Finite Element Modeling of Multibody Contact and Its Application to Active Faults

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Abstract

Earthquakes have been recognized as resulting from a stick-slip frictional instability along the faults between deformable rocks. An arbitrarily shaped contact element strategy, named as node-to-point contact element strategy, is proposed and applied with the static-explicit characters to handle the friction contact between deformable bodies with stick and finite frictional slip and extended here to simulate the active faults in the crust with a more general nonlinear friction law. Also introduced is an efficient contact search algorithm for contact problems among multiple small and finite deformation bodies. Moreover, the efficiency of the parallel sparse solver for the nonlinear friction contact problem is investigated. Finally, a model for the plate movement in the Northeast zone of Japan under gravitation is taken as an example to be analyzed with different friction behaviors.

Key words: Multibody contact, finite element method, nonlinear frictional contact, parallel sparse solver, active faults, earthquake.

1. INTRODUCTION

Japan is located in one of the world's most earthquake-prone zones and has suffered the loss of many valuable human lives in the earthquake history. To further investigate the occurrence of earthquake and to predict it in the future, as a part of the Earth Simulator Project of Japan, a finite element software system for large-scale computation of the earthquake process is being developed in RIKEN, including tectonic CAD/Database and mesh generation, static analysis and dynamic analysis. Only the static analysis is introduced here, which aims to calculate the accumulation of stress around active faults induced by a subduction of plates in a long time span.

The earthquakes can be regarded as a contact between deformable rocks with a special friction law along the active faults (e.g. Brace, 1966), it includes three kinds of main nonlinearities: the material, the geometrical and the contact along the faults. Contact problems are characterized by contact constraints, which are imposed on contacting boundaries. In the current FEM analysis, both the dynamic-explicit FEM and the static-implicit FEM are available corresponding to the different problems. However, convergence is still a problem in implicit analysis, especially when three-dimensional large deformation contact problems with sliding friction are encountered. This is partly due to the iteration solution method and its corresponding serious requirement, such as no drastic change of the contact state and the deformation state, more smooth contact surface definition (e.g. Nagtegaal, 1991; Ling, 1997). Although many efforts have been made as above, there still exist problems to be overcome (e.g. Parisch, 1997; Zhong, 1993). Thus dynamic-explicit FEM seems to be used increasely, even for problems, which are characterized as static or quasi-static ones, but it is also well known that it is quite time consuming and also difficult for dynamic-explicit FEM to predict the stress distribution of a quasi-static problem with a high accuracy (e.g. Bathe, 1996). Thus, an arbitrarily shaped isoparametric contact element strategy with the static-explicit integration algorithm, named as the node-to-point contact element strategy, was proposed by the authors to handle the static or quasi-static friction contact between deformable bodies with stick and finite frictional slip (Xing, 1998a, 1998b, 2000). Moreover, the friction behaviour in the practical engineering and the active faults is quite complicated, it depends on the slip velocity, the state, the contact pressure, the material property and so on. This paper will focus on how to extend our algorithm to simulate it. In addition, to meet the practical requirement of a stable and large-scale calculation, the parallel sparse solver is also investigated for the nonlinear friction contact problem and applied to simulate the active faults. Finally, a model for the plate movement in the Northeast zone of Japan under gravitation is taken as an example to be analyzed with different friction behaviors.

2. GENERAL CONSIDERATION AND NOTATION

Consider two bodies B^1 and B^2 with surfaces S^1 and S^2, respectively, to contact on an interface S_c, given by $S_c = S^1 \cap S^2$. The size of S_c can vary during the interaction between the two bodies. The part of S^α that belonged to S_c is designated S_c^α, that is $S_c^\alpha = S_c \cap S^\alpha$, and assume $S_c^\alpha = S_c$, where superscript $\alpha = 1, 2$ refers to body B^α (as shown in Fig. 1). Let the union of the two bodies be denoted by B: $B = B^1 \cup B^2$, $\boldsymbol{n}$ be the unit normal vector of the contact surface, $\boldsymbol{s}$ be the unit tangential vector along the relative sliding direction on the contact surface, and $\boldsymbol{t} = \boldsymbol{n} \times \boldsymbol{s}$. Thus $\boldsymbol{s}$ and $\boldsymbol{t}$ form a tangential plane to the contact surface.

