Multithreaded approach for dynamic load balancing of parallel adaptive PDE computations

Nikos Chrisochoides*
Northeast Parallel Architectures Center
and Computer Science Department
Syracuse University, Syracuse, NY 13244-4100

Abstract

We present a multithreaded model for the dynamic load-balancing of parallel adaptive PDE computations. Multithreading is used as a means of exploring concurrency in the processor level and for tolerating: (i) long memory latencies, and (ii) synchronization costs inherent to traditional load-balancing methods. We show that under certain values of the parameters (i.e., number of threads, and context switch time) of the model, multithreaded load-balancing systems are expected to perform better than existing software systems based on traditional load-balancing methods.

1 Introduction

One of the difficulties in parallel programming is attaining good performance when solving problems with irregular and dynamic (adaptive) computations. Many of the early successes of parallel processing were obtained on relatively regular problems (e.g., structured, static grids). The need to solve real-life problems increased the necessity to address issues related to the parallelization of irregular and adaptive computations such as adaptive finite-element computations for fluid

*This work was supported by an Alex Nason Prize Award, by NSF ASC 93 18152/ PHY 93 18152 (ARPA supplemented), and by NASA Contract No. NAS1-19480, while the author was in residence at ICASE, NASA Langley Research Center, Hapton, VA 23681.

flows and structures. One way to deal with these problems is to explore the use of concurrency within a processor. A processor executes many computational actions or threads of control; each thread is typically dependent on results of local or remote threads. Instead of scheduling these threads in a static and predefined way, we allow them to be dynamically scheduled based on availability of the data they depend on.

Traditional load-balancing methods for single-threaded computations under-utilize multiprocessor resources such as CPU, memory, and network. Because the load-balancing of single-threaded computations is carried out in sequential phases that require global synchronizations [1], [2]. As an alternative to the traditional load-balancing methods we propose a multithreaded model for developing runtime support systems for parallel adaptive computations. Multithreading is used as a mechanism for : (i) tolerating long memory latencies, and (ii) concurrently executes both tasks required for load-balancing—such as information dissemination, decision making, and data migration—and tasks required for the computation of the actual application. Multithreading improves CPU and network utilization by masking the inherent synchronization delays involved in traditional single-threading load-balancing methods. We prove that under certain values of the parameters (i.e., number of threads, and context switch time) of the model, multithreaded load-balancing systems are expected to perform better than existing single-threaded load-balancing software.

The rest of this paper is organized as follows: Section 2 provides an overview of exiting load-balancing methods and a brief introduction to lightweight threads. In Section 3 we describe a set of geometric abstractions that we use in the rest of the paper. Section 4 outlines the basic principles of our load-balancing algorithm based on the multithreaded model. Section 5 presents a comparison of the load-balancing algorithm with existing direct and incremental load-balancing methods. Finally, in Section 6 we outline a number of advantages and disadvantages of the multithreaded approach for parallel numerical computing and we conclude with future work and directions.

2 Background

Parallelism, for data-parallel PDE solvers, is achieved by decomposing the underline geometric (i.e., grids) or algebraic (i.e., linear systems of equations) data structures. The data decomposition of grids or sparse coefficient matrices is equivalent to a graph partitioning problem which is

an NP-Complete problem. During the last 8 to 10 years many interesting heuristics have been proposed to compute sub-optimal data distributions for static PDE problems. In this section first, we present an brief overview of the most commonly used heuristics and introduce basic concepts of threads which we will be using throughout the paper. Threads were used for many years in disciplines such as real-time systems, and distributed operating systems very successfully. Threads are used here as a mechanism to explore concurrency within a processor and tolerate long memory latencies and synchronization costs inherent to traditional load-balancing methods.

2.1 Load-Balancing: An Overview

The objective of any load-balancing system is to minimize the execution time of the slowest processor. Existing systems assume no overlapping of communication with calculations and balance the processors' work loads by minimizing an objective function that is equal to the summation of the processors' work loads and communication costs[1] [23]:

$$OF = \max_{1 \le i \le \mathbf{P}} \{ \; W(m(D_i)) \; + \sum_{D_j \in \kappa_{D_i}} C(m(D_i), m(D_j)) \; \} \tag{1}$$

where, for data-parallel PDE solvers, $m : \{D_i\}_{i=1}^{\mathbf{P}} \rightarrow \{P_i\}_{i=1}^{\mathbf{P}}$ is a function that maps the grid points (grids) of submesh D_i to the processors; $W(m(D_i))$ is the computational load per iteration of the processor $m(D_i)$, which is proportional to the number of grids in D_i; $C(m(D_i), m(D_j))$ is the cost of communication required (per iteration) between the processors $m(D_i)$ *and* $m(D_j)$; and, finally, κ_{D_i} is the set of submeshes that are adjacent to D_i. These heuristics are classified into two classes: direct (or clustering) and iterative (or incremental). A list of very interesting results for direct and incremental methods, which is by no means complete, appears in [22], [23], [24], [25] [13], [41], [42], [43], [44], [50], [1], [45], [46], and [47].

The data-clustering algorithms are based on grouping mesh points into clusters such that the points within a cluster have a high degree of "natural association" among themselves, while the clusters are "relatively distinct" from each other. In our case, "natural association" is expressed in terms of the locality properties of the finite element and finite difference stencils that are used to approximate a continuous PDE operator. The "relative distinction" is expressed in terms of the address space that is associated with the unknowns of the mesh or grid points of the same cluster. Most of these algorithms are computationally expensive and very successful in solving

[1] We assume homogeneous processors.

the load-balancing problem for static PDE computations [13], [48], [49]. They require a complete global knowledge of the graph (mesh) and, therefore, these methods are not suitable for adaptive methods in which the topology and/or geometry of the mesh change any time we perform an h-refinement throughout the PDE solution process. In addition, some of these methods are sensitive to small perturbations (h-refinement) and often lead to heavy data migration [44], [45].

On the other hand, existing incremental (single-threaded) methods are not as expensive as clustering methods. Flaherty's group at PRI have shown in [1], [50] that these methods are very successful in load-balancing the computation of adaptive PDE methods on distributed-memory MIMD machines. Incremental methods—as is true for direct methods—decompose the parallel adaptive PDE computations into three phases: (1) *computation*, (2) *balancing* and (3) *data-migration*. The *computation* phase corresponds to the actual computation and inter-processor communication required by the PDE solver, while the *balancing* and *data migration* phases correspond to the calculation and inter-processor communication required to solve and enforce the solution of the minimization problem defined by equation (1). A global synchronization barrier guarantees that all processors reach the balancing and data-migration phases at the same time [1].

