Status Report on Grand Challenge Activities

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1 Apparent Horizons

1.1 Finding Apparent Horizons

The apparent horizon is a 2-surface (with the same topology as a 2-sphere) in the 3-d spatial hypersurface at which the expansion function, Θ, is zero. The expansion function is

$$\Theta = \nabla_i s^i - K + K_{ij} s^i s^j \tag{1}$$

where s^i is the spatial vector that is unit normal to the surface, K_{ij} is the extrinsic curvature, and K is the trace of the extrinsic curvature.

1.1.1 NCSA

The NCSA effort for finding apparent horizons is summarized in the paper gr-qc/9412058. First, they write down a general surface as an expansion using certain basis functions (e.g. spherical harmonics). Specifying the coefficients in that series is equivelant to specifying a surface. They then search for coefficients that minimize the integral of the square of the expansion function,

$$\int dS^2\, \Theta^2 = 0 \tag{2}$$

If the apparent horizon is smooth, then only a few terms in the expansion will only be necessary. However, apparent horizons can be highly distorted, so in practice, more terms will be needed.

1.1.2 University of North Carolina, Chapel Hill

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1.2 Boundary Conditions at Apparent Horizons

Once the Apparent Horizon has been found on a certain t = constant slice, the inside (where the singularity is forming) is cut out, which leaves a boundary on the computational grid. The remaining question is how to deal with this boundary.

1.2.1 NCSA

In the document gr-qc/9412069 (submitted to Phys. Rev. D) " Horizon Boundary Condition for Black Hole Spacetimes ", NCSA puts forth its "boundary without a boundary" implimentation. The formalism presented is based in spherical symmetry. Because of this, some of what is presented will be able to be implimented in full 3D case, and some will not. The basic scheme has two key steps:

- Lock the position of apparent horizons to a fixed coordinate location:

  $$\left(\frac{\partial\Theta(r)}{\partial t}\right)_{r=r_{AH}} = 0 \tag{3}$$

  Here, Θ is called the expansion function, whose zero defines the location of the apparent horizon.

- Prevent pathological behavior of coordinate system (grid points may be "sucked in" and then crash into apparent horizon. NCSA presents four different ways to choose the shift such that the coordinate system stays nice:
  - distance freezing shift : choose shift vector so that the proper distance between grid points is constant.
  - area freezing shift: keep the area of surfaces of constant radial coordinate constant (may not be robust enough for full 3D case)
  - expansion freezing shift : keep expansion function constant
  - minimal distortion shift : minimizes the global coordinate shear (although this method is more complex numerically, it has a geometrical basis, and can be implimented in any coordinate system)

Having carried out the above, the causal structure at (or inside) the apparent horizon allows a differencing scheme that requires no information from inside the apparent horizon in order to update the points on the apparent horizon. This is called the "boundary without a boundary condition".

