Apparent Horizons: What Are They and Why Do We Need Them?

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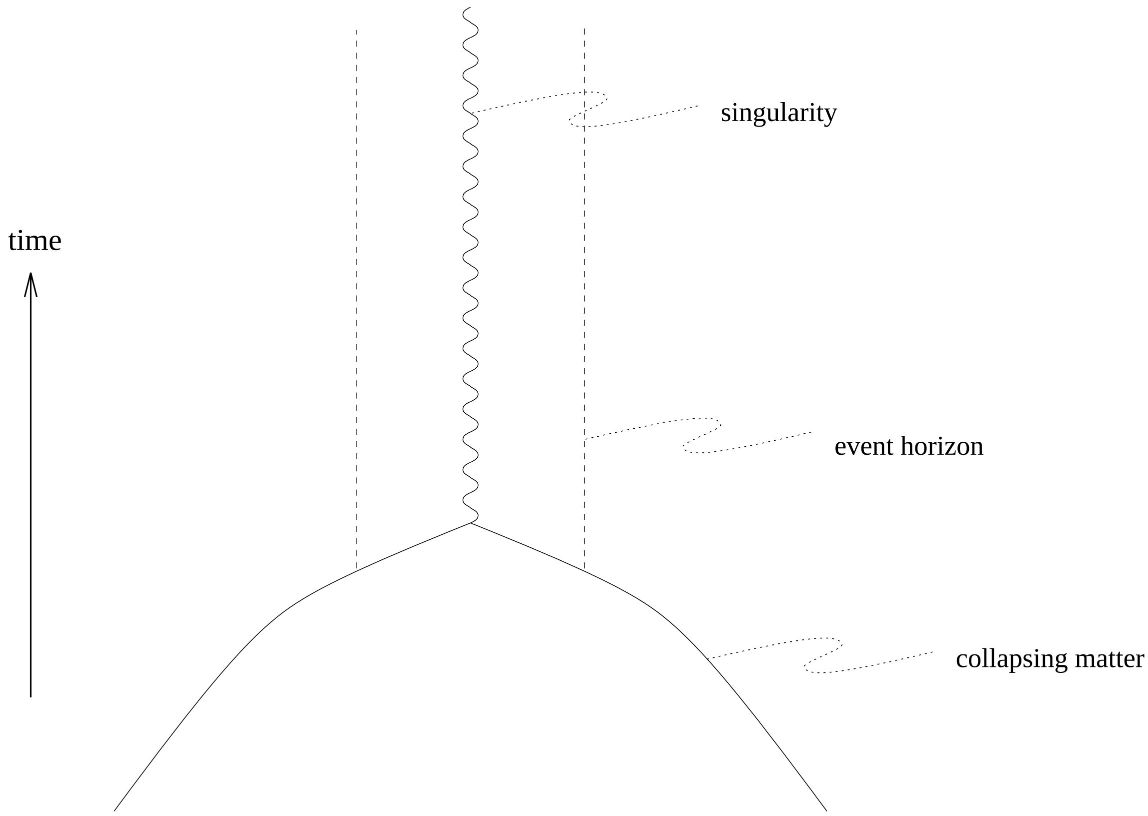
Introduction

A seemingly generic property of General Relativity is the formation of spacetime singularities via. gravitational collapse. A spacetime singularity is a region of spacetime that gives infinite answers to physically reasonable questions concerning the geometry in that region. It is in this region that the classical theory of gravity breaks down.