2.2 Threads

A thread is an independent set of instructions that executes within the context of a UNIX process (see [8], [9], [10], [11], and [12]). Threads in multithreaded programs run logically concurrently. For multicomputers, the decomposition of a coarse-grained computation into finer grained, logically concurrent executable threads is needed for three reasons : (1) to overlap in time logically separate tasks that use different resources (i.e., network, CPU, disks), (2) to simplify parallel programming, and (3) to load balance computations. Typical examples for the logically concurrent execution of threads are (1) the overlapping of calculation with communication [14], and (2) the overlapping of load balancing and actual computation phases [15].

During execution each thread can be in one of the following states: *new, ready, running, blocked, dead*. The state of a thread is defined by its current activity. When a thread is created it is given a function to run and it is set to a *new* state. A thread in the *new* state consists of various data structures that describe its context. Once the data structures are allocated and the thread is registered with the system, the thread moves to the ready queue and its state is set

to *ready*. If a thread is selected to execute, then its state changes to *running*. While a thread is in the running state it may decide[2] to wait for a condition or for outstanding receives to be signaled, in which case its state changes to *blocked*. Finally, after a thread completes its execution or decides to terminate its state, it changes to *dead*. The use of these states will become clear in Sections 4.

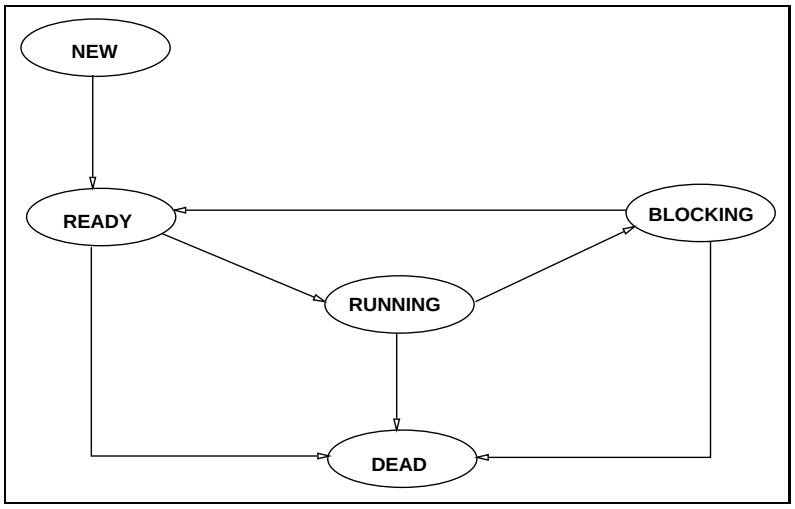

Figure 1: Thread state diagram.

For the type of parallel computations we address in this paper we need non-preemptive scheduling of the threads (i.e., threads run to completion or voluntarily yield the CPU). An advantage of such a scheduling strategy is the reduction of overhead by keeping context-switches[3] at a minimum. Threads are classified into *heavy- or light-weight* threads based on the amount of context (weight) that needs to be saved/restored when a thread is removed or reinstated from/to CPU. In this paper we deal with light-weight threads.

The computation of multithreaded data-parallel programs consists of two phases, namely, *computation* and *thread-scheduling* phases. During the computation phase, the processor concurrently performs the operations needed to execute the actual computation: (1) it initializes send/write operations, (2) it polls and notifies threads of outstanding messages, (3) it performs actual calculations, and (4) it provides services such as get/put data to/from remote proces-

[2] In the case of non-preemptive systems.

[3] Switching from one thread to another requires a certain amount of time for administration (saving and loading registers and memory maps, updating various tables and lists).

sors, check/change status of non-local threads, data migration, etc. Finally, during the thread-scheduling phase the processor does the bookkeeping required for the administration and the execution of threads. This is an additional overhead that does not appear in the single-threaded data-parallel paradigm.

3 Basic abstractions

We break the original load-balancing problem into many simpler problems by defining a hierarchy of geometric abstractions: *domains, blocks, subdomains and regions*. Regions are mapped to scalar objects called threads. Threads execute in a loosely synchronous mode. Based on computation and synchronization requirements, threads are grouped into distributed objects: *strings, ropes and nets*. Threads that correspond to regions of the same subdomain belong in the same string (see Figure 2). Threads that belong in the same string execute the same code on different data (SPMD model). Strings that correspond to the subdomains of the same block belong in the same rope. Threads on different ropes might compute the solution for different PDEs, use different grid types, apply different solution methods, and they may therefore have different computational requirements and synchronization points (MPMD model). Finally, ropes that correspond to blocks of the same domain and handle the computation associated with the same application belong in the same net (see Figure 2).

Processor work-load is balanced by: (i) migrating threads from overloaded processors to underloaded ones that handle strings from the same rope, and (ii) by migrating strings from overloaded processors to underloaded ones that handle ropes from the same net. The thread and string migration is non-preemptive and, therefore, instead of thread migration we perform data-migration. The data are migrated so that subsequent communication for the actual parallel computation of the PDE solver is minimized.

For each subdomain $S_{i,j}$ we identify two types of regions: *interface* regions and *interior* regions. A region is considered to be interface if there is a grid point in the region that has at least one of its adjacent neighbor points residing in a different context (traditionally non-local memory). Non-interface regions are considered to be interior (see Figure 3). Computations on interior regions of different subdomains can be performed independently. To each interface or interior region we correspond (create) one *thread*, t (see Figure 3). The size, $|t|$, of a thread is analogous to the number of grid points of the corresponding region—many times in this paper we denote

