

# Finite Element Modeling of Multibody Contact and Its Application to Active Faults

H.L. Xing\* and A. Makinouchi

Materials Fabrication Lab, The Institute of Physical and Chemical Research (RIKEN)  
2-1 Hirosawa, Wako, Saitama, 351-0198, Japan

Corresponding author: H.L. Xing<[xing@postman.riken.go.jp](mailto:xing@postman.riken.go.jp)>

## Abstract

Earthquakes have been recognized as resulting from a stick-slip frictional instability along the faults between deformable rocks. An arbitrarily shaped contact element strategy, named as node-to-point contact element strategy is proposed to handle the friction contact between deformable bodies with stick and finite frictional slip and extended to simulate the active faults in the crust with a more general nonlinear friction law. Also introduced is an efficient contact search algorithm for contact problems among multiple small and finite deformation bodies. Moreover, the efficiency of the parallel sparse solver for the nonlinear friction contact problem is investigated. Finally, a model for the plate movement in the northeast zone of Japan under gravitation is taken as an example to be analyzed with different friction behaviors.

## 1. INTRODUCTION

Japan is located in one of the world's most earthquake-prone zones and has suffered the loss of many valuable human lives in the earthquake history. To further investigate the occurrence of earthquake and to predict it in the future, as a part of the Earth Simulator Project of Japan, a finite element software system for large-scale computation of the earthquake process is being developed in our laboratory, including CAD and mesh generation, static analysis and dynamic analysis. Only the static analysis is introduced here, which aims to calculate the accumulation of stress around active faults induced by a subduction of plates in a long time span and to further predict the earthquake occurrence.

The earthquakes can be regarded as a contact between deformable rocks with a special friction law along the active faults (e.g. Marone C. 1998), it includes three kinds of main nonlinearities: the material, the geometrical and the contact along the faults. Contact problems are characterized by contact constraints, which are imposed on contacting boundaries. In the current FEM analysis, both the dynamic-explicit FEM and the static-implicit FEM are available corresponding to the different problems. However, convergence is still a problem in implicit analysis, especially when three-dimensional large deformation contact problems with sliding friction are encountered. This is partly due to the iteration solution method and its corresponding serious requirement, such as no drastic change of the contact state and the deformation state, more smooth contact surface definition (e.g. Nagtegaal & Taylor 1991, Ling et al 1997). Although many efforts have been made as above, there still exist problems to be overcome (e.g. Parisch 1997, Zhong 1993). Thus dynamic-explicit FEM seems to be used increasingly, even for problems, which are characterized as static or quasi-static ones, but it is also well known that it is quite time consuming and also difficult for dynamic-explicit FEM to predict the stress distribution with a high accuracy. Thus, an arbitrarily shaped isoparametric contact element strategy with the static-explicit integration algorithm, named as node-to-point contact element strategy, was proposed by the authors to handle

the static or quasi-static friction contact between deformable bodies with stick and finite frictional slip (Xing and Makinouchi et al 1998). Moreover, the friction behaviour in the practical engineering and the active faults is quite complicated, it depends on the slip velocity, the state, the contact pressure, the material property and so on. This paper will focus on how to extend our algorithm to simulate it. In addition, to meet the practical requirement of a large-scale calculation, the parallel solver is also investigated for the nonlinear friction contact problem and applied to simulate the active faults. Finally, a model for the plate movement in the northeast zone of Japan under gravitation is taken as an example to be analyzed with different friction behaviors to show the efficiency, stability and usefulness of this algorithm.

## 2.GENERAL CONSIDERATION AND NOTATION

### 2.1 Kinematics

Consider two bodies  $B^1$  and  $B^2$  with surfaces  $S^1$  and  $S^2$ , respectively, to contact on an interface  $S_c$ , and  $S_c = S^1 \cap S^2$ ,  $S_c^\alpha = S_c \cap S^\alpha$ , where superscript  $\alpha = 1, 2$  refers to body  $B^\alpha$  (as shown in Fig. 1). Let the union of the two bodies be denoted by  $B$ :  $B = B^1 \cup B^2$ ,  $\mathbf{n}$  be the unit normal vector of the contact surface,  $\mathbf{s}$  be the unit tangential vector along the relative sliding direction on the contact surface, and  $\mathbf{t} = \mathbf{n} \times \mathbf{s}$ . Thus  $\mathbf{s}$  and  $\mathbf{t}$  form a tangent plane to the contact surface. When contact occurs, the following conditions should be satisfied for unilateral contact

