Parallel Differential-Algebraic Equation Solvers for Power System Transient Stability Analysis*

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Abstract

Real-time or faster-than-real-time power system transient stability simulations will have significant impact on the future design and operations of both individual electrical utility companies and large interconnected power systems. The analysis involves solution of extremely large systems of differential and algebraic equations. Differential-Algebraic Equation (DAE) solvers have been used to solve problems similar in nature to the transient stability analysis (TSA) problem. This paper discusses the possibility of the use of the existing DAE solvers to solve the transient stability analysis application. We also discuss our research in developing a scalable, parallel DAE solver for use by the power system community and in related applications [13].

1 Introduction

In engineering and science, many problems give rise to smooth and/or discontinuous systems of differential equations coupled with nonlinear algebraic equations. These coupled systems of differential and algebraic equations are commonly known as differential/algebraic equations (DAEs). Research in the field of DAEs started about a decade ago and is still in its early stages. A number of engineering and science problems have been solved with the help of DAE solvers developed by [2, 3, 5, 10, 13]. But to this date, there has been no investigation of the application of DAE solvers to power systems problems such as the Transient Stability Analysis (TSA) [1].

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Transient Stability Analysis is a compute-intensive problem requiring the solution of nonlinear, discontinuous systems of DAEs for the purpose of simulating such electrical power systems phenomena as network component and transmission line failures. Existing DAE solvers often use higher order implicit Runge-Kutta integration techniques or Backward Differentiation Formulas (BDF), techniques that yield more accurate solutions or permit larger time-steps. DAE solvers have been especially developed to handle discontinuities in functions, a condition encountered in transient stability calculations.

This paper describes research on the use of existing DAE solvers to solve the power system TSA problem. In section 2, a detailed description of the power system problem is given. In section 3, we discuss various existing DAE solvers and their potential for application to the TSA problem. Our ongoing research in the benchmarking of various sequential solvers and the development of parallel solvers is discussed in section 4. Lastly, in section 5, our conclusions on the application of DAE solvers for TSA problem are discussed.

2 Power System Transient Stability Analysis Problem

Transient Stability Analysis (TSA) examines the dynamic behavior of a power system for as much as several seconds following a power-flow disturbance. The general nature of the transient stability problem is explained in [11] as follows: Consider a simple mechanical analog where a number of masses, representing the generators in an electric power system, are suspended from a “network” consisting of elastic strings representing the electric transmission lines. Now one of the strings is suddenly cut, corresponding to a sudden loss of a transmission line. Thereafter the masses will experience transient coupled motion, and the forces in the strings will fluctuate. This sudden disturbance may cause one of the following two effects:

1. The system will settle down to a new equilibrium state, characterized by a new set of string forces corresponding to the power voltages in the electricity case.
2. Because of the transient forces, one or more additional strings will break, causing a weaker network, resulting in a chain reaction of broken strings and total collapse of the system. If the system has the strength to survive the disturbance and settle into a new steady state, it is referred to as “transient stable for the fault in question.”

The transient stability analysis problem can be described by a system of DAEs where the generators are represented by differential equations and the power system network interconnecting the generators is represented by nonlinear algebraic equations. The reaction of a single generator to variations in its load can be modeled using the following Ordinary Differential Equations (ODEs):

$$E\ + x_d I_d = E_q^{'} + x_d^{'}$$

$$E_d + x_q I_q = E_d^{'} + x_q^{''} I_q$$

$$\tau_{q0}^{'} \dot{E}_d^{'} = -E_d^{'} - (x_q - x_q^{'}) I_q$$

$$\dot{E}_q^{'} = \frac{1}{T_{do}^{'}}(E_{FD} - E)$$

$$T_e = E_d^{'} I_d + E_q^{'} I_q - (L_q^{'} - L_d^{'}) I_d I_q$$

$$\tau_j \dot{\omega} = T_m - D\omega - [E_d^{'} I_d + E_q^{'} I_q - (L_q^{'} - L_d^{'}) I_d I_q]$$

$$\dot{\delta} = \omega - 1$$

The equations described above roughly describe the “structure” of the differential equations involved in the transient stability analysis DAEs. The notation used in the above equations is as follows:

- E is the magnitude of source voltage of the generator,
- I is the field current of the generator,
- δ is the rotor angle,
- T_e and T_m are the electrical and mechanical torques respectively,
- ω is the shaft angular velocity,
- x is the reactance of the generator,
- L is inductance involved,
- τ_j and $\tau_{q0}^{'}$ are the time constants involved in the derivation of these equations.

