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Baby Universes in $4d$ Dynamical Triangulation

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Abstract

We measure numerically the distribution of baby universes in the crumpled phase of the dynamical triangulation model of $4d$ quantum gravity. The relevance of the results to the issue of an exponential bound is discussed. The data are consistent with the existence of such a bound.

One of the more promising approaches to understanding the nature of four-dimensional quantum gravity has arisen through models based on summing classes of simplicial manifolds - the dynamical triangulation (DT) models [1, 2]. The manifold is approximated by a set of equilateral simplices whose edge lengths are taken to constitute an invariant cutoff. Quantum fluctuations of the geometry are incorporated by constructing a partition function which sums over all possible ways of assembling these simplices into a piecewise linear manifold

$$Z(\kappa_0, \kappa_4) = \sum_{T(S^4)} e^{\kappa_0 N_0 - \kappa_4 N_4} . \quad (1)$$

Here the class of triangulations has been restricted to that of spherical topology. The coupling κ_4 constitutes a bare cosmological constant conjugate to the total number of four-simplices (volume) N_4 . Similarly, κ_0 plays the role of an inverse Newton constant coupled to the total number of zero-simplices (nodes) N_0 in the triangulation.

Assuming that we wish to remove the edge length cutoff it is necessary to find points in the parameter space of the model at which the mean volume $\langle N_4 \rangle$ diverges. To see how this may happen consider expanding the *grand canonical* partition function Eq. (1) as a power series in $e^{-\kappa_4}$,

$$Z(\kappa_0, \kappa_4) = \sum_{N_4} \Omega(\kappa_0, N_4) e^{-\kappa_4 N_4} . \quad (2)$$

The coefficients in this expansion are the microcanonical partition functions for the system at fixed volume N_4 . It is these quantities which are estimated in Monte Carlo simulations. In two dimensions it is known rigorously that the analogous coefficients behave as $\Omega(N_2) \sim e^{\kappa_2^c N_2}$ - that is there is an exponential bound on the number of triangulations composed of N_2 triangles provided we restrict the global topology sufficiently¹. The existence of this bound ensures that the expansion has a finite radius of convergence determined by the critical coupling κ_2^c . The mean volume can then be shown to diverge in power-like fashion as this critical coupling is approached. This is the basis for taking the continuum limit.

In dimensions greater than two the volume dependence of $\Omega(\kappa_0, N_4)$ even when restricted to the four sphere is, in principle, unknown. In a previous paper we pointed out that the behaviour for small volume is consistent with a super-exponential growth $\Omega(N_4) \sim e^{\beta N_4 \log N_4}$ [3]. This would, at least naively, render a continuum limit impossible. Since then two other groups have examined the issue on larger lattices and claim strong evidence for an exponential bound [4, 5]. In light of this we have both extended our calculations to larger volumes and in addition looked at alternative quantities such as the distribution of baby universes. The latter is very sensitive to the volume dependence of $\Omega(\kappa_0, N_4)$ and might thus be useful in resolving this issue.

¹In two dimensions this restriction amounts to fixing the genus of the surface.