$$g_n = 0, \dot{g}_n = \dot{\mathbf{u}}^1 \cdot \mathbf{n}^1 + \dot{\mathbf{u}}^2 \cdot \mathbf{n}^2 = 0 \quad \text{on } S_c \quad (1)$$

$$\mathbf{f}^\alpha \cdot \mathbf{n}^\alpha \leq 0, \mathbf{f}^1 + \mathbf{f}^2 = 0 \quad \text{on } S_c \quad (2)$$

where  $g_n$  is the gap normal to the contact surface and  $\mathbf{f}^\alpha$  is the contact stress on  $S_c^\alpha$ .

The so-called slave-master concept is widely used for the implementation of contact analysis.

Assume that one of the bodies,  $B^1$ , is the slave and the material points on its contact surface are called slave nodes; and the other body  $B^2$  is the master and the material points on its contact surface are called master nodes. Contact (master) segments that span master nodes cover the contact surface of the master body. Therefore, the above problem can be regarded as a contact between a slave node and a point of a master segment. And a slave node makes contact with only one point on the master segments, but one master segment can make contact with one or more slave nodes at the same time. This is the basic assumption of the node-to-point contact element strategy.

### 2.2 Constitutive Equation for Friction Contact

#### 2.2.1 Normal contact stress

We choose the penalty method to treat the normal constraints when contact occurs. For a slave node,

$$f_n = E_n \text{sign}(g_n) g_n = -E_n g_n. \quad (3)$$

Here  $E_n$  is the penalty parameter to penalize the penetration (gap) in the normal direction, and  $g_n = \mathbf{n} \cdot (\mathbf{x}_s - \mathbf{x}_c)$ , here  $\mathbf{x}_s$  and  $\mathbf{x}_c$  are the position coordinates of a slave node  $s$  and its corresponding contact point  $c$  (as shown in Fig.2), respectively.

#### 2.2.2 Friction Stress

Friction is by nature a path-dependent dissipative phenomenon that requires the integration of the constitutive relation. The analogy to plasticity can be founded in Michalowski & Mroz's work (Michalowski & Mroz 1978). In this study, a standard Coulomb friction model, with an additional

limit on the allowable shear forces, is applied in an analogous way to the flow plasticity rule. The basic formulations are summarized below. (*Note: A variable with  $\sim$  on top stands for a relative component between slave and master bodies, and  $l, m=1,2; i,j, k=1,3$  in this paper.*)

Based on experimental observations, an increment decomposition is assumed

$$\Delta\tilde{u}_m = \Delta\tilde{u}_m^e + \Delta\tilde{u}_m^p, \quad (4)$$

where  $\Delta\tilde{u}_m^e$  and  $\Delta\tilde{u}_m^p$  represent the sticking (reversible) and the sliding (irreversible) part of  $\Delta\tilde{u}_m$ , respectively. In addition, the slip is governed by the yield condition

$$F = \sqrt{f_m f_m} - \bar{F}, \quad (5)$$

where  $\bar{F}$ , the critical frictional stress, has three choices:  $\bar{F} = \mu f_n$ ,  $\bar{F} = F_{limit}$  and  $\bar{F} = \min(\mu f_n, F_{limit})$ ;  $F_{limit}$  is an allowable value of shear stress;  $\mu$  is the friction coefficient, it may depends on the normal contact pressure  $f_n$ , the equivalent slip velocity  $\dot{\tilde{u}}_{eq}^{sl}$  and the state variable  $\varphi$ , i.e.  $\mu = \mu(f_n, \dot{\tilde{u}}_{eq}^{sl}, \varphi)$ .

If  $F < 0$ , contact is in the sticking state, and

$$f_m = E_t \tilde{u}_m^e = E_t \Sigma \Delta\tilde{u}_m^e, \quad (6)$$

where  $E_t$  is a constant in the tangential direction.

When  $F=0$ , the friction changes its character from sticking to sliding, and

$$\Delta\tilde{u}_m^p = \Delta\bar{u}^p f_m / \bar{F}, \quad (7)$$

where  $\Delta\bar{u}^p$  is the ‘equivalent relative slip increment’.

From Eqs. (4) and (6),

$$f_m = E_t (\tilde{u}_m - \tilde{u}_m^p) = f_m^e - E_t \Delta\tilde{u}_m^p. \quad (8)$$

where  $f_m^e = E_t (\tilde{u}_m - \tilde{u}_m^p|_0)$ , and  $\tilde{u}_m^p|_0$  is the value of  $\tilde{u}_m^p$  at the beginning of this step.