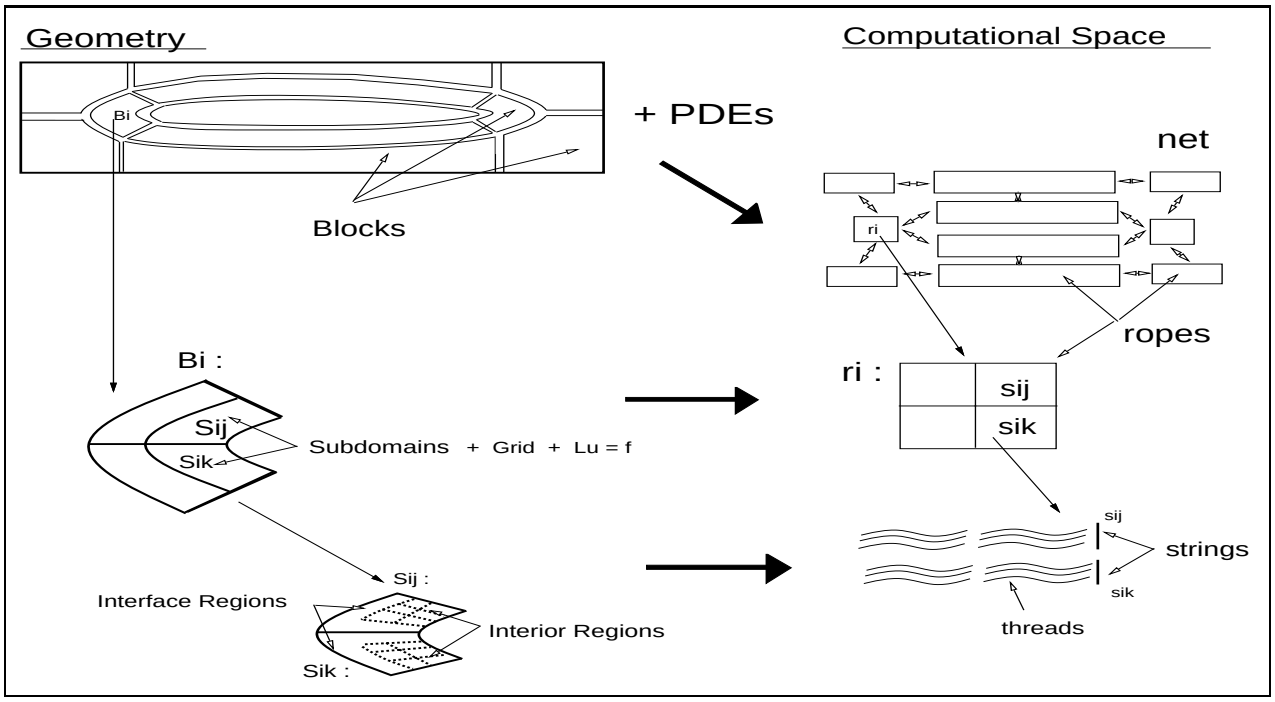

Figure 2: Geometric and parallel abstractions.

by t the mesh that corresponds to the thread t. All threads for h-refinement PDE methods are of the same size. The size of a thread can change during the computation in order to achieve better balancing of processors' work-loads (i.e., each thread can be split into two or more threads, depending on the required load-balancing resolution; such a resolution can be achieved within a small number—order of $\log_2 |t|$ or $\log_4 |t|$— of iterations compared to the number of iterations required by incremental load-balancing algorithms [1]).

Interior threads execute exclusively on data residing in the memory of the processor on which the threads execute, while interface threads require the access of non-local data. Threads corresponding to regions of the same subdomain belong in the same process (context) and communicate using the shared-memory (user-address space for the process) model, while interface threads that correspond to regions from different subdomains communicate through message passing (for more details, see [17]). Two types of local communication are identified: *inter-block (or inter-group)* and *intra-block (or intra-group)*. Scheduling between these communication types can reduce network traffic. Such schedulings can be achieved by grouping threads of the same block into *ropes* [26], [27], [28]. Ropes can use group synchronization and collective communication mechanisms and more than one rope can share resources such as CPU, network, and

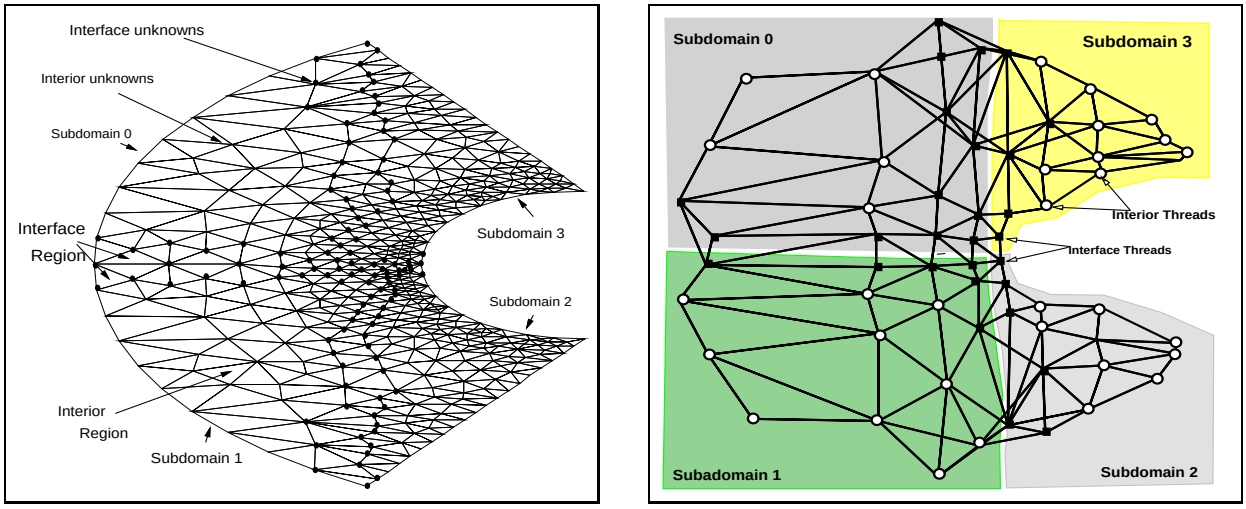

Figure 3: Left) Block, B_i, in middle row second from the left of Figure 2; B_i is partitioned into four subdomains, $\{S_{i,B_i}\}_{i=1}^{4}$. Right) Global thread graph and its partition into four subgraphs that correspond to $\{S_{i,B_i}\}_{i=1}^{4}$. Threads with data dependencies (edges) that cross the internal boundary of the subdomains are interface threads, otherwise they are interior threads.

memory. For more details on ropes, see a companion paper on an object-oriented approach for load-balancing [29] (also, see [27], [26]).