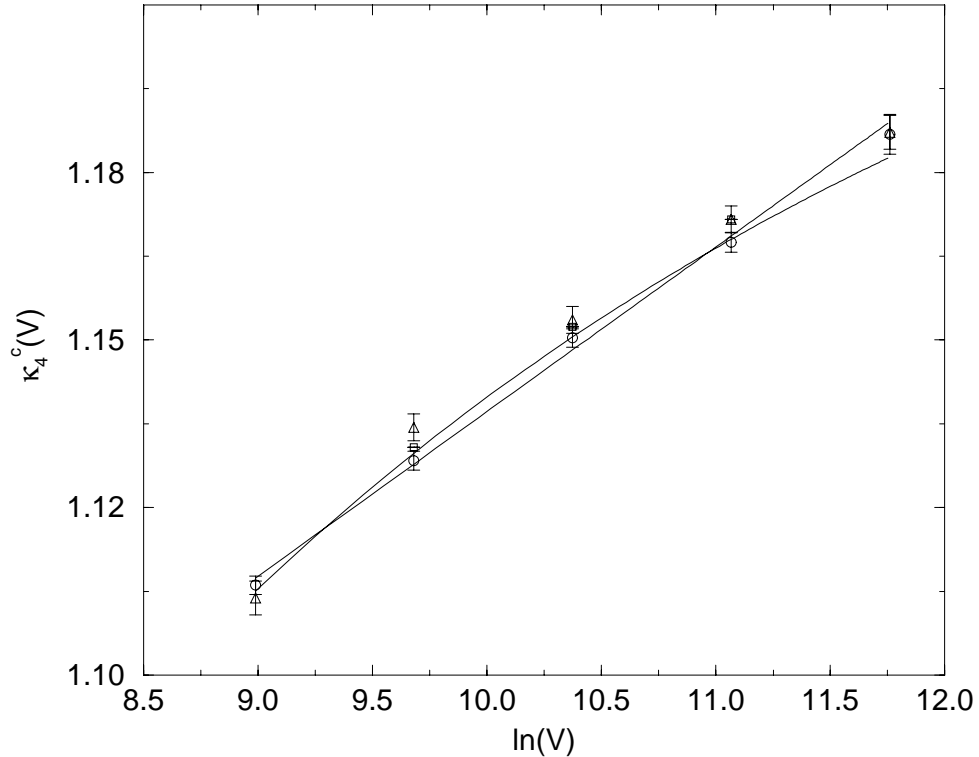


Figure 1: The critical coupling $\kappa_4^c(V)$ together with fits assuming power-law convergence (curve) and super-exponential growth (straight line).

The usual way in which an exponential bound is observed is by looking at the volume dependence of the quantity $\kappa_4^c(\kappa_0, V) = \frac{1}{V} \log \Omega(V, \kappa_0)$ which is a numerical estimate for the effective critical cosmological coupling κ_4^c at volume V . In Fig. 1 we plot it as a function of the logarithm of the volume to expose any logarithmically divergent component to the critical coupling. We show both our data (circles) together with the data published in [4, 5]. Clearly, these different simulations are in agreement within statistical errors. We then fit our data from volumes $V = 8000 - 128000$ using two different functional forms. The straight line represents a least square fit to a super-exponential form

$$\kappa_4^c(V) = \alpha + \beta \log V , \quad (3)$$

Fit	$\alpha (\alpha')$	$\beta (\beta')$	$\chi^2/d.o.f$
Power $\gamma = 0.25$	1.242(3)	-1.23(4)	2.4/3
Power $\gamma = 0.1$	1.389(8)	-0.68(2)	0.8/3
Log	0.894(8)	0.025(1)	3.1/3

Table 1: The optimal fit parameters as the data for $\kappa_4^c(V)$ is fitted to either super-exponential behaviour or a weak power-law convergence.

while the curve corresponds to a weak power-law convergence to an exponential bound of the form

$$\kappa_4^c(V) = \alpha' + \frac{\beta'}{V^\gamma} . \quad (4)$$

The fit parameters and quality of the fits are shown in Table 1. Since there is so little data we have chosen to do the power fit with two different *fixed* powers $\gamma = 0.25$ and $\gamma = 0.1$. Arguments for the former choice are made in [4] and it corresponds to the curve plotted in Fig. 1. It is clear that both types of fit can equally well describe the data. The quality of the fit with $\gamma = 0.1$ appears somewhat superior but since the log fit has χ^2 of order one this should not be taken as significant. In [5] a $\gamma = 0.25$ fit over the same volume range was claimed to be substantially better than the logarithm. Our data do not seem to support this and we interpret this as simply pointing to the delicacy of deciding between similar fits with rather limited data. It is quite possible that many runs at intermediate volumes would be useful to resolve this issue.

Thus while we see that an exponential bound is certainly consistent with the existing numerical data at large volumes it is not *strongly* preferred over the logarithmic divergence. In light of this we have turned to an analysis of other quantities to try to settle the issue. The distribution of baby universes is one such observable [6]. A baby universe is defined as a section of a d -dimensional triangulation connected to the bulk only through a so-called minimal neck which consists of $d + 1$ $(d - 1)$ -simplices or faces constituting a boundary of a simplex *not* already present in the triangulation. In four dimensions this is a set of five tetrahedral faces which make up the surface of a new simplex and divide the triangulation into two pieces. The volume of the baby universe is defined to be the number of simplices in the smaller piece.