Subscripts d, q, F, and D represent the respective d-axis, q-axis, F-axis and D-axis components of the elements described. Detailed explanation on the notation involved in the described equations can be found in [1].

There are multiple generators supplying power to the network and the generators are coupled through an electrical power network. The power network can be modeled by nonlinear algebraic equations as follows:

$$I = V\ Y$$

where I is the network current, V is the network voltage and Y is the network admittance. Apart from these two sets of equations, TSA involves equations for loads (which can be differential or algebraic) and equations for the generator control (that are differential). All these equations together form a system of DAEs that must be solved simultaneously.

Structure of the TSA DAEs: The transient stability analysis DAEs are non-symmetric in nature. They are of bordered-block-diagonal form wherein blocks of generator equations along the diagonal are coupled with the power system distribution network. The admittance matrix involved is extremely large, complex and sparse. Research is being conducted [9] to reorder this matrix into block-diagonal-bordered form in order to exploit the structure for parallelism.

Order of the TSA DAEs: The number of equations involved in the transient stability analysis solution is extremely large. Consider an example of 20 differential equations to describe each generator and two equations to describe the complex voltage/current at each network bus. Then an interconnected power system with 2000 buses and 300 generators could generate a sparse, unsymmetric system of 10,000 nonlinear algebraic equations that must be solved simultaneously. For regional power systems, the number could be five times higher.

Index of the TSA DAEs: The differential index of a system is the minimal number of analytical differentiations needed such that the differential equations of the DAE can be transformed by algebraic manipulations into an explicit ODE system. The transient stability problem leads to an index-one or higher-index DAE system with the possibility of the ODEs involved being stiff in nature.

Algebraic Equations of TSA: The network equations of the TSA are complex in nature (the voltages and currents involved are both complex). These equations are also nonlinear. Research in developing a solver that can be used directly to solve a system of complex, nonlinear algebraic equations has been limited and leaves the field wide open for further research.

Scalability: Since a regional power network can consist of many individual power systems, it is highly desirable that the DAE solver used for the transient stability analysis be scalable. Also, the power networks can vary in size depending on the region under consideration.

3 Differential-Algebraic Equation Solvers

The power system community has been solving the transient stability problem approximately as a decoupled system of differential and algebraic equations. As explained in [2], this method of reduction of large DAE system to a system of explicit ODEs not only destroys the sparsity of the system but also prevents the exploitation of the system structure (symmetry of the Jacobian). Several sequential DAE solvers exist that use various numerical techniques for their solution. Prominent methods among the existing solvers are as follows:

- Extrapolation methods [3, 5],
- Implicit Runge-Kutta methods [5],
- Backward Differentiation Formulas (BDF) methods [2, 6, 7, 8],
- Rosenbrock Methods [5].

Our research is concentrated on analyzing the following DAE solvers:

- **LIMEX** [3] uses an extrapolation technique. As discussed in [2], one-step methods such as extrapolation techniques have an inherent advantage over BDFs methods when applied to DAEs with frequent discontinuities. LIMEX is a code developed for semi-explicit index one DAEs. It implements an extrapolation of the semi-implicit Euler method and can handle stiff ODEs. This code has been successfully used in the solution of problems arising in combustion modeling where there are frequent discontinuities in time. Because of its ability to handle stiff differential equations and frequent discontinuities, LIMEX is an excellent choice for the power system TSA solution.
- **SODEX** [5] and **SEULEX** [5] also use extrapolation technique for the solution of DAEs.
- **RADAU5** [5] uses an Implicit Runge-Kutta (IRK) method of order five. Research on solution of stiff ODEs has focused a great deal on Implicit Runge-Kutta methods. RADAU5 is designed to solve index one, two, and three systems. IRK methods have a definite advantage over multistep methods such as BDFs methods when the DAEs exhibit frequent discontinuities. Because of their one-step nature, IRK methods can be started at a higher order after every discontinuity, whereas multistep methods must be restarted, usually at a low order, after every discontinuity. This fact helps the IRK methods to be more efficient. Also, with IRK methods, it is possible to construct high order A-stable or nearly A-stable IRK formulas, which is important while solving stiff ODEs with eigenvalues lying close to the imaginary axis. The generator ODEs in the transient stability analysis may be of this type. Since the TSA DAEs can be of index higher than one, RADAU5, with its ability to handle higher index systems makes an excellent choice for the transient stability analysis problem.
- **DASSL** [2] uses a multistep, backward differentiation technique. It uses a variable step-size, variable-order fixed-leading-coefficient implementation of BDFs formulas to advance solution from one time step to the next. DASSL can solve DAEs of index zero and one. BDFs methods suffer no order reduction for index one systems. Also, BDFs methods achieve the same order of convergence for this class of DAEs as they do for ODEs. DASSL has a robust order selection strategy for DAEs with eigenvalues close to the imaginary axis in the complex plane. DASSL can also be used to solve semi-explicit index-two systems of DAEs. DASSL by itself cannot handle discontinuities, but a variation of DASSL, DASRT, has root finding capabilities to locate discontinuities when they are sufficiently large that DASSL cannot integrate through without intervention. Re-

