Principal Components Analysis

Graphical representation and analysis of higher-dimensional data becomes a very difficult problem when the dimensionality of the feature space (in which the objects are represented as patterns or points) is increased beyond three. As a consequence of this limitation of human visual perception, visual identification of relationship among data points, for example, identifying clusters, is not possible. Several alternative approaches such as graphical techniques and projection methods have been proposed to visualize and analyze high dimensional data. The graphical techniques although novel in their approach, do not yield useful results when the number of patterns and features are large. Projection algorithms on the other hand provide an effective representation in a lower dimensional space, while preserving the information content as much as possible. The projection technique chosen for a particular application depends on whether the approach is *linear* or *nonlinear* and on the availability of category information (*unsupervised* or *supervised*). Irrespective of the approach chosen, the aim is to retain as much structural information of the original high-dimensional data as possible in the lower dimension.

The category information is not available in a majority of applications. The linear eigenvector projection method also called the principal component analysis (PCA) approach, is the most commonly used approach in such (unsupervised) cases. This method involves computation of covariance matrix of the input patterns, its eigenvalues and eigenvectors. The eigenvectors define a linear transformation which not only expresses the new features as a linear combination of the original features, but in doing so also:

- makes the new features uncorrelated
- provides the largest spreads (variance)
- minimizes the mean-squared error
- maximizes the scatter

Due to its simplicity, linearity, well understood mathematical properties, data visualization (projection) and feature extraction capabilities, this method is the most favored approach. It may however be noted that the entire process is *sequential* in nature because of which the time and complexity involved in the computations increases rapidly as the dimensionality of the feature space increases.

Neural network architectures have been developed for a number of applications in both supervised and unsupervised cases. This approach is particularly useful and convenient to the eigenvector problem because there is no *explicit* computation of the covariance matrix, its eigenvalues and eigenvectors. The network 'learns' on its own based on the input patterns presented, and after 'learning', the parameters (weights) of the network specify the eigenvectors. This method not only simplifies the computational burden but also leads to possible hardware implementation. Analog circuit implementation being dependent only on the time constants of the circuits would then eliminate the time complexity involved thereby making it suitable for real-time applications. A number of neural network models have been proposed to compute the eigenvectors directly but none of them leads to a compact circuit realization. A recent interest in the field of analog VLSI is to provide circuit implementation for various existing algorithms.

Computing the largest eigenvector also called the maximal principal component is the first major step towards such implementation, because it contains the maximum information pertaining to the original data. We at the SCANN lab have formulated a new model for the computation of principal components that is tailored for circuit implementation. The model uses MOS elements and differential pairs which operate in the subthreshold regime of MOS operation. Consequently, power consumption is substantially reduced.

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