The distribution of these baby universes can be computed by considering the number of ways a volume V triangulation can be built from a baby of volume B and a mother of size $V - B$ by attaching the baby to the mother at some point. This gluing operation is effected by identifying one simplex in the baby with another on the mother. Thus the distribution takes the form

$$P(B) = \frac{(V - B) \Omega(\kappa_0, V - B) B \Omega(\kappa_0, B)}{\Omega(\kappa_0, V)}. \quad (5)$$

Strictly speaking the factors $\Omega(\kappa_0, B)$ should be replaced with one point functions but we shall ignore this unimportant technicality here. The important thing to notice is that any exponential factor in Ω cancels out in this formula and $P(B)$ only depends on *sub-leading* corrections - that is it is maximally sensitive to the finite volume corrections to coefficients $\Omega(\kappa_0, V)$.

With this in mind we have measured the distribution $P(B)$ numerically in the crumpled phase of the model when $\kappa_0 = 0$. The true volume of the triangulation space is most easily estimated here since all triangulations contribute with equal weight to the partition sum. Indeed we do not believe it is safe to try to estimate the behaviour of $\Omega(\kappa_0, V)$ from simulations at large κ_0 . At such node couplings the dominant triangulations correspond to branched polymers whose mean node number varies linearly with volume. Such configurations are known to possess an exponential bound. The crumpled configurations which predominate at small κ_0 in contrast have mean node numbers scaling as some fractional power of the volume. At large κ_0 these latter configurations will receive large (as $V \rightarrow \infty$) exponential suppression relative to the branched polymers from the node term in the action.

We have simulated the model at four different volumes; 500, 1000, 4000 and 8000 simplices, using runs of length 10 million sweeps². We will see that the measured distribution falls off exponentially fast which necessitated such high statistics runs. This precluded the use of significantly larger lattice volumes in this study. Using our previous parameterizations of the finite volume corrections to $\Omega(V)$ we have attempted to fit the data with functional forms corresponding to either logarithmic divergence or weak power law convergence

$$\log P(B) = a + \beta((B + \delta) \log(B + \delta) + (V - B + \delta) \log(V - B + \delta)), \quad (6)$$

$$\log P(B) = a' + \beta' \left((B + \delta)^{1-\gamma} + (V - B + \delta)^{1-\gamma} \right). \quad (7)$$

The constant δ is inserted as a phenomenological parameter to reflect sub-leading finite size corrections and a and a' reflect an ambiguity in overall normalization. In practice we have removed the largest contribution to the latter by dividing the measured number of baby universes by the volume V .

Figure 2 shows the distributions together with a series of curves resulting from least-square fits assuming the logarithmic scenario Eq. (6). The fit to the largest volume yields $a = -2.92(3)$, $\beta = +0.056(1)$ and $\delta = -7(1)$ with $\chi^2 = 9.6/6$ (per d.o.f.). Fits to the other volumes give consistent results. Notice that we have fitted baby universes with size $B = 4(n + 1)$ only (n integer). Baby universes of size $B = 4(n - 1)$ lie on a curve which while yielding consistent fits for β is shifted by a constant with respect to the first. This effect has been observed before [7] and

²One sweep corresponds to V attempted elementary local moves.