4 Multithreaded approach for load-balancing

In this section we describe a multithreaded model for developing load-balancing (sharing) algorithms. We've seen in Section 2.1 that the traditional single-threaded approach for load-balancing of PDE computations leads to (1) under-utilization of multiprocessor resources such CPU and network and (2) in some cases intensification of problems like network contention — due to the fact that all processors perform data migration simultaneously. In this Section we propose a new approach for load-balancing that explores concurrency within the processor in order to maximize utilization of multiprocessor resources without sacrificing program complexity. Our approach, in contrast to the direct and incremental single-threaded load-balancing methods, attempts to ensure that no processor is waiting idle while more than one thread remains to be executed on other processors. Each processor, when the need arises, requests work (threads) from a subset (neighborhood) of processors that are overloaded or slow.

Independently off the approach (traditional or new) we use, to load-balance PDE computations we should concern about three fundamental issues, namely: *(1) memory latency, (2) synchronization cost, and (3) convergence rate.* In this paper we address the memory latency and synchronization cost; it is difficult to uncouple these issues and therefore we have to consider the convergence rate of PDE solvers whenever we deal with message passing and scheduling latencies. In the rest of the paper we address issues related to convergence rate of PDE solvers only whenever is absolutely necessary.

4.1 Memory Latency

Parallel computers introduce a new level in the storage hierarchy; in addition to registers, cache and memory, there is remote memory that is accessed across an interconnection network. In this paper our objective is to distribute the computation and data so that we not only balance processors' work loads but also minimize overheads due to message passing (i.e., memory latencies). In order to achieve our objective: (1) we minimize the access of non-local unknowns by minimizing the number of grid points that reside on the interfaces of the subdomains (see Figure 3) and (2) we mask memory latencies due to access of non-local data by overlapping calculations with communication. We use threads to solve the above minimization problems without increasing the software complexity. Threads here are used as a mechanism to explore concurrency in the processor level in order to tolerate memory latency.

For multithreaded parallel PDE computations we identify two types of communications: first, the communication between interior and interface threads that belong in the same context (local threads), and second, the communication between interface threads that belong in different contexts (non-local threads). The efficient communication of interior and interface threads is critical for the overall performance and success of the multithreaded approach. Next we briefly describe the different communication mechanisms between local and non-local threads. For more details on the implementation of these mechanism, see [36], [37], [38], [20], [39], and [40].

The communication between local threads can be implemented using one of the following three approaches: (1) *message-passing*, (2) *shared variables*, or (3) $\mathcal{W}$*-buffer scheme* [36]. An advantage of the first approach is that it treats the communication of both interior and interface threads in a uniform way, thereby simplifing programming. A disadvantage is that its implementation (with some exceptions e.g., Mach) requires copies of local data-structures (to threads) that

belong in the same address space.[4] The second approach eliminates additional copy operations by using shared global data-structures. A disadvantage of this approach is that it requires the use of synchronization mechanisms (mutexes) to protect unwanted reads/writes from/to shared variables. The implementation of this approach increases program complexity and requires drastic code re-structuring of existing single-threaded programs.

Finally, the third approach we present in [36] is a communication scheme specific to PDE computations[5] and is based on $\mathcal{W}$ (≥ 2) copies of the shared-variables. The idea of this scheme is to use "rondeau" memory locations in such a way that a thread (destination in the case of message passing) always reads the correct values that its partner (source) just wrote. Read and write operations between threads that share these copies of variables are interleaved (odd/even —*mod* $\mathcal{W}$— iterations in the case $\mathcal{W} = 2$) in a way that the overwriting (by one thread) of useful values (to another thread) is prevented. Therefore, this scheme preserves the integrity of shared variables without substantially increasing program complexity (in the case of single-copy shared-variables) and without introducing overheads by unnecessary and expensive copy operations (in the case of message passing) on local data structures. This approach can be implemented on top of the existing single-threaded data-parallel codes with minimum modifications. A disadvantage of this communication scheme is that the storage complexity is increasing by $O\left(\mathcal{W} \cdot \sqrt{|t|}\right)$.

The communication between non-local threads (i.e., reside in the memory of different processors) can be implemented on top of existing message-passing such as MPI [19], p4 [30], and PVM [31]. Unfortunately, most of the currently available message-passing software do not provide support for sending/receiving messages to (from) a specific entity (function) of a process. To provide thread-to-thread communication mechanism we use an idea similar to the AMs which is described next. For more details on the implementation of the thread-to-thread communication mechanism, see [37], [38], [20] and [40].

An interface thread that executes for the first time sends its messages to other threads, then posts all its receives[6] and voluntarily yields the CPU to the dispatcher. The dispatcher does the proper bookkeeping and schedules the next interface thread—if any. After all the interface

[4] This is additional overhead that does not appear in the single-threaded approach.

[5] However it can be generalized for many other similar computations.

[6] If a non-blocking receive operation is available, it first posts its receives and then sends its messages to other threads, which increases the probability of saving a local copy of the message from the system buffer to user address space.

threads from the ready queue are exhausted the dispatcher schedules the first interior thread. Interior threads require only local data and therefore execute until completion. This process is repeated until all interior threads are also exhausted from the ready queue.

During the time interior threads perform their own computations, non-local data is arriving from the network (see Figure 4). The non-local data is stored in memory locations at the user space. In [37] we describe a mechanism where a specific function (message-handler) is activated (on message arrival the process is interrupted by a hardware signal) and performs the following operations:

- Parses the header of the message and identifies the destination thread.
- Decrements thread counter that indicates the number of outstanding receives and, when it becomes zero, changes the state of the thread from blocking to ready and then moves the thread from the blocking stack to the ready stack (see Figure 4). At this point, interface threads have all the data (local and non-local) they need to perform their computations. Notice that while interface threads are waiting for incoming messages (through the network), the CPU is utilized by the interior threads, and thus calculations and communication are overlapped.

4.2 Synchronization Cost

Load-balancing operation is a special case of the *producer-consumer* operation. Consumer-producer operations like forks and joins and mutual exclusion in parallel programming require synchronization. In parallel adaptive PDE computations the synchronization cost appears in the form of waiting time due to unbalanced processor work-loads. Our objective is to minimize this time and mask if possible inherent delays involved in the traditional load-balancing methods. Again we do not want to sacrifice program complexity. Our approach, in contrast to the direct and incremental single-threaded load-balancing methods, attempts to ensure that no processor is waiting idle while more than one threads remain to be executed on other processors. Each processor, when the need arises, requests work (threads) from a subset (neighborhood) of processors that are overloaded or slow.