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1.2.3 Cornell

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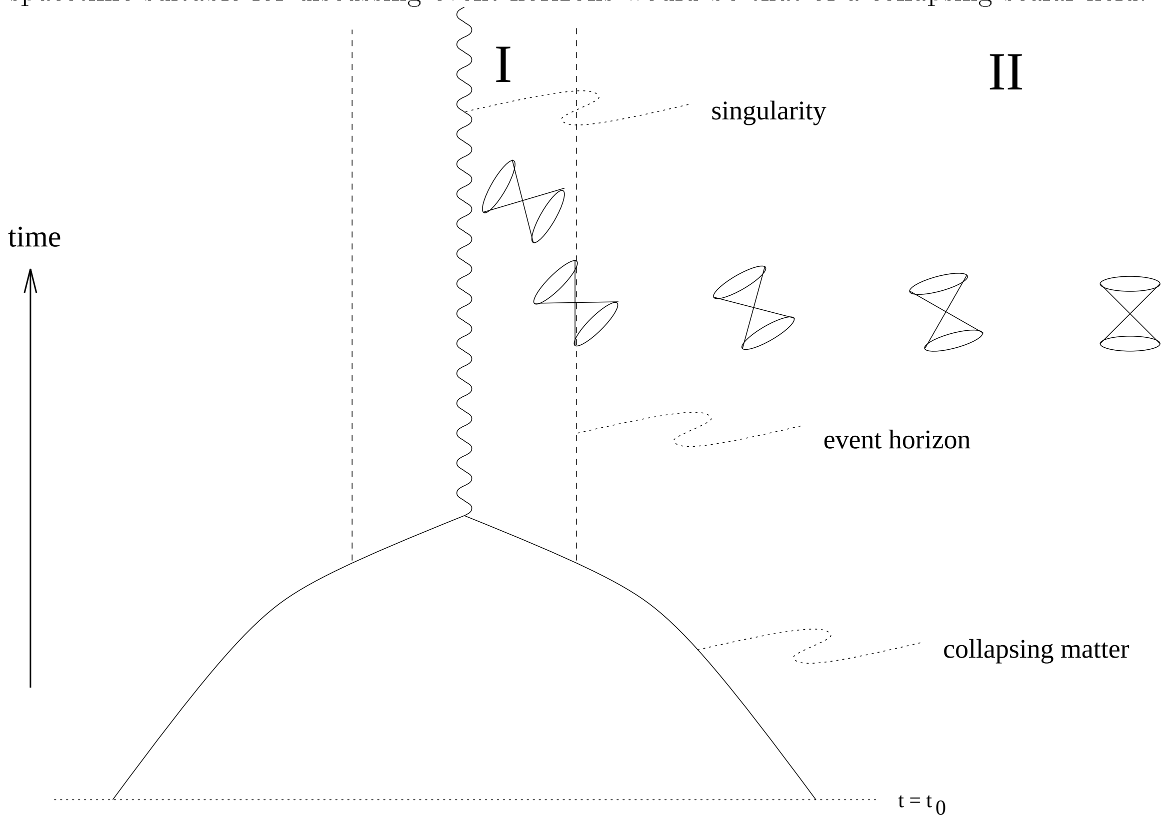
2 Event Horizons

Unlike apparent horizons, which can be found on a 3-dimensional hypersurface, an event horizon is a global property of a spacetime. Not all spacetimes that are solutions to Einstein's equations contain event horizons, although the ones we are interested in (black hole spacetimes) do. A typical spacetime suitable for discussing event horizons would be that of a collapsing scalar field.



The effect of the "gravitational field" can be interpereted as the "tipping over" of the light cones at each point in space. Stronger gravity means a more tipped light cone. As one approaches the collapsed scalar field, the light cones get tipped so much that it would be impossible for anything to "escape to infinity." This "region of no escape" is labeled region I. The region where the light cones are not tipped enough to trap observers is labeled region II. The boundary between region I (can't escape to infinity) and region II (able to escape to infinity) is the definition of the event horizon. Passing through region II into region I can be thought of as passing the "point of no return." No information (electomagnetic waves, mass, message in a bottle) can ever cross the event horizon from region I into region II.

2.1 Finding Event Horizons

In principle, an event horizon can be found by tracing light rays through time. Outward going light rays that are emittted just outside the event horizon will escape to infinity, diverging away from it. Outward going light rays that are emitted just inside the event horizon will fall away from it, towards the singularity.

2.1.1 NCSA

The NCSA group has devoloped a method for finding the event horizons of a spacetime (http://jean-luc.ncsa.uiuc.edu/Papers/htmls/BlackHoles.html). Instead of integrating individual light rays forward in time, they integrate null surfaces backward in time. There are many advantages to this scheme, including

- The method is stable: once the surface becomes the event horizon, it connot drift away from it.
- Since the method only requires knowledge of the metric (and not its derivatives), it is more accurate than evolving photons.

2.1.2 Cornell

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3 Full 3-D Cauchy Evolution

3.1 NCSA

3.2 University of Texas at Austin

4 Adaptive Mesh Refinement

4.1 Cornell

4.2 NCSA

4.3 University of Texas at Austin

5 Cauchy Evolution Coupled to Null Surface Evolution

5.1 University of Pittsburgh

5.2 Penn State

6 Elliptic solvers

6.1 Cornell

6.2 NCSA

6.3 Penn State

7 Perturbative approach to black hole collisions

7.1 Cornell

7.2 University of North Carolina, Chapel Hill