Towards High Order Constraint Representations for the Frequency Assignment Problem

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Abstract

The assignment of frequencies to radio transmitters in a mobile telecommunications network in such a way as to provide total coverage for a designated area is a problem of enormous interest to the telecommunications industry. This Frequency Assignment Problem has traditionally been modelled with binary constraints making it analogous to generalised GRAPH COLOURING (A well known problem from discrete mathematics).

In this paper we examine the use of non-binary high order constraints for the frequency assignment problem. The high order constraint representation allows small groups of interfering transmitters to be constrained by a single constraint that would have otherwise been constrained by a collection of binary constraints forming a clique. These higher order constraints are then solved by a high arity constraint solver.

The results show that the application of high order constraints to the Frequency Assignment Problem introduces some flexibility at a local level that is not possible with the binary constraint model. This is the first time that high order constraints have been used to model and find solutions to the Frequency Assignment Problem and the results in this paper show that this novel technique may provide a useful new tool for frequency planning.

Keywords : *non-binary constraints, frequency assignment, constraint satisfaction, modelling*

1 Introduction

A Constraint Satisfaction Problem consists of a set of variables, some subsets of which are constrained to take only certain simultaneous values [13]. A solution is an assignment to all of the variables such that each constrained subset takes an allowed set of values.

This is a straightforward definition and many real world questions can be expressed naturally as constraint satisfaction problems. However, the representation of a real-world question as a constraint problem sometimes requires considerable artistry and skill. It is not always clear from the original formulation of the problem what the variables should be, what their domains of possible values might be, which sets are to be constrained, or indeed, how to specify the constraints.

It has often been observed [16] that any constraint formulation is equivalent to one in which the maximum number of constrained variables in any constraint is two. Such a problem is called a *binary* constraint problem. The simplicity of their structure makes binary constraints very attractive, and indeed, much constraints work has concentrated on the world of binary constraints.

The wealth of theory that has been produced around the solving of binary constraints means that it is often seen as most natural and effective to model any real-world question with binary constraints. It is our contention that such a simplification is not always helpful, and may sometimes lead to poor models with sub-optimal solutions. We shall argue for this conclusion from a study of the Frequency Assignment Problem.

1.1 The Frequency Assignment Problem

In the classic Frequency Assignment Problem (FAP) we have a geographical area (say England) that we wish to serve with some broadcast service (say television). To achieve this we will erect transmitters that broadcast this service using radio transmission. The strength of any radio signal decreases with distance from the transmitter, so the first obvious requirement is that every member of our intended audience should be close enough to one of our transmitters to receive adequate signal strength.

What makes the problem interesting is that the signals from different transmitters can interfere with each other. If I am in an area that receives signals from two nearby transmitters then my receiver must be able to pick up one signal without undue interference from the other. This imposes constraints on the allowable assignment of frequencies to transmitters. The natural solution to this problem is to assign widely different frequencies to all of the transmitters so that each receiver can tune into its desired signal and filter out all the others.

However, radio spectrum is a limited natural resource, and widely separated frequencies are not always available. In fact, it is increasingly the case that broadcasters have to pay a regulatory body for the radio spectrum that they wish to use [1], so it is in their interest to use fewer frequencies. This means that in many situations it is desirable to re-use frequencies as often as possible over the network of transmitters. It is this requirement - to minimise the number of distinct frequencies assigned to the whole network of transmitters - that makes the FAP hard.

To summarise, in the frequency assignment problem we have to assign frequencies to transmitters in such a way as to provide adequate service without excessive interference over a given geographical area, using the smallest possible range of frequencies.

1.2 Choice of Models

The requirements of the FAP could be modelled in a number of different ways, each leading to distinct solution strategies. The approach we consider in this paper is to model the FAP as a Constraint Satisfaction Problem (CSP) in which the variables are the transmitters, the domain of possible values for each variable is the set of available frequencies for the transmitters, and the constraints restrict the values which can be assigned to some subsets of transmitters.

