

Figure 3: Expectation value of R/V for $n = -5, 0, 5$, $N_4 = 4000, 16000$; curves ordered as in figure 1.

Figure 4: Expectation value of d for $n = -5, -1, 0, 1, 5$, $N_4 = 16000$; curves ordered as in figure 1.

Figure 1: Expectation value of R/V for $n = -5, -1, 0, 1, 5$. The curves are ordered from the smallest n at the bottom upwards.

Figure 2: Expectation value of d for $n = -5, -1, 0, 1, 5, N_4 = 4000$; curves ordered as in figure 1.

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reabsorb in a simple renormalization of the critical coupling. Always keeping in mind that a precise finite size study is required before making quantitative statements [12], we believe there are two basic possibilities. The first possibility is that there is only one universality class, and that all the theories we have studied do asymptotically show the same critical behavior. In this case the rate of approach to the continuum limit is strongly influenced by n . We will select the theory with faster convergence to the continuum.

The second possibility (which is the most interesting one) is that the measure factor changes the universality class. Our results, albeit preliminary, seem to hint in this direction. In this case we could have a critical value of n_c , and transitions belonging to different universality classes. This is a very appealing scenario, and here the lattice discrete theory could make its own original contribution. It could be possible to pick out the correct measure, on the lattice, by requiring a particular expectation value and scaling behavior of some physical observable. Such a prescription would be a powerful tool, turning the discrete version of the theory from a source of indetermination into a completely determined scheme.

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3-simplex faces that connect a pair of 4-simplices and average over all 4-simplices and random manifolds.

Both figures show that the measure operator has a pronounced effect. Increasing the coupling of the measure term leads to a continuous, monotonous deformation of the curves. Notice that the curves are not just shifted. In the case of R/V , the singularity seems stronger for $n \simeq 0$, where the jump in R/V is quite sharp. The distance d has a sharper jump for $n = 1$, where it seems to jump from one constant value to another constant. Smaller values of n show a slower increase in d .

For large absolute values of n , especially for $n = -5$, the plots show a weaker singularity. The profile of R/V hints less at a sharp jump than the former cases, and the distance increases very smoothly from a critical value of k_2 , $k_2^c(n)$ on. When n increases to the value of 5 the system seems to loose criticality on an absolute scale. Its behavior through the crossover is quite smooth.

A critical value of k_2^c can be defined, for example, as the point where the distance value starts to change. But for the $n = 5$ case the transition point is not very clear. Let us note that such a value of k_2^c changes its sign as a function of n .

Figure 3 and 4 show results for $N_4 = 16000$ to indicate what kind of finite size effects are present. As evident from figure 3, larger volumes amplify the effect of large absolute n for $k_2 < k_2^c$. Figure 4 for the average distance d displays the same qualitative behaviour as figure 2 for $k_2 < k_2^c$, but for large k_2 the average distance does not remain constant, which might already have been guessed in figure 2. The explanation is that the measure term S_M for positive n introduces a bias towards smaller $o(a)$ which increases d , but for large enough k_2 and k_4 the contribution of S_E dominates and d approaches its $n = 0$ value. This may be the case for $k_2 > k_2^c$, while for $k_2 > k_2^c$ the critical value $k_4^c(k_2)$ is such that $S_E \approx 0$, and at least for the range of negative k_2 considered here S_M is relevant and d remains large for $n = 5$. The same holds for R/V when $n = 5$.

Our conclusion is that the measure term has a strong effect, which seems difficult to

where $k_4 = \lambda + 10/G$ and $k_2 = 2\pi/\alpha G$.

In the discrete quantum theory there exists a critical line $k_4 = k_4^c(k_2)$ such that if k_4 is different from k_4^c for a given value of k_2 then the random walk tends to either zero or infinite volume. All measurements are made for $k_4 = k_4^c(k_2)$.

We have selected not one but a family of measures in order to investigate the influence of the measure in a rather general setting. Our choice is guided by diffeomorphism invariance of the measure [10] but ignores more sophisticated arguments like BRST invariance. We have studied, as a function of n , a measure contribution of the form

$$\prod_x g^{n/2} , \quad (5)$$

i.e. in the triangulated theory $S_E[T]$ is replaced by $S[T] = S_E[T] + S_M[T]$, where

$$S_M = -n \sum_a \log \frac{o(a)}{5} . \quad (6)$$

The sum runs over all 0-simplices (sites) of the manifold, and $o(a)$ is the number of 4-simplices which include the site a . We considered n in the interval from -5 to 5 . The case $n = 0$ repeats simulations with the trivial, uniform measure, which can be compared with previous results.

Let us summarize our results. We confirm the fact that the phase transition can be of second order, and that it is plausible that we will be able to define a sensible theory. We find that the measure factor plays an important role, and that the critical behavior does depend on n . This is very different from $2d$ quantum gravity (see for example [11]), where modifications of the measure factor of the same kind we use here do not have any non-trivial effect on the critical behavior. Varying n does not only change non-universal quantities, like for example the value of the critical coupling, but changes the actual (pseudo-)critical behavior.

In figure 1 we plot the average curvature R/V for $V \equiv N_4 = 4000$ as a function of the coupling k_2 for different values of n , $n = -5$ for the lowest curve, then $n = -1, 0, 1$ and $n = 5$ for the upper curve. In figure 2 we plot the average distance (in the internal space) of two 4-simplices. We count the minimum number of steps from 4-simplex to 4-simplex across

observation certainly warrants further careful investigations.

These first simulations have been run with uniform measure, where all triangulations have the same weight in the sum which defines the path integral of the quantum theory. There are no particularly good reasons for this choice to be the correct one, and in the following we investigate the changes introduced by defining simplicial quantum gravity with a non-trivial measure.