Let us assume that our processors are homogeneous and our PDE solver does not share the resources of the system with any other application. We define as *computational graph* $G_C(E_C, V_C)$,

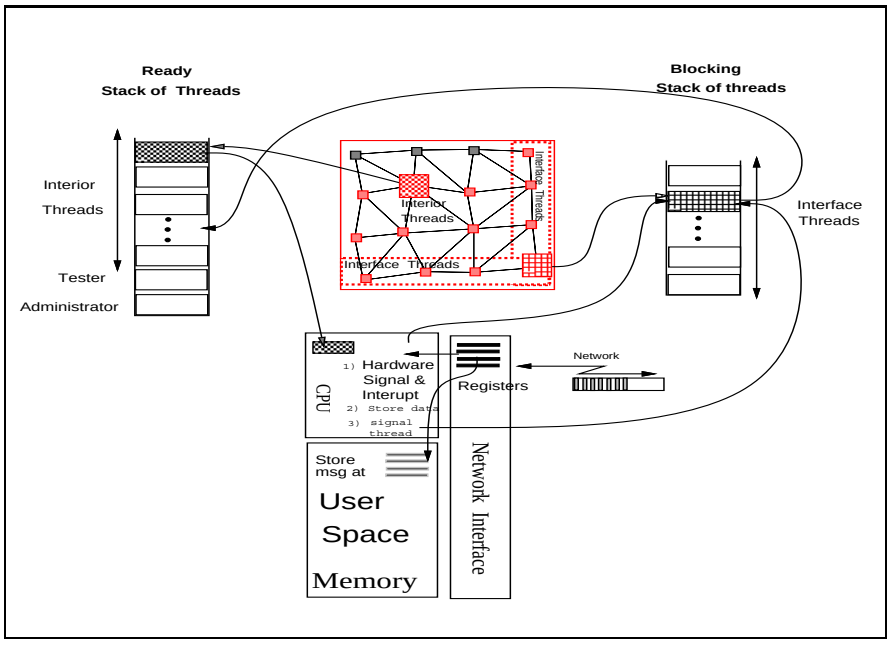

Figure 4: Overlapping of computation with communication; while interface threads are blocked in the *blocking stack* waiting for the arriving non-local data, interior threads—from the ready stack—utilize the CPU, performing computations on local data.

and denote as G_C throughout this paper, the graph whose vertices, v_i, correspond to the submeshes D_i, and the edges $e_{i,j}$ connect two vertices (v_i, v_j) if $D_i \cap D_j \neq \emptyset$. The weights, w_i, on the vertices v_i of the graph correspond to the computation associated with the submesh D_i and they are analogous to the number of mesh points, $|D_i|$. Figure 5–right depicts the computational graph of the refined mesh (Figure 5–left); the weights w_i indicate the number of threads per subdomains (i.e., context or processor).

During the computation of adaptive PDE methods the mesh is refined in areas where the resolution of the solution is larger than a given tolerance (see Figure 5). After the mesh refinement is completed, new threads are created (or old ones are destroyed) at runtime. All threads are of the same size (h-refinement). Processor computation is balanced by migrating interface threads. The thread migration is non-preemptive (i.e., threads migrate before they start execution) and, therefore, instead of thread migration, (save-and-migrate threads context, registers-values, etc.) we perform data-migration of mesh-points only. The data are migrated so that subsequent communication for the actual parallel computation of the PDE solver is minimized. Figure 6 depicts the first four migration phases required to share processors work loads. Notice that,

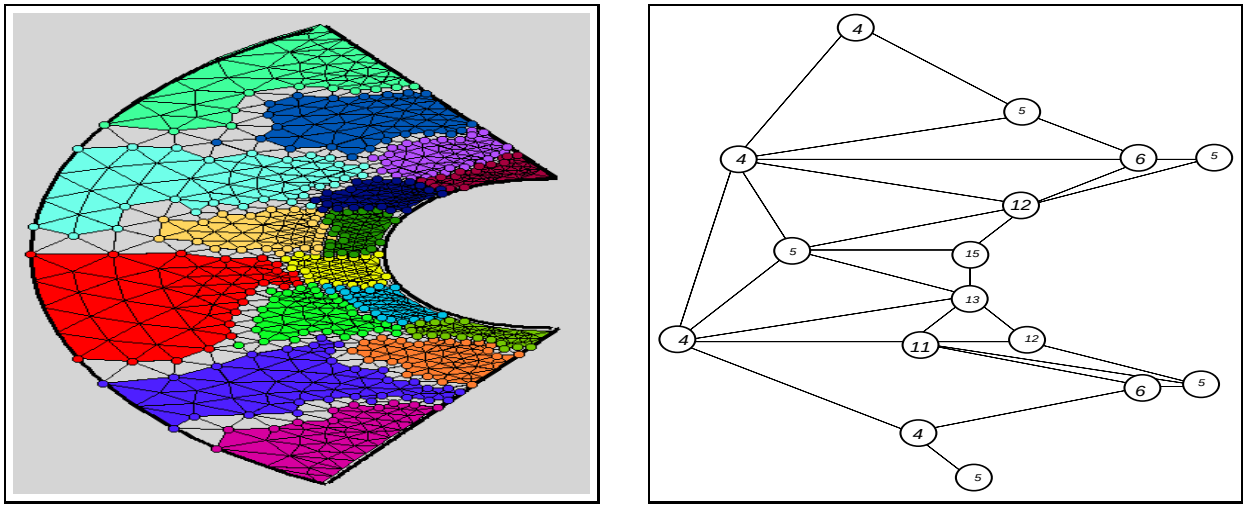

Figure 5: Left) h-refinement and a 16-way partition of the block B_i of domain defined in Figures 2. Right) Computational graph with uneven number of threads per subdomains (i.e., context or processor).

in contrast to incremental methods, load sharing of processors is completed within the first iteration (i.e., before any global reduction operation is required for error checking or update of global variables).