Firstly we observe that the need to avoid interference can be expressed as a requirement that nearby transmitters should use frequencies which are widely enough separated that they will not cause excessive interference. This implies that the nearer transmitters are to each other the further apart their assigned frequencies must be. One obvious way to model this requirement is by using binary constraints on nearby pairs of transmitters which force them to be assigned frequencies at least a certain distance apart [3, 8, 10, 11, 12, 17]. This formulation has had considerable appeal since it represents the problem using a graph in which the edges are labelled with these binary constraints, and when it is formulated in this way the problem is reminiscent of the well-studied GRAPH COLOURING problem. This similarity has been pointed out many times [2, 3, 4, 14, 19].

We note that a number of recent studies have raised concerns about the adequacy of the standard binary CSP model for some versions of the FAP [3, 5, 15] and have proposed that the requirements of the problem can be more adequately expressed by allowing the use of both *hard* and *soft* binary constraints. This means that the problem is reformulated as an optimisation problem, or partial CSP, but all of the constraints are still binary.

A rather different approach, which has so far received much less attention, is to look at small groups of transmitters (containing more than two transmitters) and determine from the original problem specification which assignments are valid for these subsets of transmitters. This information can then be expressed as a constraint on each of these subsets of transmitters, and standard constraint solving techniques can be used to find an assignment to all of the transmitters which satisfies all of these higher order constraints simultaneously. The possibility of modelling the FAP in this way was first suggested in [6].

So, as with many other problems, we can model the FAP as a binary CSP, or as a higher order CSP. The binary model is very well-established, seems very natural, and is similar to a classic problem from discrete mathematics. On the other hand, the higher order model has been considered by very few researchers, and there has been almost no discussion in the literature about how to choose which sets of transmitters to constrain, how to determine the appropriate constraints, or how to represent these constraints efficiently.

In the following sections we will describe one way in which the higher order approach can be carried through, and present some evidence to show that it deserves to be investigated further. We do this by examining a small example problem and showing that even a naive “guess” at the structure of the required high order constraints allows us to find good solutions which are not so easily found using the standard binary model.

2 Defining a FAP

In this section we give a more precise definition of the particular type of FAP that we shall consider.

In general, a radio communications network consists of a set of transmitters and receivers in a given region, where each transmitter has the following properties:

(a) A position in the region.

(b) An output signal which has an associated

   - frequency spectrum,
   - power,
   - directional distribution.

Each receiver responds to the signals from each of the transmitters, to a greater or lesser extent. Some of these signals are generally "wanted" or "carrier" signals, which are used for communication, and others are regarded as "interference".

For the frequency assignment problems we consider in this paper we make the following standard simplifying assumptions, which are appropriate to model systems such as mobile telephone networks and public broadcast services.

(a) The positions of transmitters are fixed.

(b) The output signal from each transmitter has

   - a single frequency to be assigned from a set of uniformly spaced channels,
   - a fixed power (constant across all transmitters),
   - a fixed and uniform directional distribution.

(c) The receivers must be able to function at any point in the region.

(d) The desired carrier signal at each receiver is the strongest signal that is available at that receiver, and all other signals are regarded as interference.

The problem we are then faced with is to assign frequencies to the given transmitters in order to achieve some specified minimum acceptable value for the ratio of the carrier signal strength to the combined strength of the interference signals, at all positions in the region of interest. This ratio is generally referred to as the "carrier-to-interference ratio" (CIR).

The strength of the carrier and interference signals at any given position depends on the following factors.

- **Signal propagation**. The signal strength available from a given transmitter at a given point is dependent on the power and direction of the transmitted signal, the distance from the transmitter, and the terrain in between, as well as unpredictable factors such as atmospheric conditions.

  In the simplified model used below we shall assume that the signal strength can be approximated as a simple function of distance.

- **Signal Frequency**. As each receiver may be tuned to respond most strongly to particular frequencies, the effective signal strength from any transmitter depends on the frequency of the transmitted signal.