From the last two equations,

$$f_m = \eta_m \bar{F} \quad \text{and} \quad \eta_m = f_m^e / \sqrt{f_l^e f_l^e}. \quad (9)$$

The linearized form of the Eq. (9) can be rewritten as

$$\begin{aligned} df_l &= \frac{\bar{F} E_t}{\sqrt{f_l^e f_l^e}} (\delta_{lm} - \eta_l \eta_m) d\tilde{u}_m + \eta_l \mu \left( df_n + \frac{\partial \mu}{\partial f_n} df_n \right) + \eta_l f_n \left( \frac{\partial \mu}{\partial \dot{\tilde{u}}_{eq}^{sl}} d\dot{\tilde{u}}_{eq}^{sl} + \frac{\partial \mu}{\partial \varphi} d\varphi \right) \quad (\text{if } \bar{F} = \mu f_n) \\ df_l &= \frac{\bar{F} E_t}{\sqrt{f_l^e f_l^e}} (\delta_{lm} - \eta_l \eta_m) d\tilde{u}_m \quad (\text{if } \bar{F} = F_{limit}). \end{aligned} \quad (10)$$

In summary, from Eqs.(3), (6) and (10), the contact stress acting on a slave node can be described as

$$\dot{f}_i = G_{ij}\dot{\tilde{u}}_j + \dot{f}_{\varphi i}, \quad (11)$$

where  $\mathbf{G}$  is the frictional contact matrix;  $\dot{f}_{\varphi i}$  is from the contribution of the terms related with  $\varphi$ , when it is not a function of  $\dot{\tilde{u}}$ ; If  $d\varphi$  only is the function of the unknown variable  $d\tilde{u}$ ,  $\dot{f}_{\varphi i} = 0$ , i.e. all its contribution can be included in  $\mathbf{G}$  at current state.

### 3. FINITE ELEMENT FORMULATION

#### 3.1 Variational Principle

The updated Lagrangian rate formulation is employed to describe the nonlinear problem. The rate type equilibrium equation and the boundary at the current configuration are equivalently expressed by a principle of virtual velocity of the form (Xing and Makinouchi 1998)

$$\int_V \{(\sigma_{ij}^J - 2\sigma_{ik}D_{kj})\delta D_{ij} + \sigma_{jk}L_{ik}\delta L_{ij}\}dV = \int_{S_T} \dot{F}_i \delta v_i dS + \int_{S_c^1} \dot{f}_i^1 \delta v_i dS + \int_{S_c^2} \dot{f}_i^2 \delta v_i dS, \quad (12)$$

where  $V$  and  $S$  denote respectively the domain occupied by the total body  $B$  and its boundary at time  $t$ ;  $S_T$  is a part of the boundary of  $S$  on which the rate of traction  $\dot{F}_i$  is prescribed;  $\delta \mathbf{v}$  is the virtual velocity field which satisfies the boundary  $\delta \mathbf{v} = \mathbf{0}$  on the velocity boundary;  $\mathbf{L}$  is the velocity gradient tensor,  $\mathbf{L} = \partial \mathbf{v} / \partial \mathbf{x}$ ;  $\mathbf{D}$  and  $\mathbf{W}$  are the symmetric and antisymmetric parts of  $\mathbf{L}$ , respectively.

The small strain linear elasticity and large strain rate-independent work-hardening plasticity are assumed, from which the elasto-plastic tangent constitutive tensor  $C_{ijkl}^{ep}$  is derived

$$\sigma_{ij}^J = C_{ijkl}^{ep} D_{kl} = C_{ijkl}^{ep} L_{kl}. \quad (13)$$

Substitution of Eq.(13) into Eq.(12) reads to the final form of the virtual velocity principle

$$\int_V \sum_{ijkl} L_{kl} \delta L_{ij} dV = \int_{S_T} \dot{F}_i \delta v_i dS + \int_{S_c^1} \dot{f}_i^1 \delta v_i dS + \int_{S_c^2} \dot{f}_i^2 \delta v_i dS, \quad (14)$$

where  $\sum_{ijkl} = C_{ijkl}^{ep} + (\sigma_{jl}\delta_{ik} - \sigma_{ik}\delta_{jl} - \sigma_{il}\delta_{jk} - \sigma_{jk}\delta_{il})/2$ .