The policy for thread migration is based on a *consumer-initiated consumer/producer (C/P) paradigm*. That is, every processor $P_i = m(D_i)$ (consumer), after it completes its computation (when counter of ready and blocking stacks is zero), searches its neighborhood

$$N(P_i, l) \;=\; \{P_j, P_j \;=\; m(D_j)\; and\; D_j \in N(D_i, l)\;, l = 1, ...,\; diameter(G_C)\;\;\}$$

to identify neighboring processors that are overloaded.[7] Since our model attempts to assure that no processor remains idle the consumer sends interrupt-driven messages to its neighbors (see [37] for implementation details) and requests the migration of one or more threads.

After an overloaded processor $P_j \in N(P_i, l)$ is identified, P_j (producer) interrupts its computation and sends a thread (data) to P_i. The thread that is migrated from the producer to consumer who has initiated the request is likely to have data dependencies with other threads that already

[7] Due to changes on the demand of computation, in the case of adaptive methods, or due to external loads of the processors in the case of time-sharing heterogeneous workstations.

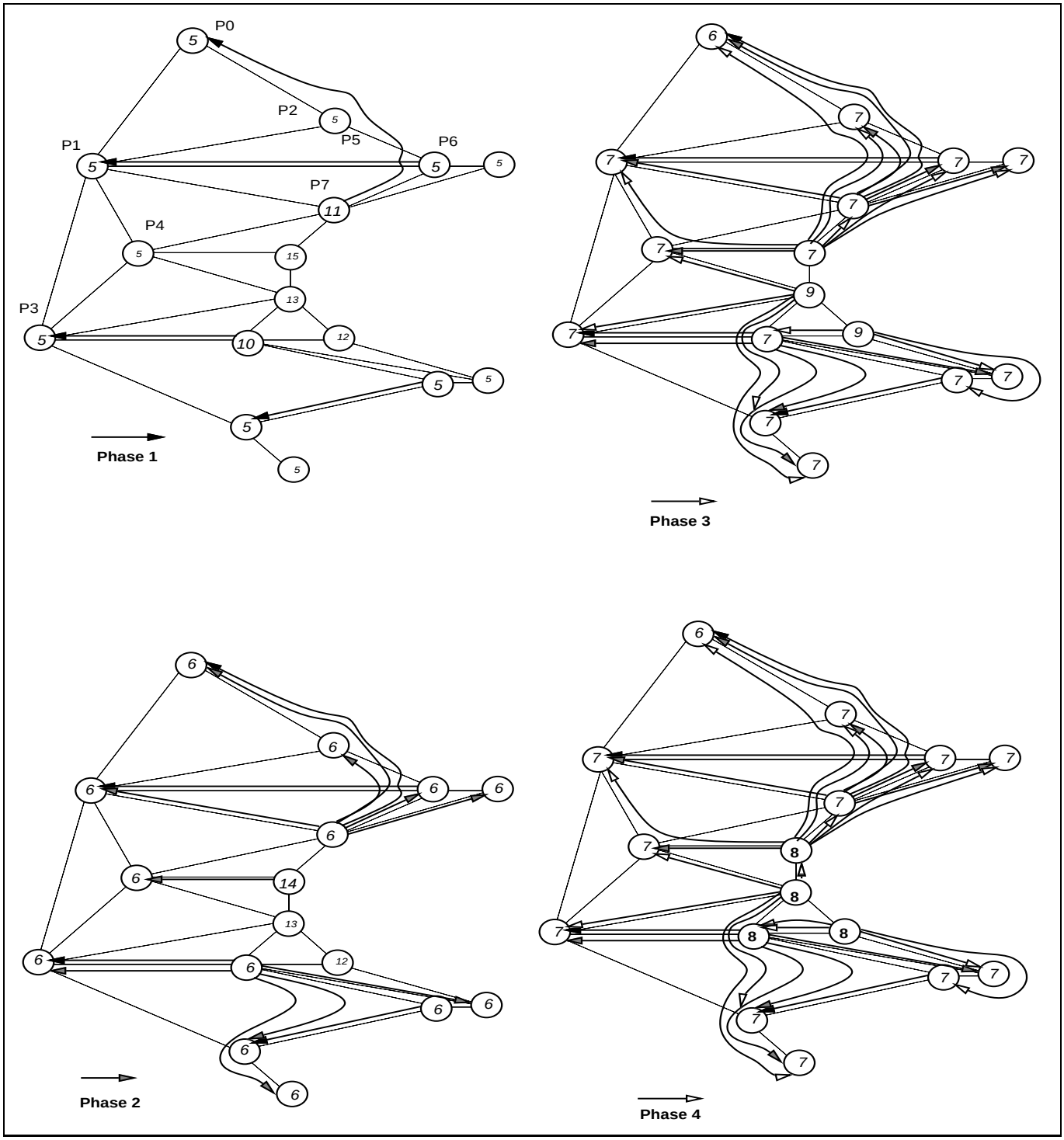

Figure 6: The four phases required to migrate threads from overloaded processors to underloaded ones, for the example of Figure 5. All four phases take place before the end of the first iteration or occurrence of a global barrier.