  In the simplified model used below we shall assume that the receiver is tuned to the frequency of the desired carrier signal, and the effective signal strength of all signals at different frequencies is reduced by a "filtering" factor, which depends on the (absolute) difference in frequency from the frequency of the carrier signal, and on the design of the receiver.

With these assumptions, the type of problems we consider consist of:

- A region - R.
- A set of transmitters - $\{T_1, T_2, \ldots, T_n\}$ at fixed positions.
- A set of available frequency channels - $\{1, 2, \ldots, k\}$.
- A collection of rules governing signal propagation and receiver response.
- A required minimum carrier-to-interference ratio - σ.

The rules for signal propagation and receiver response which we shall use throughout the paper are based on values commonly used in the literature for analogue mobile telephone networks [18, 7]:

1. The signal strength at any point is given by

$$\frac{1}{d^4} \tag{1}$$

   where d is the distance from the point to the transmitter.

2. The strength of all signals on a frequency channel which is separated by an absolute difference of s channels from the wanted signal is filtered at the receiver by a factor of

$$\begin{cases} -15(1 + \log_2 s) \text{ dB} & \text{if } s > 0 \\ 1 & \text{if } s = 0 \end{cases} \tag{2}$$

3. The total strength of the interference signals is obtained by summing the interfering signal from each transmitter.

Finally, we shall assume that the required minimum carrier-to-interference ratio, σ, at all points in the region of interest is 15dB.

2.1 Example - A hexagonal grid of transmitters

The running example which we shall refer to throughout the paper consists of a collection of 37 transmitters arranged in a regular hexagonal grid, as shown in Figure 1.

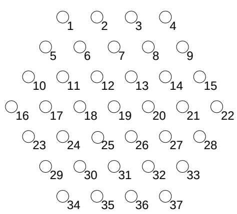

Figure 1: A regular grid of transmitters

Each of the transmitters in Figure 1 is situated in the centre of a hexagonal *cell*, as indicated in Figure 2. For any receiver within this cell the desired carrier signal is the signal from the central transmitter.

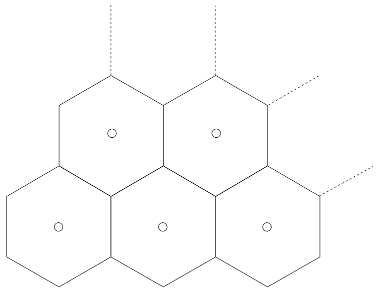

Figure 2: Some transmitter cells

To find an assignment of frequency channels to all of these transmitters which achieves the required carrier-to-interference ratio of 15dB everywhere we can simply check each possible assignment to see whether it satisfies this requirement. One assignment using 5 channels is illustrated in Figure 3, where the number in each cell represents the frequency channel assigned to the transmitter in that cell.

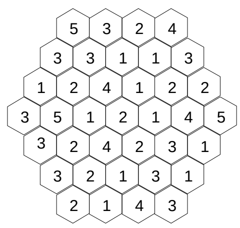

Figure 3: A frequency assignment for the example problem

When we calculate the carrier-to-interference ratio resulting from this assignment at all points within the region of interest, using the propagation rules and receiver characteristics described above, we find that there are many points at which this ratio falls below the required value of 15dB. This is illustrated in Figure 4, in which the dark areas indicate points at which the calculated carrier-to-interference ratio is below 15dB. In fact, if we consider the central rectangular region between the top and bottom rows of transmitters, then the proportion of points at which the calculated carrier-to-interference ratio is above the required value of 15dB is only 60%. This figure will be referred to as the *coverage percentage* of the solution.

In order to cut down the search space of possible assignments, by eliminating poor solutions of this kind, we can impose constraints on some subsets of transmitters, and we now examine possible ways in which this can be done.

