 641 (1961).

[2]M. Nauenberg and A. Pais, Phys. Rev. Letters 8, 82 (1962); R. J. Oakes, Phys. Rev. Letters 12, 134 (1964).

[3]We emphasize that this statement applies to elastic isobar-meson scattering. If, instead, the process is inelastic, isobar + meson → isobar′ + meson′, with the masses of these particles suitably related, namely, $a+c<B<B_c$ where $B_c^2=(a+c)^2+(c/b)[(b-a)^2-C^2]$, the lower P singularity of Fig. 1(d) may be near the physical region. See P. V. Landshoff, Phys. Letters 3, 116 (1962); I. J. R. Aitchison, Phys. Rev. 133, B1757 (1964). The effect may be especially large when $b \approx a+c$: F. R. Halpern and H. L. Watson, Phys. Rev. 131, 2674 (1963).

[4]C. J. Goebel, University of Wisconsin report, 1962 (unpublished); the result was already implicit in reference 8.

[5]R. C. Hwa, Phys. Rev. 130, 1580 (1963).

[6]P. K. Srivastava, Phys. Rev. 131, 461 (1963).

[7]I. J. R. Aitchison and C. Kacser, Phys. Rev. 133, B1239 (1964).

[8]G. Barton and C. Kacser, Nuovo Cimento 23, 593 (1961).

[9]I. P. Gyuk and S. F. Tuan, Nuovo Cimento 32, 227 (1964).

[10]R. F. Peierls, private communication.

[11]D. R. Harrington, Phys. Rev. 127, 2235 (1962).

[12]S. Mandelstam, J. E. Paton, R. F. Peierls, and A. Q. Sarker, Ann. Phys. (N.Y.) 18, 198 (1962).

[13]R. F. Peierls and J. Tarski, Phys. Rev. 129, 981 (1964).

[14]That is, the scatterer is not only static in space (mass → ∞) but also in spin, charge, etc., so that the interaction with a meson does not change its space, spin, charge, etc., coordinates.

[15]That is, provided that the interaction of the two mesons does not remain independent, as it was in the original production amplitude, Eq. (3).

[16]References may be found in H. Jahn, Phys. Rev. 124, 280 (1961).

[17]It should be observed that for p-wave static models, the "strong-coupling" regime is characterized by the Δ_n being small compared to the cutoff energy; the isobars are not necessarily bound (unlike the case for s-wave static models). It should be noted that all the arguments and conclusions presented here are equally valid for bound, as well as unstable, isobars. The reason is that Imf, at energies small compared to the cutoff, is dominated by the δ functions at bound states and by the peaks at resonances; insofar as the resonance peaks can be approximated by δ functions, the resonances are equivalent to bound states. Specifically, the following are equally valid for bound and unstable isobars: (a) the absence of the singularities of Fig. 1(a) from the physical region for the processes of Figs. 1(c) and 1(d), (b) Peierls' argument (reference 10) for the induction of an isobar-meson higher isobar, (c) the solution, Eq. (3), of the Peierls-Tarski model, and (d) strong-coupling results.

[18]C. Goebel, Phys. Rev. 109, 1846 (1958).