Ph.D. Qualifying Exam
Algorithm Design and Analysis

Computer Science Department
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The algorithms part of this exam has one problem.

Problem

Consider the problem of optimally breaking a paragraph into lines. The input text is a sequence of n words. Suppose that we're give $LC_{i,j} \geq 0$ as inputs where $LC_{i,j}$ is the cost of having exactly words i through j on a single line. We want to break the text into lines so that the total cost of the text is minimized.

Let $c(j) \geq 0$ be the cost of an optimal arrangement of line breaks for words 1 through j. It can be defined recursively as:

$$c(j) = \begin{cases} 0 & \text{if } j = 0 \\ \min_{1 \leq i \leq j}(c(i-1) + LC_{i,j}) & \text{otherwise} \end{cases} \tag{1}$$

You are asked to do the following:

1. Show that direct calculation of $c(n)$ with equation (1) results in a running time of $O(n2^{n-1})$. You may need the following fact:

$$\sum_{i=1}^{n-1} ix^{i-1} = \frac{nx^{n-1}}{x-1} + \frac{1-x^n}{(x-1)^2} \tag{2}$$

2. Discuss strategies to improve the running time for computing $c(n)$?

3. Give an algorithm that computes $c(n)$ efficiently.

4. What are the time and space complexities of your algorithm?