#### 3.2 Contact Stress on A Slave Node

Assume that contact segment surfaces are described by  $\mathbf{x} = \mathbf{x}(\xi_m)$ , a slave node  $s$  has made contact with a master segment on point  $c$  (as shown in Fig. 2), and the contact stress acting on it in Eq.(14) can be described in the local contact coordinate system as follows

$$\dot{\mathbf{f}} = \dot{f}_i \mathbf{e}_i + f_i \dot{\mathbf{e}}_i. \quad (15)$$

Here  $\mathbf{e}_i$ , the base vector on the contact segment, is specified by

$$\mathbf{e}_i = \mathbf{e}_i(\xi, \eta) = \mathbf{e}_i(\xi_m) \text{ and } \dot{\mathbf{e}}_i = \frac{\partial \mathbf{e}_i}{\partial \xi_m} \dot{\xi}_m = E_{ijm} \mathbf{e}_j \dot{\xi}_m, \quad (16)$$

in which  $E_{ijm} = \mathbf{e}_{i,m} \cdot \mathbf{e}_j$ .

From Eqs. (15) and (16),

$$\dot{\mathbf{f}} = \dot{f}_i \mathbf{e}_i + f_i E_{ijm} \mathbf{e}_j \dot{\xi}_m. \quad (17)$$

Assuming that the tangential surface is spanned by the tangents to the parameter lines,

$$\mathbf{e}_m = \frac{\partial \mathbf{x}_i}{\partial \xi_m}, \quad \mathbf{e}_3 = \mathbf{e}_1 \times \mathbf{e}_2 \quad \text{and} \quad C_{ml} = \mathbf{e}_m \cdot \mathbf{e}_l. \quad (18)$$

Considering the normal projection of the slave node onto the tangential plane, the coordinates of the contact point  $\mathbf{x}_c$  should satisfy

$$\mathbf{e}_m \cdot (\mathbf{x}_s - \mathbf{x}_c) = 0. \quad (19)$$

Linearize the above equation with the unknowns, and note that  $\mathbf{x}_c = \mathbf{x}_c(\mathbf{u}_c, \xi_m)$ , we have

$$\dot{\mathbf{e}}_m \cdot (\mathbf{x}_s - \mathbf{x}_c) + \mathbf{e}_m \cdot (\dot{\mathbf{x}}_s - \dot{\mathbf{x}}_c) = 0, \quad (20)$$

where  $\dot{\mathbf{x}}_s = \dot{\mathbf{u}}_s$ ,  $\dot{\mathbf{x}}_c = \dot{\mathbf{u}}_c + \mathbf{e}_m \dot{\xi}_m$ ,  $\dot{\mathbf{e}}_m = \mathbf{e}_{m,l} \dot{\xi}_l + \dot{\mathbf{u}}_{c,m}$ .

Solving the above system yields

$$\dot{\xi}_m = \left\{ g_n^0 D_m + (D_m \mathbf{n} + \bar{C}_{ll} \mathbf{e}_m - \bar{C}_{ml} \mathbf{e}_l) \cdot \dot{\mathbf{u}} \right\} / h \quad (\text{no sum on } m, l), \quad (21)$$

where  $\bar{C}_{ml} = C_{ml} - g_n^0 \mathbf{n} \cdot \mathbf{e}_{m,l}$ ,  $h$  is the determinant of  $\bar{C}_{ml}$ ,  $g_n^0$  is the penetration of the previous increment step;  $D_m = \bar{C}_{ll} \dot{\mathbf{u}}_{c,m} \cdot \mathbf{n} - \bar{C}_{ml} \dot{\mathbf{u}}_{c,l} \cdot \mathbf{n}$ ;  $\dot{\mathbf{u}} = \dot{\mathbf{u}}_s - \dot{\mathbf{u}}_c$ , while  $\dot{\mathbf{u}}_c$  is the velocity of material position  $c$  on the segment,  $\dot{\mathbf{u}}_c = N_\gamma \dot{\mathbf{u}}_\gamma$ , and  $\dot{\mathbf{u}}_\gamma$  is the nodal velocity on the segment, where  $N_\gamma$  is the shape function of the segment.