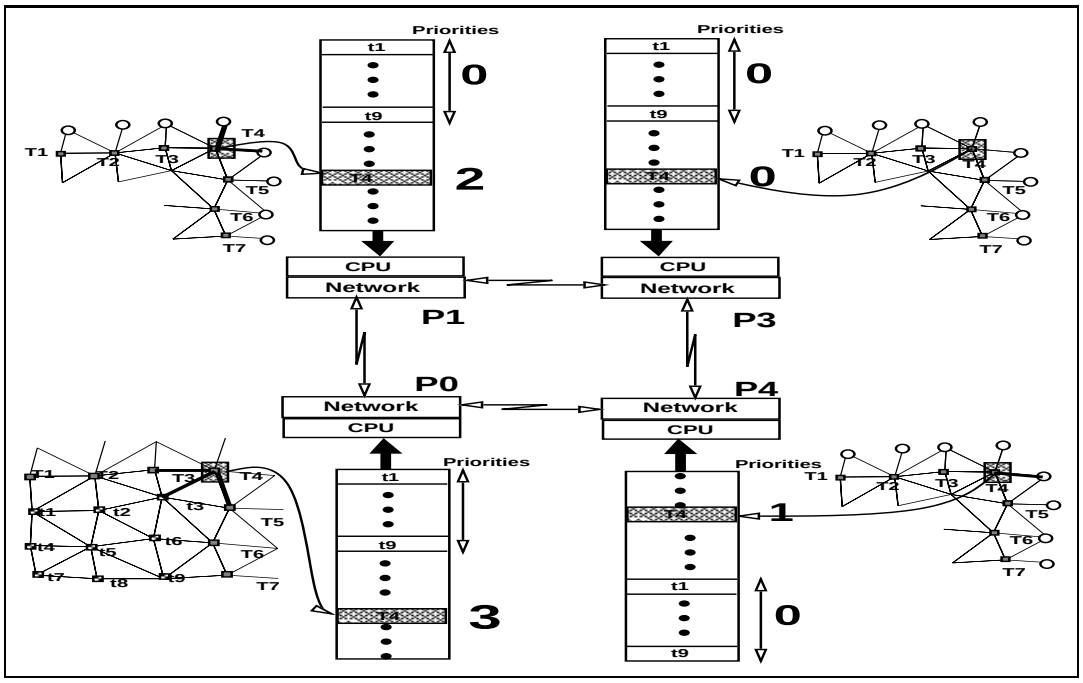

Figure 7: Threads T_4 migrates from processor P_0 to processor P_1; T_4 has most of its data dependencies with the threads of processor P_1. The thread migration is based on the principle of minimizing the communication of the actual computation.

reside on the consumer's side. The producer's decision to migrate an interface thread, t_i, is based on priorities, Π_{t_i}, that are computed at runtime using the following equation:

$$\Pi_{t_i} = \sum_{t_j \in N_{t_i}} (1 - \chi(t_i, t_j))$$

where

$$\chi(t_i, t_j) = \begin{cases} 0 & \text{if } m(t_i) = m(t_j) \\ 1 & \text{if } m(t_i) \neq m(t_j) \end{cases}$$

and N_{t_i} is the set of all threads t_j with data dependencies on t_i. Figure 7 depicts the different priorities for an interface thread T_4

A consumer (processor) might get data from more than one producer. In this case it creates and executes non-local threads one at a time, using FIFO policy. Before a non-local thread is created, the consumer uses a remote service request, Check_Thread_State, to check the state of the threads. If the state of a non-local thread for execution is *ready*, then the thread is scheduled for execution. The Check_Thread_State is also an interrupt-driven RSR (see [37] for implementation details). Notice that in contrast to existing load-balancing methods, no

synchronization is required for load sharing between producer and consumer processors. Also, most of the computation required for decision making in sharing loads among processors takes place at the consumer's side. Since any consumer would stay idle, we can use it for the extra work required for the load-sharing. Also, notice that we use interrupt-driven remote service requests so that the consumer can get data and schedule non-local (migrated) threads as soon as possible. Existing load-balancing methods are based on global synchronization barriers that have to be reached by all processors. In the case of single-threaded incremental methods, this implies that some processors have to wait idle, since the processors' loads are balanced gradually.

5 Analysis

Consider the computation graph of Figure 5 and let T_{st} be the total execution time required by the PDE solver—whose computation is balanced by an incremental algorithm—to perform N iterations until the next mesh refinement occurs. An incremental method will balance the computation in K iterations. For each iteration i, with $1 < i < K$, let W^i_{max} denote the maximum work-load over all processors. Let T_{slb} be the summation of time needed for the decision making, communication, and packing data to be migrated from an overloaded processor to an underloaded one. The decision making takes place at the beginning of each iteration i (for $i < K$), and it therefore delays further the execution process of the overloaded (slower) processor. Once the processors decide on the data to be migrated, they send/receive the additional data we denote this time as T_{migr}.

Taking into account that the slowest (most overloaded) processor dominates the execution time at each iteration, we can compute the total execution time between two mesh refinements by:

$$T_{st} = \sum_{i=1}^{K} W^i_{max} + (N-K)W^K_{max} + K \cdot (T_{slb} + T_{migr}). \qquad 2$$

Let $W^i_{max} = W^K_{max} + \Delta_i$ where Δ_i depends on the application and effectiveness of the incremental algorithm and let $C_1 = \sum_{i=1}^{K} \Delta_i$, Δ_i, and subsequently of C_1, be relatively large constants.[8] Then (2) can be written as :

$$T_{st} = K \cdot W^K_{max} + C_1 + N \cdot W^K_{max} - K \cdot W^K_{max} + K \cdot (T_{lb} + T_{migr}) \Rightarrow$$

[8] For the examples that appear in [51], Δ_i varies from 10 to 50 units, and K varies from a few tens of iterations to a few 100's of iterations.

$$T_{st} = N \cdot W^K_{max} + K \cdot (T_{slb} + T_{migr}) + C_1. \quad 3$$

Now, consider again the same computation (Figure 5) as above and let T_{mt} be the total execution time required by the PDE solver—whose computation is load balanced by a multithreaded load-sharing algorithm—to perform N iterations until the next mesh refinement occurs. Then T_{mt} is equal to :

$$T_{mt} = \sum_{i=1}^{N} (W^t_{avr} + N_t \cdot T_{ctxt}) + L \cdot (T_{mlb} + T_{put}) \Rightarrow$$

$$T_{mt} = N \cdot W^t_{avr} + N \cdot N_t \cdot T_{ctxt} + L \cdot (T_{mlb} + T_{put}) \quad 4$$

where W^t_{avr} is the average load in threads, L is the number of times the slowest processor has to migrate data, and N_t is the maximum number of threads. Since we adjust (reduce) the size of the threads in order to get better load resolution (unit-wise, where a unit can be an element for FEM or a grid point for FD), we can say that after a small number, M, of iterations that depends on the size of the thread we can have $W^t_{avr} \simeq W_{avr}$, which is very close to perfect load balance, unit-wise. For example, for threads with 100's of units (elements of grid points), by reducing each time the thread size, $|t|$, by half, we can achieve perfect balance in fewer than ten iterations. Also, let $\delta_i = W^t_{avr} - W_{avr}$ for $i < M$ and $C_0 = \sum_{i=1}^{M} \delta_i$ be a small constant. Then (4) can be written as :

$$T_{mt} = N \cdot W_{avr} + N \cdot N_t \cdot T_{ctxt} + L \cdot (T_{mlb} + T_{put}) + C_0. \quad 5$$

