# IU/CSD Qualifier Exam — FOUNDATIONS
## September 1996

Instructions: No questions to the proctor are permitted. If you should perceive an ambituity or an error on the exam, you should point out the difficulty at the top of your answer before you attack the question. Then state a *reasonable* reinterpretation of it that clears your difficulty; then answer the reinterpreted question. Any reinterpretation that trivializes a question is not a reasonable one.

1. Choose a programming language, and identify three "keywords" of your choice from its syntax. Excluding *only* these keywords (standing for finitely many other keywords), give a definition of the set of identifiers in that language as a formal language.

2. Two examples of conditional constructs in the language C appear below to illustrate the following challenge. Give a formal grammar specifying both conditional constructs and compound constructs in that language. You may use `<stat>` as a terminal in your grammar to represent any C statement.

```
if (a==0) a=1; else {a=2; b=3;}
if (d=e)  {c=5; d+=d;}
```

3. Prove in detail that $\{1^p \mid p \text{ is prime}\}$ is not a regular language. State clearly the style of proof that you use and state any theorem you need before you use it.

4. **MegaSoft Consortium** at Cow College has noticed that a lot of computer time is spent running computer-virus-detection programs that crash with floating-point exceptions before giving any output. It has submitted a proposal to the Department of Antiterrorism for a project to build a pre-compiler that identifies program–input pairs that exhibit this behavior. You have been asked to review this proposal for the institute-trained engineers in charge of antiterrorism.

Sight unseen, give a careful projection for the success of this project that can be understood by the department. Without appealing to any theorem, justify your projection, using programs one could presume available at the time MegaSoft delivers.

5a. Distinguish between the propositional calculus and the predicate calcu-

lus. Define what it means for a predicate-logic formula to be in conjunctive normal form.
5b. Consider the problem to discover the satisfiability of an arbitrary formula in the propositional calculus that is in conjunctive normal form. Give a bound on the time necessary to answer this question: yes or no, as a function of the number of variables and conjuncts.

6. Consider the problem to discover the satisifiability of an arbitrary statement in the predicate calculus, where all quantified clauses are in conjunctive normal form. Give a bound on the time necessary to answer this question: yes or no, as a function of the number of variables and conjuncts. Provide an argument to support your claim.

7. Let $p$ be any function programmed in your favorite programming language (a good computational model, of course!) Comment on the recursive enumerabilty of each of the following sets of programs $p$ and justify your claims.

- $A_1 = \{p|\ \text{codomain } p = \emptyset\}$;
- $A_2 = \{p|\ \text{codomain } p \neq \emptyset\}$;
- $A_3 = \{p|\ \text{domain } p = \emptyset\}$;
- $A_4 = \{p|\ \text{domain } p \neq \emptyset\}$;
- $A_5 = \{p|\ p \text{ terminates on all input}\}$.

8. All residents of the town of Anglopolis hold jobs selected from engineer, firefighter, grocer, hairstylist. All firefighters are engineers, but no grocers are engineers. No grocer is a hairstylist, and anyone who is not a hairstylist is an engineer. The spouse of the town's only engineer also lives there. Every firefighter is a grocer.

Express each of the statements above in some succinct logical formalism.

What is the profession of the engineer's child, who lives there too. Give a formal proof supporting your claim.

9a. Define the complexity class LOGSPACE and give two examples of problems in LOGSPACE, arguing why they are there.
9b. Prove that LOGSPACE is closed under composition.
9c. Prove that PTIME contains LOGSPACE.