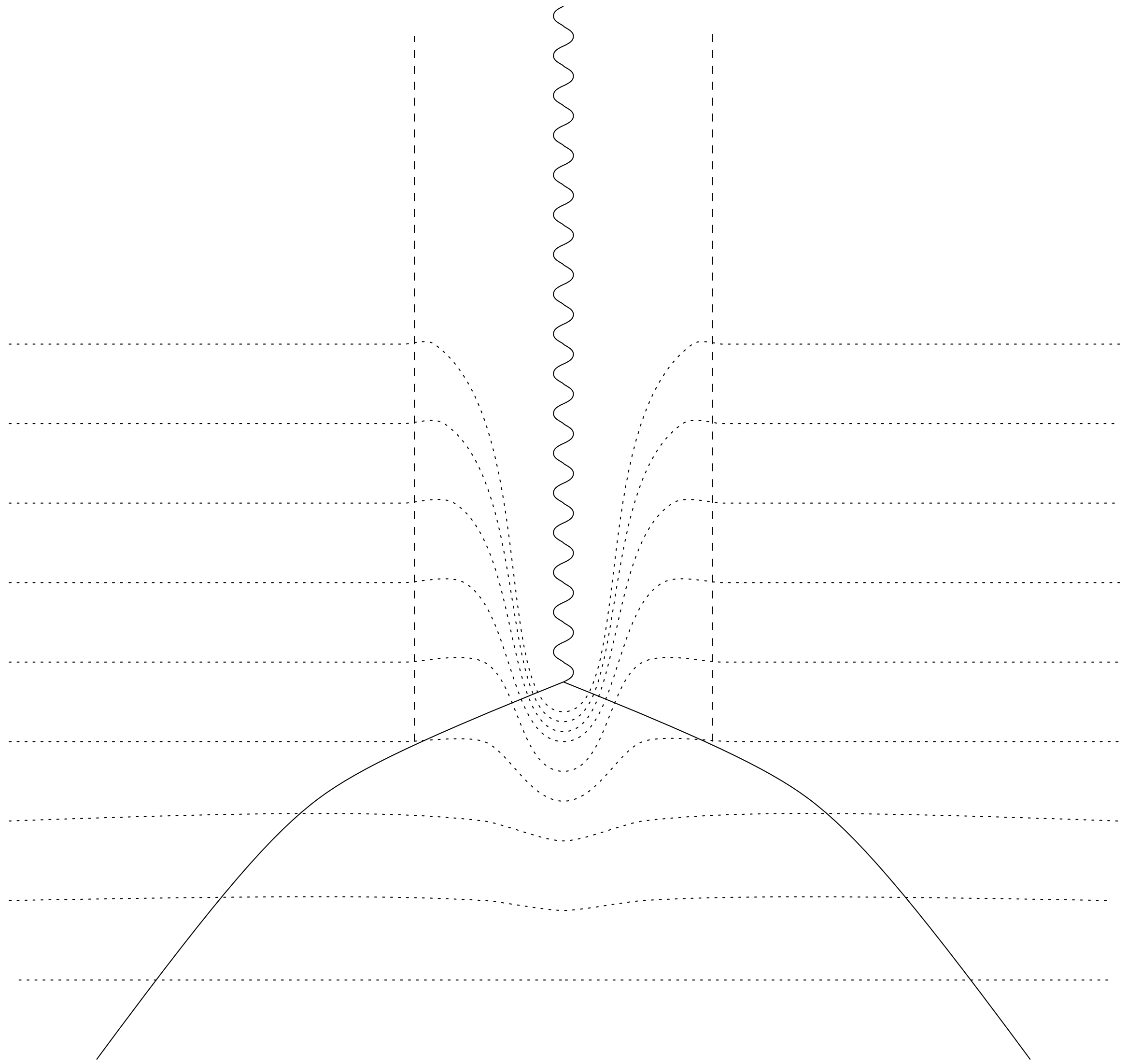
Needless to say, singularity formation wreaks havoc in numerical relativity, especially if the goal is to simulate the spiraling and coalescence of a binary black hole system. If one is using vacuum General Relativity to evolve the black holes (instead of introducing matter), then we *know* that there are singularities hiding inside. The author of this note is aware of only three general ways of dealing with singularity formation in numerical relativity.

1 Singularity Avoiding Slicing

Imagine a physical system where a shell of, say, gravitational waves is collapsing to form a singularity. A spacetime diagram might look like this (two of the three spatial dimensions have been supressed):



Now imagine that we are using numerical relativity via General Relativity's initial value formulation to evolve the spacetime, starting at the $t = t_0$ slice. How do we hope to avoid running into the singularity? Remember that in the process of evolving data in General Relativity, there are four functions, called the lapse (1 function) and shift (3 functions), that are freely specifiable during evolution. These functions can be chosen in a way to evolve "around" the singularity, thus "avoiding" it. This "singularity avoiding slicing" can be viewed as follows:



There are many ways of implementing singularity avoiding slicing. One way that is independent of the coordinates chosen on the spatial slice is to hold the trace of the extrinsic curvature, $K = K_{ij}h^{ij}$, constant on each slice. This choice (called maximal slicing) works very well in avoiding singularities. Unfortunately, its implementation involves solving a global elliptic-type equation at each evolution step. Other singularity avoiding gauge choices that involve algebraic functions of the lapse and shift and other (coordinate dependent) metric variables are also currently being studied by many numerical relativity groups.

