Qualifying Exam in Analysis of Algorithms
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1. A tree is defined as a connected acyclic directed graph. Let $G = (V, E)$ be a acyclic directed graph with node set $V = \{v_1, v_2, \ldots v_n\}$ and edge set E. G is stored as an array of length n, with each component containing a linked list of neighbors. Give an algorithm that creates the parent array P for the tree, such that $P[k] = i$ if v_i is the parent of v_k. Let $P[j] = 0$ if v_j is the root.

   What is the best, worst, and average complexity of your algorithm? How much computer memory does it use? Justify how you measure complexity.

   Give an algorithm that takes two nodes v_j and v_k and determines if v_j is an ancestor of v_k using the parent array. Analyze and discuss the complexity of this algorithm.

   The best solution should have efficient algorithms and simple, clear analyses. If you cannot analyze your best algorithms, you might also propose simpler algorithms that you can analyze.

2. The following situation is a simplification of a popular computer game. On each turn a city of size n produces $a(n+1)$ units of food and the people in the city eat $2n$ units of food. Any food not eaten is put in a bin, and if there is a shortage of food, the needed food is taken from the bin. If the bin has no food and there is a shortage, then the city has size $n-1$ on the next turn, and the bin is still empty. If the bin has $10(n+1)$ units of food or more, then the city has size $n+1$ on the next turn and the bin is empty. On turn 0, the city has size 1 and the bin is empty.

   a. Write recurrence equations for the city size and for the amount of food in the bin.

   b. Consider positive integer values of a. Solve the equations exactly in those cases that you can. The cases $a = 1$, 2, and 10 are particularly simple. Say something short and useful about the solutions for all a.

   c. For which values of a is the growth of the city (exactly) linear? For which values is the growth rate exponential?

   d. Are there any values of a for which the long term growth rate is approximately linear? (You do not need to repeat the values for which it is exactly linear.) In what sense is the growth rate approximately linear?