One of us has described in [8] the structure of the Monte Carlo simulations, and how the efficiency of the numerical procedure can be optimized. The programming of dynamical triangulations is difficult since a dynamical data structure is required, but it is very relevant in many practical applications. For the same reason the implementation of DTRS on parallel computers is a hard problem. Since a set of ergodic moves preserving the volume for canonical simulations is not known, one has to consider a Markov chain which sweeps out the space of different volume simplicial manifolds. We have used the quasi-canonical method introduced in [9], which allows us to control the systematic distortion arising from the *ad hoc* potential that keeps the system close to a given number of 4-simplices [8]. All the details about the system we study and about the numerical procedures are given in [8].

In the continuum the euclidean Einstein-Hilbert action for a metric $g_{\mu\nu}$ has the form

$$S_E[g] = \int d^4x \sqrt{g} \left(\lambda - \frac{R(g)}{G} \right), \quad (1)$$

where $R(g)$ is the Ricci scalar and λ and G are the cosmological and gravitational constants, respectively. We consider a fixed S^4 topology. On a triangulation T we discretize according to

$$V = \int d^4x \sqrt{g} \rightarrow N_4[T], \quad (2)$$

$$R = \int d^4x \sqrt{g} R(g) \rightarrow \frac{2\pi}{\alpha} N_2[T] - 10 N_4[T], \quad (3)$$

where $\alpha \simeq 1.318$, and $N_i[T]$ is the number of i -simplices of the triangulation T . The discrete action is then

$$S_E[T] = k_4 N_4[T] - k_2 N_2[T], \quad (4)$$

without, for example, curvature squared terms. Even though the partition function formally diverges, at fixed volume the local curvature is bounded both from below and from above. Therefore we can study the theory at fixed (or better quasi-fixed, see later) volume, and look for the existence of a stable fixed point in the large volume limit. A second order phase transition with diverging correlation lengths, in the statistical mechanics language, would allow us to define a continuum limit which is universal and is not influenced by the details of the underlying discrete lattice structure. Precisely this scenario constitutes one of the best hopes we have to find a consistent quantum theory of gravity. If euclidean quantum gravity based on the Einstein action does have non-perturbative meaning, then we can exhibit it in this way.

The second problem is to determine the measure one should use to define the quantum theory. This problem, far from being solved in the continuum, is completely open in the lattice approach (for a good review see for example [3]). In quantum Regge calculus the influence of the measure has been examined in [4] but there is no direct relation to DTRS. The problem of the measure is the main point we address in this note, and we want to suggest that the DTRS approach may be powerful enough to solve it.

Recently two groups have pioneered Monte Carlo simulations of DTRS [5, 6, 7]. Numerical simulations (even if on quite a small scale) turned out to be feasible, and lead to very non-trivial results. One clearly observes a phase transition structure. Although quantitative statements are not easy to make, given the limited statistics and the small lattice size, it is clear that the situation is different from the $3d$ case, where the phase transition is manifestly of first order, and there is no continuum theory. In $4d$ [5] there is an open possibility that the transition is second order (although that cannot be claimed without a much more detailed finite size study). One does not observe hysteresis cycles, and the crossover is less sharp than in $3d$. More involved statements about critical exponents have to be taken at this point, we believe, *cum grano salis*, but there is evidence for the possible presence of a critical point with a non-trivial continuum theory in the phase diagram for the euclidean Einstein action. This

Dynamically triangulated random surfaces (DTRS) [1] play an important role in the efforts to develop a coherent description of quantum gravity. The (euclidian) space-time is approximated by a d -dimensional simplicial triangulation, where the link length is constant, equal to 1, but the connectivity matrix is a dynamical variable.

The most important advances have been obtained in two-dimensional quantum gravity, where DTRS are simplicial triangulations of a $2d$ manifolds. The analytic success of matrix models, which can be for example exactly solved in the case of pure $2d$ gravity [2], has strongly encouraged this approach. The results obtained in the triangulated approach and in the continuum lead to consistent predictions for correlation functions and critical exponents.

Dynamical triangulations are also potentially relevant in four dimensions. One can hope that a sensible, non-perturbative definition of the quantum gravity theory can be obtained in some *scaling* limit of the theory of $4d$ hyper-tetrahedra. This approach has much in common with Regge calculus, where the connectivity is fixed but the functional integration runs over the link lengths. The underlying principle is clearly very similar, and one could say that DTRS have the status of an improved Regge calculus. We face the usual problem inherent in discretizing a theory, i.e. the discretization scheme can break some of the continuous symmetries, which will have to be recovered in the continuum limit (if there is one). Indeed, Wilson lattice gauge theories have taught us an important lesson. The fact that gauge invariance is exactly conserved in the lattice theory, for all values of the lattice spacing a , is in that case crucial: it would have been very difficult to establish firm numerical results if one would have had to care about the presence of non gauge-invariant correction, which would disappear only in the $a \rightarrow 0$ limit. In the case of quantum gravity, diffeomorphism invariance plays such a crucial role, and DTRS are diffeomorphism invariant by construction, at least on the space of piecewise flat manifolds. Hence part of the difficulties Regge calculus has in forgetting about the lattice structure are eliminated a priori in the DTRS lattice approach.

There are two more important points to stress. The first one is that in the DTRS approach in $3d$ and $4d$, as opposed to $2d$, we can try to make sense out of the pure Einstein action,

4d Simplicial Quantum Gravity with a Non-Trivial Measure

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Abstract

We study 4d simplicial quantum gravity in the dynamical triangulation approach with a non-trivial class of measures. We find that the measure contribution plays an important role, influencing the phase diagram and the nature of the (possible) critical theory. We discuss how the lattice theory could be used to fix the quantum measure in a non-ambiguous way.

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