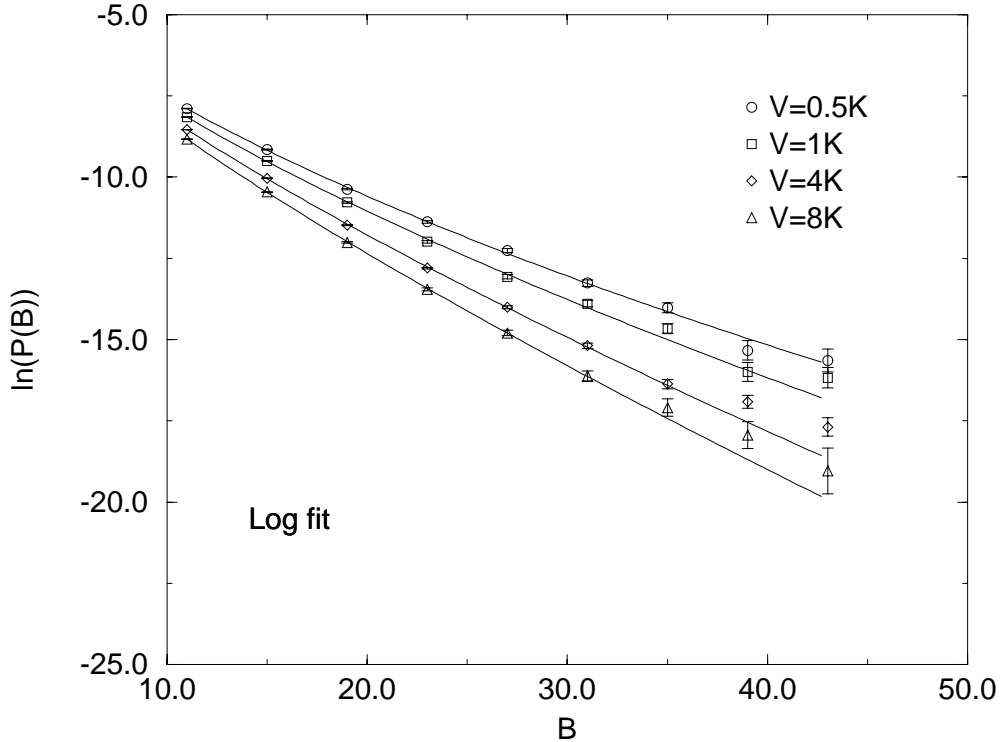


Figure 2: $\log P(B)$ vs B with a logarithmic fit.

is presumably the result of finite size effects. We fit only for $B > 10$ and truncate due to poor statistics at $B > 50$.

Figure 3 shows the same data now fitted according to the power scenario Eq. (7). The best fit in this case yields $a' = -0.2(15)$, $\beta' = -1.38(5)$ and $\delta = 3(2)$ with $\chi^2 = 6.3/6$ assuming $\gamma = 0.25$ as before. At face value then it remains hard to differentiate between the two situations. However, notice that the extracted value of $\beta = 0.056(1)$ from the log fit is more than twice its estimated value from the fits for the effective critical coupling $\beta = 0.025(1)$ (Table 1). In contrast the estimate for $\beta' = -1.38(5)$ from the power fit is quite close to its value estimated earlier $\beta' = -1.23(4)$. The relative proximity of the two estimates is particularly impressive considering that one is derived from the behaviour of baby universes with size less than 8000 simplices while the other is extracted from the critical coupling at volumes much greater than 8000. Furthermore, it is clear that the power fit would

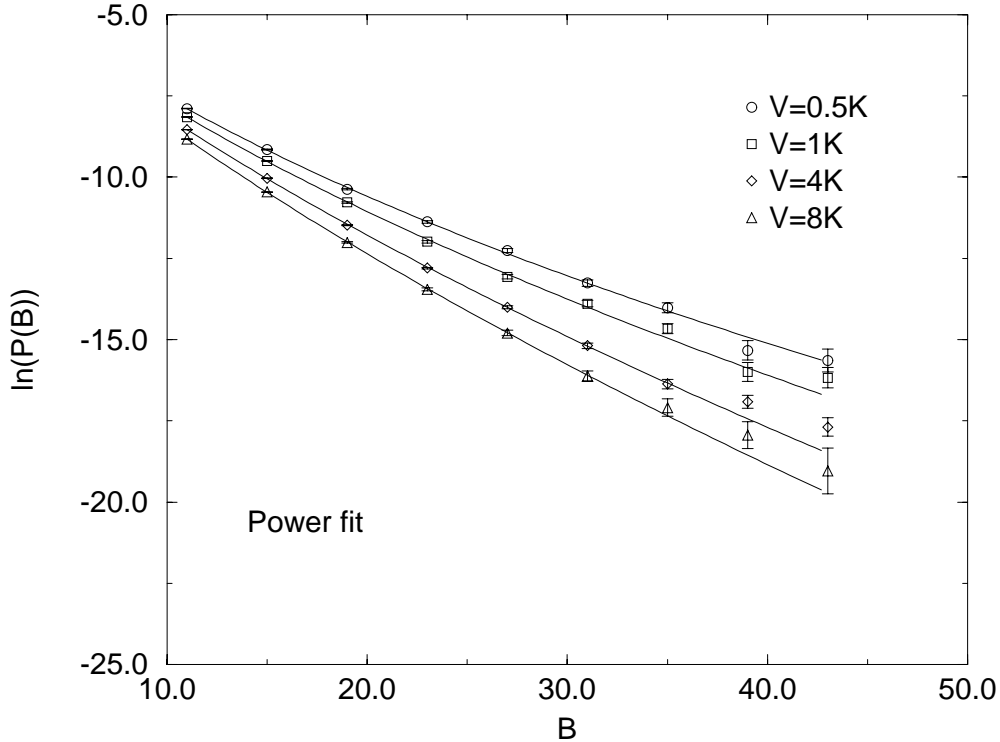


Figure 3: $\log P(B)$ vs B with a power fit assuming $\gamma = 0.25$.

still hold good if we set $\delta = a' = 0$ so that such a fit (with a truly minimal number of parameters) would do much better than the logarithm.