3 Binary Constraints for the FAP

Binary constraints for the FAP are imposed on certain pairs of transmitters and restrict the frequencies that can be assigned to those transmitters simultaneously.

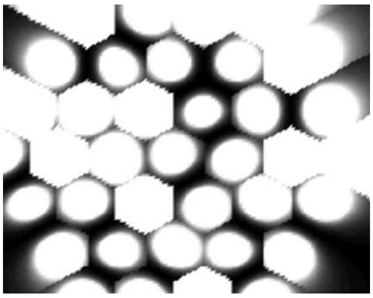

Figure 4: Results from binary constraint set one

For any pair of transmitters, T_i and T_j, we denote the frequency channels assigned to these transmitters by f_i and f_j respectively. Since the interference caused by signals from T_j in the cell surrounding T_i depends on the absolute difference between f_j and f_i, the standard form of binary constraint that is used in models of the FAP is as follows:

$$|f_i - f_j| \geq k$$

where k is a positive integer.

It is not generally necessary to impose a binary constraint on every pair, T_i and T_j, of transmitters in a network because the interfering signal from a transmitter T_i falls away with distance, so provided that T_j is far enough away from T_i there is no need for any binary constraint between them. Hence in any problem instance we need to decide which pairs of transmitters to constrain.

3.1 Binary Constraint Set One

Reconsider the example problem with 37 transmitters introduced in Section 2.1. Since the strongest interfering signals in the cell around any transmitter T_i, will come from the transmitters in the network that are closest to it, we first consider imposing binary constraints between each transmitter and each of its immediate neighbours in the grid. The positions of some of these binary constraints are illustrated in Figure 5

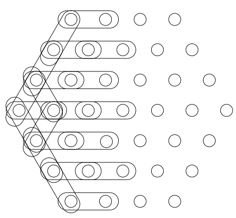

Figure 5: Placing the binary constraints

We set the binary constraint on each neighbouring pair T_i and T_j as follows:

$$|f_i - f_j| \geq 1.$$

These constraints ensure that neighbouring pairs of transmitters cannot be assigned the same frequency channel.

3.2 Solving the Binary Constraint Problem

We now have a fully defined constraint satisfaction problem to represent our original FAP, and this problem can be solved with any commercially available constraint solution package. For this study we used the package FASoft [9], which is specifically designed to deal with binary constraints of this nature.

The problem is solved quickly (less than 30 seconds) by the package and the first solution returned is shown in Figure 6.

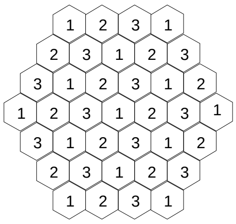

Figure 6: A solution to binary constraint set one

This is a solution to the binary constraint problem, but that does not mean that it is necessarily a solution to the original frequency assignment problem. When we calculate the carrier-to-interference ratio resulting from this assignment at all points within the region of interest, using the propagation rules and receiver characteristics described above, we find that there are still many points at which this ratio falls below the required value of 15dB. In fact, the coverage percentage is still only 68%. This is illustrated in Figure 7, in which the dark areas indicate points at which the calculated carrier-to-interference ratio is below 15dB.

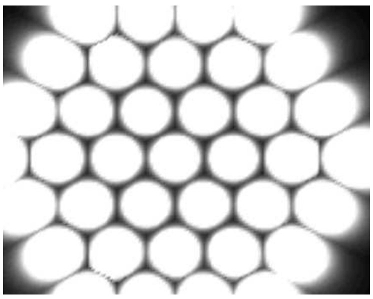

Figure 7: Results from binary constraint set one

3.3 Binary Constraint Set Two

The constraint representation described in Section 3.1 is clearly not restrictive enough. As a first step to increase the percentage coverage we shall try replacing the binary constraints defined in Section 3.1 with a more restrictive set of binary constraints.

