Modules in Numerical Relativity

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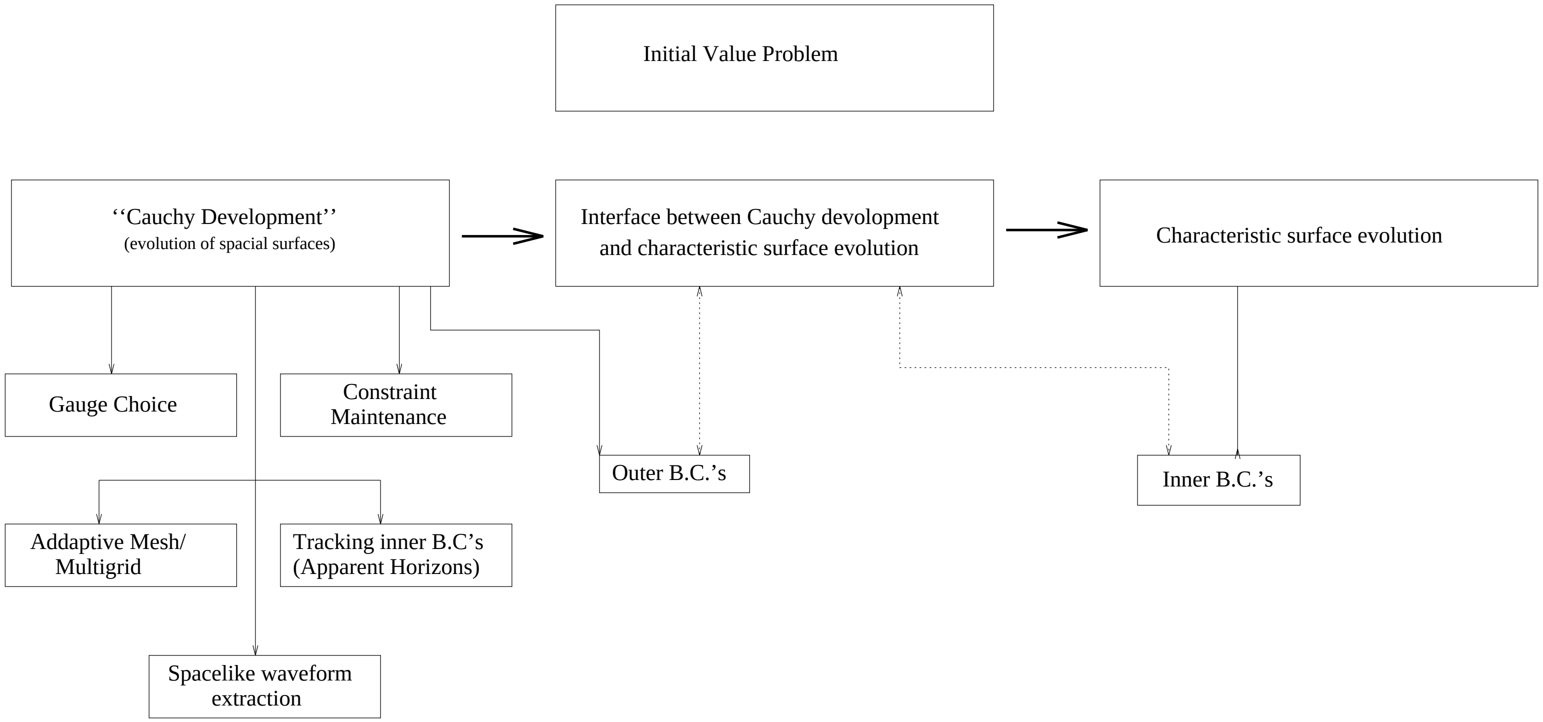

1 Choice of Physical Domain

- spherical symmetry (1 spatial dimension); spherical coordinate (r)
- axisymmetry (2 spatial dimensions); spherical polar coordinates (r, θ)
- no symmetry (3 spactial dimensions)
  - cartesian coordinates (x, y, z)
  - spherical coordinates (r, θ, ϕ)
  - others (i.e. Cadez coordinates)

2 Initial Value Problem

- specify initial data that satisfies 4 elliptic "constraint equations"
  - "copy" from known analytic solution
  - use elliptic solver to solve equations
    * identify parameters for 2 BH solutions to constraint equations

3 Cauchy Development (evolution of spatial surfaces)

- hyperbolic equations
- choice of methodology for approximating PDE solutions
  - finite differencing with structured mesh (current "method of choice")
  - finite elements (Regge Calculus)

3.1 Gauge Choice

- lapse and shift functions must be chosen in order to specify Cauchy development
  - $q^{ij}K_{ij}$ = const (Trace(K) = const) gauge
    * this choice of gauge has "singularity avoidance" properties, but the payment is solving an elliptic equation on each spatial surface (that is, for each time step)
  - Harmonic gauge (shift = 0)
    * simplification of evolution equations, but at the cost of possible numerical instabilities, grid stretching, etc.
  - Algebraic gauge
    * easy to impliment (no elliptic equation to solve, as in Tr(K) = const), but is physically hard to justify

3.2 Constraint Maintenance

- although analytic evolution maintains the constraint equations (they stay solved throughout the evolution), errors introduced by finite differencing make the constraints "stray" during evolution.
  - method for constraint enforcement (if at all)

3.3 Tracking Apparent Horizons (Inner Boundary)

- for some (but not all!!) coordinate choices, there are inner boundaries on the computational grid
  - put the inner boundary "a little beyond" apparent horizons; this means you have to track apparent horizons during evolution – this can be tricky.

3.4 Addaptive Meshes, Multigrid

- criterion for adaptation

3.5 Spacelike waveform extraction

- it is possible to extract wave information inside the Cauchy devolopment (in spatial slices)
  - requires large(r) computational grids
  - still have to worry about outer boundary conditions (outgoing waves)

4 Interface between Cauchy Development and Characteristic Surface Evolution

- IMPORTANT: must suppress numerically induced reflections while allowing physical backscattering (may be difficult)

5 Characteristic Surface Evolution

- extract waves relevant to LIGO measurements (certain frequency band)