From equations (3) and (5) and the fact that $W^K_{max} \geq W_{avr} + \alpha \cdot t_{unit}$ (usually $\alpha >> 1$ [1]), we see that a multithreaded load-sharing algorithm can be more efficient than any single-threaded incremental algorithm if:

$$N_t \leq \frac{N \cdot (W^K_{max} - W_{avr}) + K \cdot (T_{slb} + T_{migr}) + C_2 - L \cdot (T_{mlb} + T_{put})}{N \cdot T_{ctxt}} \quad 6$$

For light-weight threads, T_{ctxt} is of an order of tens of micro-seconds as well as T_{mlb} and T_{put} [33], therefore $N \cdot T_{ctxt}$ and $L \cdot (T_{mlb} + T_{put})$ are very small numbers compared to $K \cdot (T_{slb} + T_{migr})$ and $C_2 = (C_1 - C_0)$ [9] [1] that are in the order of seconds. Therefore, theoretically, a light-weight multithread system with a reasonably large number of threads N_t is capable of improving the performance of parallel adaptive PDE methods even further. A careful and very efficient implementation of such a model will be able to realize the above expectations. Moreover, for

[9] $C_0 \leq C_1$, since M usually is of the order of 10 ($log_2 100$), while K can be from a few tens to a few hundreds of iterations [1] and δ_i decreases each time by half, while Δ_i can be between a few units to tens of units.

high-order schemes or problems with many degrees of freedom per grid point (4.6) is valid without any questions, since $W^K_{max} - W_{avr}$ can be in the order of minutes. Also, this is true whenever very expensive load-balancing algorithms (with T_{slb} sometimes in the order of hours) are used.

6 Discussion - Conclusions

Existing load-balancing algorithms require that all processors enter the balancing phase at the same time—guaranteed by global synchronization barriers. This requirement leads to: (1) the under-utilization of resources such as CPU and network because many processors may wait idle until the overloaded or slow processors reach the global barrier, and (2) the intensification of problems like network contention due to exclusive use either of the network (data-migration) or of the CPU (decision-making). Concurrent execution of tasks required for load-balancing with tasks required for the actual computation is the key ingredient for developing efficient load-balancing algorithms, and thus high-performance adaptive PDE solvers on multicomputers. Here threads are used as a mechanism to explore concurrency in the processor level in order to tolerate memory latency and mask synchronization costs inherent to traditional load-balancing methods. It is important that threads tolerate memory and scheduling latencies without sacrificing program simplicity and portability.

Our preliminary experimental data using CHANT [38] indicates that up to 16 threads the overhead introduced is very small (see Figure 8–left). Moreover, applications with a large number of coarse-grain threads (up to a point) can minimize cache misses and improve performance (of course the same performance can be achieved by the re-structuring of single-threaded programs —error prune process). Also, in [54] preliminary data indicates that with up to seven threads one can overlap computation with communication and improve processor and network utilization.

Finally, further research is required in three directions: (1) performance evaluation and calibration of the model using real applications, (2) generalization to handle a larger spectrum of applications (e.g., *hp*-refinement methods, AMR algorithm), and (3) a transformation mechanism (re-usability) for converting the off-the-shelf vast scientific computing libraries for PDEs from single-threaded to multithreaded programming paradigms.

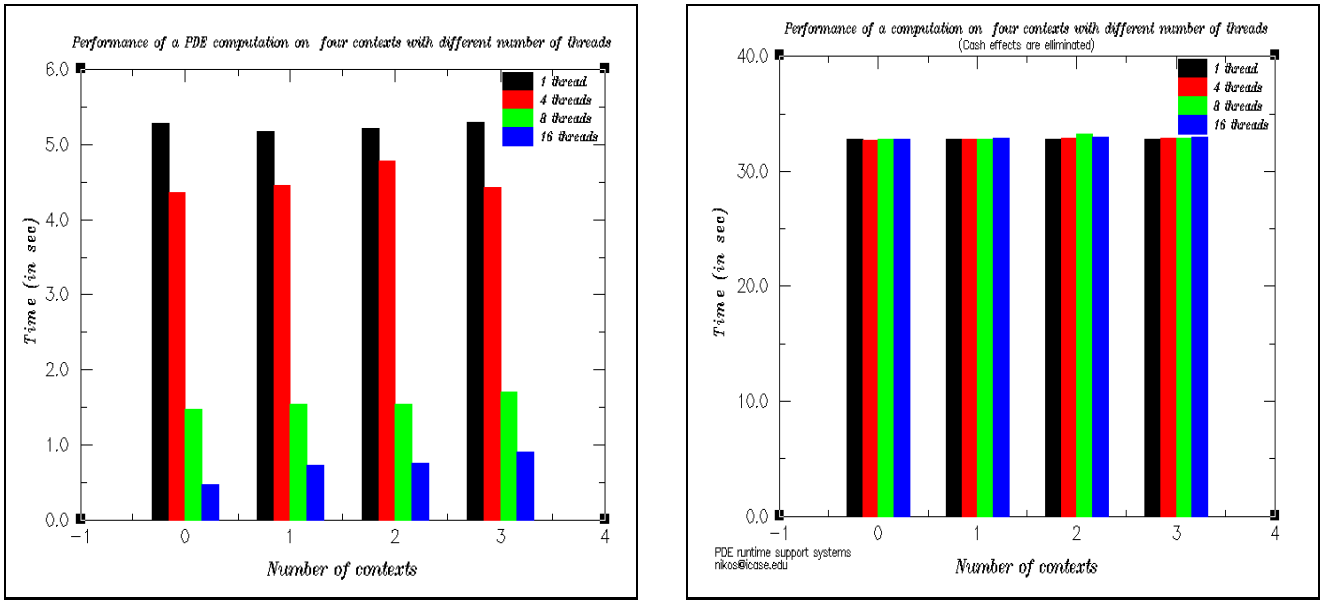

Figure 8: Left) 5-point stencil computation on SPARC-ELC. As the number of threads increase from 1 to 16 the performance of the computation improves due to less cache misses. Right) The same computation but involving only 5 variables —to eliminate cache effects. The increase of number of threads from 1 to 16 has little effect on the overall execution time.

Acknowledgements

I would like to thank Mike del Rosario, Matthew Haines, Thomas Fahringer, Piyush Mehrotra Geoffrey Fox, John Rice, David Keyes and Janusz Niemic for interesting and very helpful discussions on threads and dynamic load-balancing of adaptive computations.

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