Read the entire exam first. Use a separate blue book for your answers to this Automata/Computability/Complexity part of the exam. Also, you may answer questions in any order, but begin each answer at the top of a page, and number it clearly.

The adjectives *decidable*, *turing decidable*, and *recursive* may be used interchangeably. Similarly, *recognizable*, *turing recognizable*, and *recursively enumerable* mean the same thing.

The class **LOGSPACE** is defined as the class of languages whose words, w, may be recognized by a deterministic turing machine in *work*-space $O(\log |w|)$. **NPC** is defined to be the class of NP-complete languages.

1. A coke-pepsi machine is a finite-state automaton accepting L_{coke}, which is a subset of Σ^* where $\Sigma = \{d, q, c, p\}$

   Interpret d as a dime, q as a quarter (coins of American currency), and c as a button selecting a Coca Cola, and p as a button selecting Pepsi Cola. A string should be accepted if it ends in p or c and the coins immediately preceding such characters total more than \$0.50. Now fifty cents is cheap for a can of soda, but this machine gets to keep all excess payments. That is, $dddqp$ is accepted but $dddqpddc$ is not and the last twenty cents are lost.

   Give a definition of a coke-pepsi machine. Try to keep it small. L_{coke}.

2. Let Σ be the alphabet $\{a, b\}$. A *palindrome* over Σ is a word $w \in \Sigma^*$ such that $w = \mathbf{reverse}(w)$. For example $abba$ is a palindrome, but aab is not.

   (a) Prove that the language generated by the following grammar is precisely the language of palindromes over Σ. (Hint: use mathematical induction to make your argument formal.)

   $$\begin{aligned} S &\to \epsilon \\ S &\to a \\ S &\to b \\ S &\to aSa \\ S &\to bSb \end{aligned}$$

   (b) Prove that the language of palindromes is not a regular language.

   (c) Describe a two-tape Turing machine that decides the palindrome language in *linear* time.

3. Consider the class LOGSPACE of the recursive functions over the natural numbers (represented as binary strings).

   (a) Consider the function f such that $f(n)$ equals the number of 0's in n's binary representation. Prove that f is in LOGSPACE.

   (b) Consider the function $g(n, m) = n^m$. Argue that g is *not* in LOGSPACE.

4. Consider the class P of polynomial time decidable languages.

   (a) Prove that the class of regular languages is a subclass of P.

   (b) Prove that the language

   $$L_P = \{\langle M \rangle \mid \text{Turing machine } M \text{ decides a polynomial time language }\},$$

   where $\langle M \rangle$ is an encoding of the TM M, is not a decidable language.

   (c) Prove that there exists a recursive language that is not in P.

5. Consider the language operators

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   union
   complementation
   concatenation
   Kleene star formation
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   Select each operator under which NPC is closed and give an argument that establishes the correctness of your selection(s).

6. The theory of computation has at its core the class of Turing recognizable languages rather than the class of decidable languages. Give a philosophical argument that establishes the reason for this situation.