Hence, we now set the binary constraint on each neighbouring pair of transmitters T_i and T_j to the following:

$$|f_i - f_j| \geq 2.$$

This constraint states that the corresponding transmitters must be allocated frequency channels that differ by at least **2**, i.e., they cannot use the same frequency channel or even an adjacent frequency channel. This constraint should have the effect of reducing interference around each transmitter as the neighbouring transmitters will be contributing a reduced level of interference.

This second formulation of the problem was also solved using the FASoft [9] package. A first solution was again found quickly by the package and is shown in Figure 8.

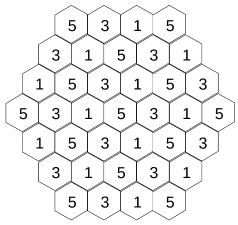

Figure 8: A solution to binary constraint set two

The coverage percentage for this solution at 73% is an improvement on the previous solution but not by a great amount. The coverage pattern shown in Figure 9 illustrates that there are still large dark areas where the interference is too high, and hence the carrier-to-interference ratio is too small.

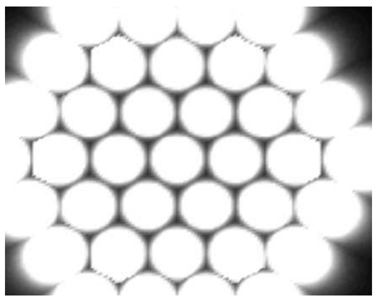

Figure 9: Results from binary constraint set two

Hence this assignment is still not acceptable as a solution to the original problem. There are a number of possible remedies which could now be adopted:

1. The binary constraints on neighbouring transmitters could be further tightened. This would have the effect of increasing the number of channels required to find a solution and may not increase the coverage percentage very significantly.

2. More binary constraints could be included to constrain transmitters at a greater distance than the immediate neighbourhood. This is the standard approach taken when only binary constraints are used. Note that even in this simple case it is not clear how to choose an appropriate mix of constraints for a variety of distances.

3. More flexibility could be introduced into the constraints on immediate neighbours by introducing higher order constraints. This approach has

not been described in any previous study, and is the approach considered in the next section.

4 High Order Constraints for the FAP

There are a number of different ways to represent a constraint over an arbitrary set of transmitters:

Absolute Values

Any constraint on an (ordered) set of transmitters, say $(T_0, T_1, T_2, T_3, T_4)$, can be defined by giving a set of lists of frequency channels that can be simultaneously assigned to the transmitters, e.g., $\{(4, 5, 6, 7, 8), (2, 6, 4, 3, 6), \ldots, (8, 8, 8, 8, 8)\}$

For most typical instances of the FAP the size of the appropriate constraints represented in this way is too large to be practicable, and for this reason we shall not represent the high order constraints used here in this way.

Relative Values

In the signal propagation model described above, the interference caused by one transmitter in the cell around another depends only on their distance apart and the absolute difference of their assigned frequency channels. Hence, a more compact way to express most constraints we might wish to impose on a set of transmitters in the FAP is to represent the set of allowable frequency channel assignments as relative values with respect to some *reference* transmitter. This method significantly reduces the number of tuples required to represent the constraint.

Using this representation, a constraint on the (ordered) set of transmitters $(T_0, T_1, T_2, T_3, T_4)$, where the reference transmitter is T_0, is represented as a set of allowed tuples of the form $(d_0, d_1, \ldots, d_4)$ where each d_i specifies the absolute difference in frequency channels between the assignment to transmitter T_i and the assignment to the reference transmitter T_0. (The first value in the tuple will always be 0 as the channel assigned to the reference transmitter will always be 0 frequency channels away from itself.)

For example, the tuple $(0, 2, 2, 2, 3)$ states that transmitters (T_1, T_2, T_3) must all be exactly 2 frequency channels away from the channel assigned to T_0, and that T_4 must be exactly 3 channels away. So, any constraint whose representation contains this tuple allows the assignment $(4, 6, 2, 6, 7)$, but not $(4, 6, 2, 6, 8)$, as in the latter case the channel assigned to T_4 is 4 channels away from the channel assigned to T_0 rather than 3.