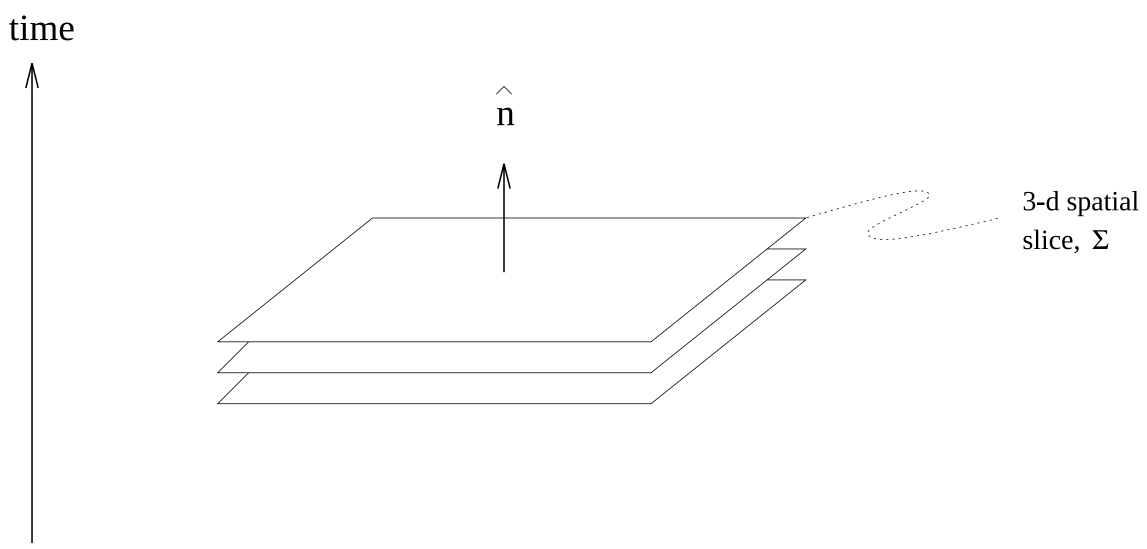
2 Apparent Horizons

In the past few years, it has been realized that the solution described above may not be the best way to avoid singularities in numerical relativity. Although the solution does work well in avoiding singularities, there are problems associated with simply approaching singularities. One problem is that gradients of the metric functions grow without bound as one approaches the singularity. Unless one greatly refines the grid in the region where a singularity is forming, the code will invariably be unstable. However, why should one spend alot of computational effort refining and evolving a region of spacetime that does not affect what we want to calculate (outgoing waveforms)? After all, what goes on inside the event horizon can have no effect on the region of spacetime outside the event horizon.