Finally, from Eqs. (11), (17) and (21), we have

$$\dot{\mathbf{f}} = (G_{ik} \mathbf{e}_i + Q_{jk} \mathbf{e}_j) \dot{\tilde{\mathbf{u}}}_k + \dot{f}_{\phi i} \mathbf{e}_i + f_i E_{ijm} D_m g_n^0 \mathbf{e}_j / h. \quad (22)$$

Here,  $Q_{jk} = f_i E_{ijm} (D_m \mathbf{n} + \bar{C}_{ll} \mathbf{e}_m - \bar{C}_{ml} \mathbf{e}_l) \cdot \hat{\mathbf{e}}_k / h$ ;  $\hat{\mathbf{e}}_k$  is the unit vector of local Cartesian coordinate system on the contact interface.

### 3.3 Reverse Contact Stress to Master Segment

From Eq. (2), the reverse contact stress acting on a node  $\gamma$  of a master segment is

$$\mathbf{f}_\gamma^2 = -N_\gamma \dot{\mathbf{f}} = -N_\gamma \left\{ (G_{ik} \mathbf{e}_i + Q_{jk} \mathbf{e}_j) \dot{\tilde{\mathbf{u}}}_k + \dot{f}_{\phi i} \mathbf{e}_i + f_i E_{ijm} D_m g_n^0 \mathbf{e}_j / h \right\}. \quad (23)$$

### 3.4 Time Integration Algorithm

The time integration method is one of key issues to formulate a nonlinear finite element method. It is well known that the fully implicit method is often subjected to bad convergence problems, mostly due to changes of contact and friction states. In order to avoid this, we employ an explicit time integration procedure as follows. It is assumed that under a sufficiently small time increment all rates in Eq. (14) can be considered constant within the increment from  $t$  to  $t + \Delta t$  as long as

there is no drastic change of states (for example, elastic to plastic at an integration point, contact to discontact or discontact to contact on the contact interface, stick to slide or slide to stick in friction on the contact interface) takes place. The  $R$ -minimum method (Yamada, 1968) is extended and used here to limit the step size in order to avoid such drastic change in state within an incremental step.

Thus all the rate quantities used to derive Eq. (14) are simply replaced by incremental quantities as

$$\Delta \mathbf{u} = \mathbf{v} \Delta t, \quad \Delta \boldsymbol{\sigma} = \boldsymbol{\sigma}^J \Delta t \quad \text{and} \quad \Delta \mathbf{L} = \mathbf{L} \Delta t. \quad (24)$$

Finally, in combination with Eqs. (22)-(24), Eq.(14) can be rewritten as

$$(\mathbf{K} + \mathbf{K}_f) \Delta \mathbf{u} = \Delta \mathbf{F} + \Delta \mathbf{F}_f. \quad (25)$$

Here  $\mathbf{K}$  is the standard stiffness matrix corresponding to body  $B$ ;  $\mathbf{K}_f$ , stiffness matrix of the contact elements, comes from the contribution of the terms related with  $\dot{\tilde{u}}_k$  in Eqs. (22) and (23);  $\Delta \mathbf{u}$  is the nodal displacement increment;  $\Delta \mathbf{F}$  is the external force increment subjected to body  $B$  on  $s_F$ ;  $\Delta \mathbf{F}_f$  comes from the contribution of all the terms except those related with  $\dot{\tilde{u}}_k$  in Eqs. (22) and (23). Note  $\mathbf{K}_f$  is unsymmetrical due to the nonlinear friction and the geometry curvature, thus the total stiffness matrix  $(\mathbf{K} + \mathbf{K}_f)$  is also unsymmetrical.

#### 4. CONTACT SEARCHING

In cases that two or more bodies come in contact with each other, the search algorithms are normally split into a global and a local search. For the global search, several methods have been proposed, such as the regular cell algorithm (e.g. Santos & Makinouchi 1993), the Hierarchy-Territory (HITA) algorithm (Zhong 1993), the position code algorithm (Oldenburg & Nilsson 1994), the bucket sorting algorithm (Benson & Hallquist 1990 and Belyschko & Lin 1987), the spherical sorting algorithm (Papadopoulos & Taylor 1993), etc. The last three methods are mainly subjected to the finite-element-type mesh description of the contact surface, and the HITA and the position code algorithms are recommended in terms of the computational efficiency. In this study, the position code algorithm is employed for the global contact search between deformable bodies. For the local search, several algorithms have also been proposed, such as the pinball algorithm (Belyschko & Neal 1991), the node-to-segment algorithm (Benson & Hallquist 1990), the inside-outside algorithm (Wang & Nakamachi 1997), etc. The latter two methods are combined and applied in this study to avoid the ‘deadzone’ problem and to obtain the precise contact position of a slave node on the segment, as shown in Fig.3. If all the  $\mathbf{V}_i$  (in Fig.3) keep the same or the reverse direction as  $\mathbf{n}_s$ , point  $c$  will locate on this segment. Then the distance between the slave node  $s$  and point  $c$  is calculated and compared with a prescribed accuracy sector. If within the prescribed zone, the slave node  $s$  is in contact with this master segment on  $c$ , and the exact location of point  $c$  and the penetration of the slave node  $s$  will be obtained and saved for further computation.