Additional information can be obtained by looking at the mean number of nodes per unit volume. It is easy to see that this quantity is related to the critical coupling through (see eg [8])

$$\left\langle \frac{N_0}{V} \right\rangle = \frac{\partial \kappa_4^c(V, \kappa_0)}{\partial \kappa_0}. \quad (8)$$

Thus finite volume corrections to the effective critical coupling result in similar finite volume corrections to $\langle N_0/V \rangle$. In Fig. 4 we show this quantity on a log-log scale together with a least-square power fit. While the χ^2 of such a fit is terrible, showing that such a simple parameterization is insufficient to describe the data in detail, the fit shows that a small power-law correction is again rather well able to account for

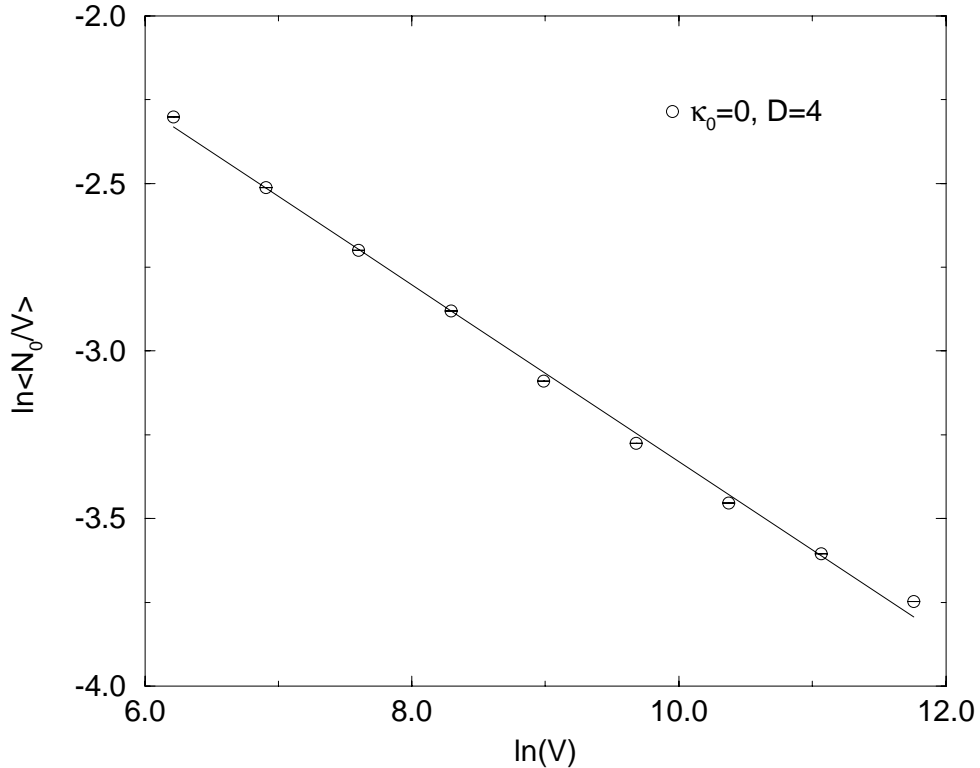


Figure 4: Mean node number per unit volume with fit assuming a weak power convergence.

the overall structure of the finite volume corrections³. Notice that any coefficient of a logarithmic piece in $\kappa_4^c(V, \kappa_0)$ will not contribute since it cannot depend on κ_0 .

In conclusion, we have presented numerical results which, although not definitive, are very consistent with the existence of an exponential bound in the dynamical triangulation model of $4d$ quantum gravity. The evidence for this comes both from fits to the volume dependence of the critical coupling, an analysis of the baby universe distribution in the crumpled phase and the scaling of the mean node number. Although individually these quantities are not very conclusive, it is remarkable how consistent results are obtained if we assume a weak power convergence. Clearly, it is important to strengthen these conclusions both by simulating intermediate lat-

³The fit yields $\gamma = 0.26$.

tice volumes and perhaps via a high statistics simulation at say volume $V = 16000$ directed at probing further into the tail of the baby universe distribution.

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