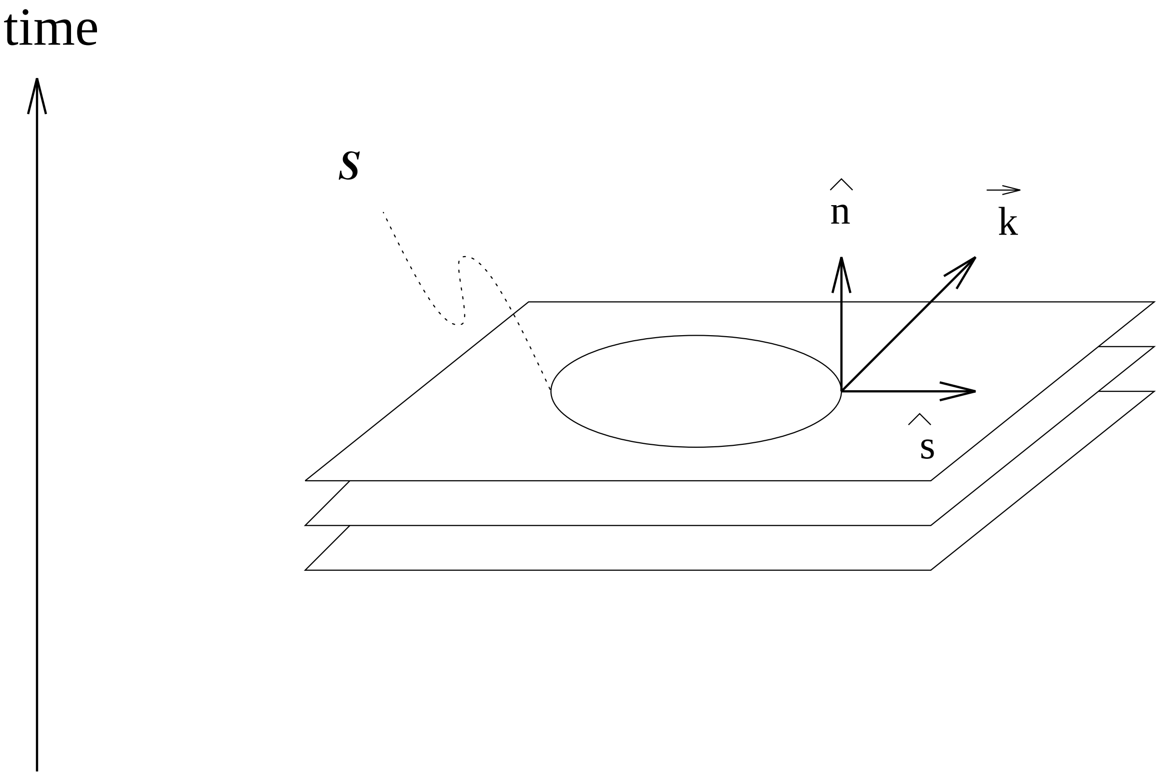
This second solution involves cutting out the offending region of spacetime, thus creating an artificial "inner boundary." The next logical question is: how much do we cut out? Clearly, the region we cut out must be inside the event horizon, or else our boundary conditions would affect the outgoing gravitational radiation. One might think that all one has to do would be to find the event horizon, and put the inner boundary there (or perhaps leave a buffer zone just a few grid points inside the event horizon). The problem with doing this, is that an event horizon is a *global* aspect of a spacetime: one can only look for an event horizon after the entire spacetime has been evolved!

What is needed is some local condition that will tell us whether or not we are inside an event horizon. This is what the apparent horizons do for us. The strategy is as follows: at each spatial slice look for an apparent horizon. If one is found, then we know that a singularity is forming, so we "cut out" the space inside the apparent horizon and treat the horizon as an inner boundary.

To understand (geometrically) what apparent horizons are, imagine we had a spatial slice that was our latest evolution step:



(In the diagram, one spatial dimension has been supressed). At every point in the spatial surface, Σ, there is a timelike unit vector $\hat{n}$ that is normal to the spatial surface. Now, imagine a closed 2-d surface embedded in the 3-d spatial slice Σ:



The spacelike unit normal to the 2-surface, $\mathcal{S}$, is called $\hat{s}$. One can now construct an outgoing null vector, $\vec{k}$, by adding the unit vectors $\hat{n}$ and $\hat{s}$:

$$\vec{k} = \hat{n} + \hat{s}$$

The closed 2-surface, $\mathcal{S}$, is an apparent horizon if the divergence of $\vec{k}$ is 0 everywhere on $\mathcal{S}$:

$$\vec{\nabla} \cdot \vec{k} = 0$$

In words, this condition is demanding that outgoing light rays do not diverge. There exist theorems in General Relativity that state (under general conditions) that apparent horizons are always "inside" or "on" event horizons. Therefore, one can use the apparent horzons as inner boundaries (in practice, a buffer zone is left inside the apparent horizon).

Some problems with using apparent horizons to avoid singularities are as follows:

- Tracking apparent horizons can be difficult: they can appear discontinuously, and their position can move at speeds faster than light.
- It might be the case that in some situations, using apparent horizons as inner boundaries may not be cutting out enough of the region inside the event horizon to make it worth the trouble of introducing an inner boundary
- It is not clear what boundary conditions should be placed at the inner boundary in order to insure stable evolution.

3 Introducing Matter

Another idea for avoiding singularities in vacuum spacetimes is to introduce matter inside the event horizon, thus halting the formation of a spacetime singularity. The author is unaware of any attempts at singularity avoidance using this method.