search is being conducted at the Northeast Parallel Architectures Center (NPAC) [10] to solve the TSA problem using DASSL.

- **LSODI** [6, 7, 8] uses BDFs. It uses a fixed-coefficient implementation of BDFs formulas to solve linearly implicit system of DAEs. The fixed-coefficient methods can be implemented efficiently for smooth problems, but suffer from inefficiency and possible instability for problems requiring frequent step-size adjustments. LSODI solves DAEs of index zero but if the coefficient matrix is singular, then it solves the resulting index one DAE system. It can solve systems of stiff and non-stiff ODEs as well. LSODI has been successfully used to solve systems arising from PDE problems. Another version of LSODI, LSODA, switches automatically between stiff (BDF) and non-stiff (Adams) methods. Because of its ability to handle stiff ODEs and frequent discontinuities, LSODI presents a viable alternative to other solvers discussed for the power system TSA problem.

- **RODAS** [5] uses an embedded Rosenbrock technique of order four. Rosenbrock methods have a big advantage over IRK methods in that they completely avoid non-linear systems of equations, at the same time providing the advantages of accurate solutions with stiff differential equations. RODAS can solve DAEs that are expressible in semi-explicit form and that are of index one. Because of the lack of data about the application of RODAS to any significant problem, and because of the varied advantages of other solvers discussed above, our research will not involve RODAS, even though it has the qualities of being a good contender.

4 Research Status

Currently at NPAC, we are involved in the following two issues: Transient Stability Analysis Benchmarks, and Parallel DAE Solvers.

4.1 Transient Stability Analysis Benchmarks

We are setting up transient stability analysis benchmarks with interfaces to different sequential solvers like LIMEX, LSODI, RADAU5 and DASSL. This is to test our empirical conclusion that these solvers are best suited for the TSA problem. We will also present a comparative review of their speed and performance. The test data being used is the standard IEEE 118-bus data but we will also benchmark larger systems at a later stage.

4.2 Parallel DAE Solvers

CDASSL (Concurrent DASSL) [13] is the first attempt that has been made at parallelizing these DAE solvers. More recently, attempts have also been made to develop a data-parallel and message-passing versions of DASPK [12] on the CM5. Our research is examining the scalability of the various DAE solution techniques. We are concentrating on selecting the most suitable solver for the power system application, and creating a scalable library for an application based on it. The parallel version of this DAE solver is being developed for the use by the power system community, and in related applications such as the chemical plant simulations (*e.g.,* [13]) and the electrical circuit simulations [2]. We are using the *Multicomputer Toolbox* [15] for the development of the parallel solver. This software is currently based on the portable message-passing system *Zipcode* [14]; the *Toolbox* will port to *MPI* [4] next year and hence our programs will run on all reasonable message-passing multicomputers.

5 Conclusions

We have discussed the power system transient stability analysis problem (TSA). We also presented a review of some of the existing DAE solvers that we consider best suited for this problem. Benchmarking of sequential DAE solvers and the development of a parallel, scalable DAE solver are in progress. This research is being closely coordinated with parallel sparse matrix solver [9] research being performed at the Northeast Parallel Architectures Center (NPAC) at Syracuse University.

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