Minimal Relative Values

For some forms of constraint it is possible to reduce the number of tuples in the representation of the constraint still further. To do this, we impose, a standard partial ordering on tuples of relative values, by setting $t \leq t'$, for any pair of tuples t and t', if $t[i] \leq t'[i]$ for each co-ordinate position i. Now we can represent the set of all allowed tuples by storing only the *minimal* allowed tuples according to this ordering. When we are using this representation, we say that an assignment is allowed by the constraint if and only if the corresponding tuple of relative values is greater than or equal to some tuple in the constraint representation.

For example, if the constraint representation contains the tuple $(0, 2, 2, 2, 3)$, then the constraint allows the assignments $(4, 6, 2, 6, 7)$, $(4, 6, 2, 6, 8)$, $(4, 7, 1, 6, 8)$, and $(3, 1, 1, 6, 7)$,(along with a great many others).

4.1 A High Order Constraint Set

Consider again the example problem with 37 transmitters introduced in Section 2.1. On which groups of transmitters would it be helpful to impose higher order constraints?

The binary constraints described in Section 3.1 constrain pairs of neighbouring transmitters, so a natural first step is to consider imposing constraints on the complete set of neighbours of each transmitter. See Figure 10.

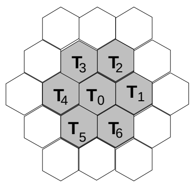

Figure 10: Immediate neighbourhood of transmitter T_0

How should the transmitters $T_0, T_1, T_2, T_3, T_4, T_5, T_6$ be constrained? It is possible to represent a mixture of the two different binary representations described above (Section 3.1 and Section 3.3) by requiring that, out of the six transmitters around the central transmitter T_0, half of them must be assigned channels at least one channel away from the channel assigned to T_0, and the other half must be assigned channels at least two channels away from the channel assigned to T_0. This constraint can easily be represented by using *minimal relative values*, giving the following set of tuples:

$$\{(0,1,1,1,2,2,2),(0,1,1,2,1,2,2),(0,1,2,1,1,2,2),(0,2,1,1,1,2,2),$$
$$(0,2,1,1,2,1,2),(0,2,1,2,1,1,2),(0,2,2,1,1,1,2),(0,2,2,1,1,2,1),$$
$$(0,2,2,1,2,1,1),(0,2,2,2,1,1,1)\}$$

Note that this set of 10 tuples is much smaller than the set of all allowed assignments for these transmitters which it represents.

We will apply this constraint to all groups of seven transmitters that are centered on a transmitter that is not on the perimeter of the hexagonal grid. See Figure 11.

It should be noticed that this constraint cannot be applied to the neighbourhood groups of *all* transmitters in the region.In particular, the transmitters on the boundary of the hexagonal grid that have only four neighbouring transmitters need a different constraint. The constraint we shall use for these groups of transmitters is a 5-ary projection of the 7-ary constraint described above, and is represented by the following set of minimal tuples:

$$\{(0,1,1,1,2),(0,1,1,2,1),(0,1,2,1,1),(0,2,1,1,1)\}.$$

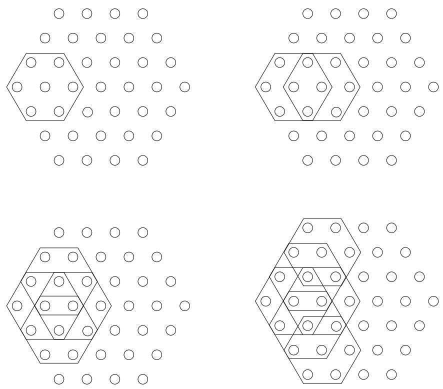

Figure 11: Placing the arity 7 constraints

This constraint will be applied to the groups of transmitters shown in Figure 12

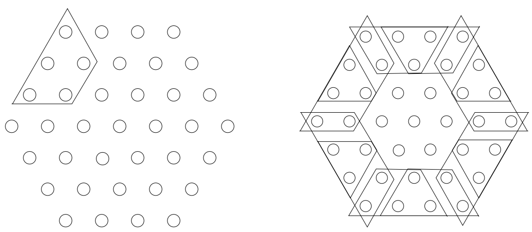

Figure 12: Placing the arity 5 constraints

Finally, the groups of transmitters on the corners of the hexagonal grid that are adjacent to three other transmitters will also require a different constraint. Again, we simply take a 4-ary projection of the 7-ary constraint described above, giving a constraint which is represented by the following set of minimal tuples:

$$\{(0, 1, 1, 1)\}.$$

4.2 Solving the High Order Constraints

We do not have access to any suitable commercial software tool for solving these high order constraints, so this representation was solved with a software tool developed by the authors specifically for the task of solving constraints of arbitrary arity. This tool solved the particular high order constraint problem described in the previous section in approximately 3 minutes. A simple backtracking method was used, incorporating some heuristic pruning methods. The first solution found is shown in Figure 13.

This solution gives a coverage percentage of 91% without any increase in the number of frequency channels required. The coverage pattern shown in Figure 14 illustrates that there are still some dark areas but the overall coverage is significantly improved. Figure 15 lists the coverage results from all of the methods we have described so far.

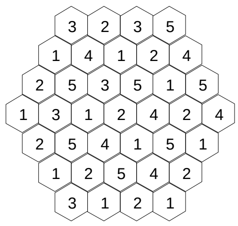

Figure 13: A solution to the high order constraints

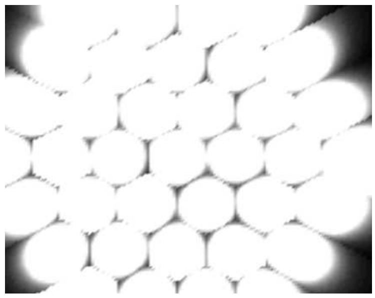

Figure 14: Results from high order constraint representation

5 Binary Constraints Revisited

Examining the solution obtained to the high order constraint model above suggests that simply breaking the symmetry of the binary constraint representation may result in a better quality of solution. To test this idea we constructed a new binary constraint representation containing a random mixture of the previously used binary constraints. The constraints were chosen as follows:

$$|f_i - f_j| \geq \begin{cases} 1 & \text{if } Rnd > 0.5 \\ 2 & \text{otherwise} \end{cases}$$

where Rnd is a random number in the range $[0..1]$, generated for each constraint.

One such randomized binary constraint representation was solved with FASoft and gave a coverage percentage of 90%, which represents a considerable improvement over both previous binary representations. However the effect of introducing this random variation is naturally rather unpredictable. When the experiment was repeated many times, the coverage percentages ranged in value from 76% to 90%.

Representation	*Coverage (Percentage)*
Binary Set One	68
Binary Set Two	73
High Order	91

Figure 15: Table of coverage statistics

6 Conclusions

In this paper we have begun to explore the use of higher order constraints for representing the FAP. We have chosen one simple candidate for a higher order constraint which is a natural generalisation of the standard binary constraints, and has the merit of being simple to represent and to manipulate. Hence we have overcome the objection that high order constraints would be hard to generate and represent.

A second objection to the use of high order constraints has been the belief that solutions would take considerably longer to find for a network of high order constraints than they do for networks of simple binary constraints. Our work does not support this view. In fact both representations were computationally tractable.

We have obtained significantly better results using our naive high order representation than we obtained using a standard binary model. We have used this information to feedback into the choice of binary models, by using a randomised mixture of simple binary constraints to simulate the "balance" of the high order representation. Our success here motivates further research into finding optimal binary constraints.

Clearly further work is needed before a fair comparison of high order and binary models for the FAP can be made. What we have done here is established the feasibility of the high order approach and provided a first piece of motivation for that research.

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