The following measures are also taken for contact search:

1). Contact candidates. The candidates of contact segments and slave nodes are marked during the pre-processing, then only these marked elements are considered during the contact searching and the calculation to save the computation cost.

2). Automatic extensions of master surfaces. To meet the requirement of the contact territory, the master surfaces can be extended automatically along the surface perimeter after one or some increment steps.

## 5. PARALLEL SOLVER

In the analysis of the practical engineering and the active faults in the crust, complex geometry has to be taken into account, thus a large-scale calculation is necessary. Recently, the parallel sparse solver is widely used in the engineering analysis for a large scale computing due to its stability, but no results on nonlinear frictional contact analysis reported. Thus, the efficiency of the parallel sparse solver for nonlinear frictional contact analysis is investigated here.

A tube and tubesheet assembling is one of the most important processes for a heat exchanger. The assembling process of 37 tubes to a tubesheet using hydraulic expansion is taken as an example to be computed. All the tubes are fixed to the tubesheet with welding at the bottom; this is modeled with 'tied' (or stick) algorithm in our code (as shown in Fig. 4). Due to symmetry, only one-twelfth of the structure has been considered, being discretized into 30024 nodes (90072 DOF) and 21247 elements. The tubesheet is only supported along the central line direction of the tubes at the outside nodes of the bottom edge; the tube is restrained along the central direction at the position  $D$  (see Fig.4). The case that all the tubes are hydraulically bulged at the same time and to be gradually contacted with the tubesheet is investigated here using the parallel unsymmetrical sparse solver PSLDU on the SGI Onyx2 computer. In which multiple CPUs are used to solve the linear equations, but only one of them is used also for other works, such as contact search and stiffness matrix assembling. Fig. 5 shows the relationship between the average time of this CPU per step and the numbers of contact nodes when 4 CPUs are used to calculate the assembling process. From this, the parallel sparse solver is powerful for such a scale calculation, but it is very sensitive to contact node numbers, and the time cost rises rapidly with the increase of contact node numbers. This needs further related research in the future.

## 6. APPLICATION TO ACTIVE FAULTS

A part of Northeast fault model with the subducting Pacific plate around Japan (Kanai, 2000) is taken as an example to be computed here with a scale of 1:100,000, as shown in Fig.6. The displacement constraints used are also shown in the above figure except that the plate is fixed along the  $x$  direction at the position A and B (see Fig. 6). As for the loading condition, the combined action of the self-gravity and the hydraulic pressure of water is investigated. And the widely accepted rate and state dependent friction law proposed by Dieterich (1978,1979) and Ruina(1983) is applied here to describe the complex phenomenon along the interface between the active faults. The calculated results (as shown in Fig.7) demonstrate that the friction coefficient along the active fault interface has obvious influences on their relative movement. The bigger the friction coefficient is, the less the relative movement along the interface is. Also this is affected by the distribution of different friction coefficients due to the relative slip velocity along the interface.

## 7. CONCLUSIONS

A static-explicit FEM code has been developed to simulate the static or the quasi-static 3-dimensional friction contact between multi-elasto-plastic bodies and extended to extended to simulate the active faults in the crust with a more general nonlinear friction law. An arbitrarily shaped contact element strategy, named as node-to-point contact element strategy is proposed and applied according to the static-explicit characters, which overcomes the main convergence problems existing in the implicit treatment of contact. Meanwhile, an efficient contact-searching algorithm for the multi-deformation-body contact problem has been implemented in our code. Moreover, the parallel sparse solver is very powerful for the nonlinear friction contact problem, but its efficiency decreases much with the increase of the contact node numbers, this may need further research. Finally, a model for the plate movement in the northeast zone of Japan under gravitation is taken as an example to be analyzed with different friction behaviors, which demonstrate the stability, efficiency and usefulness of this algorithm.

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## Figures

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