The so-called slave-master concept is widely used for the implementation of contact analysis. Assume that one of the bodies, B^1, is the slave and the material points on its contact surface are called slave nodes; and the other body B^2 is the master and the material points on its contact surface are called master nodes. Contact (master) segments that span master nodes cover the contact surface of the master body. Therefore, the above problem can be regarded as a contact between a slave node and a point on a master segment (Here, this point may locate at a node, an edge or an interior surface of a master segment, but no special attention is necessary when the local contact searching algorithm in section 5 is applied). And a slave node makes contact with only one point on the master segments, but one master segment can make contact with one or more slave nodes at the same time. This is the basic assumption of the node-to-point contact element strategy (Xing, 1998a, 1998b, 2000).

Based on the above assumption, the normal vector and the tangential vector are defined on the contact surface S_c^α of each body as follows

$$\boldsymbol{n} = \boldsymbol{n}^2 = -\boldsymbol{n}^1 \quad and \quad \boldsymbol{s} = \boldsymbol{s}^2 = -\boldsymbol{s}^1. \tag{1}$$

Let $\boldsymbol{f}^\alpha$ be the traction vector acting on the contact surface S_c^α, then normal component $\boldsymbol{f}_n^\alpha$ and the tangential component $\boldsymbol{f}_s^\alpha$ are given by

$$f_n^a = \boldsymbol{f}^a \cdot \boldsymbol{n}^a, \; \boldsymbol{f}_n^a = f_n^a \, \boldsymbol{n}^a \quad \text{and} \quad \boldsymbol{f}_s^a = \boldsymbol{f}^a - f_n^a \, \boldsymbol{n}^a . \tag{2}$$

When contact occurs, the following conditions should be satisfied on the contact interface S_c for the unilateral contact:

1). The momentum has to be balanced,

$$\boldsymbol{f}^1 + \boldsymbol{f}^2 = 0 . \tag{3}$$

And let $\boldsymbol{f} = \boldsymbol{f}^1 = -\boldsymbol{f}^2$ in this paper.

2). No tensile traction can occur on the contact interface,

$$\boldsymbol{f}^a \cdot \boldsymbol{n}^a \le 0 . \tag{4}$$

3). The contact points move with the same displacement and velocity in the direction normal to the contact surface during contact, that is

$$\boldsymbol{u}^1 \cdot \boldsymbol{n}^1 = \boldsymbol{u}^2 \cdot \boldsymbol{n}^2 \quad \text{and} \quad \dot{\boldsymbol{u}}^1 \cdot \boldsymbol{n}^1 = \dot{\boldsymbol{u}}^2 \cdot \boldsymbol{n}^2 . \tag{5}$$

This is usually called as the impenetrability condition.

3. CONSTITUTIVE EQUATION FOR FRICTION CONTACT

3.1 Normal Contact Stress

We choose the penalty method to treat the normal constraints when contact occurs. When $g_n < 0$, the contact occurs. For a slave node *s*,

$$f_n = \boldsymbol{f} \cdot \boldsymbol{n} = E_n g_n \quad (\neq 0 \text{ only for } g_n < 0) \tag{6}$$

here E_n is the penalty parameter to penalize the penetration (gap) in the normal direction, and $g_n = \boldsymbol{n} \cdot (\boldsymbol{x}_s - \boldsymbol{x}_c)$, here $\boldsymbol{x}_s$ and $\boldsymbol{x}_c$ are the position coordinates of a slave node *s* and its corresponding contact point *c* (as shown in Fig. 2), respectively.

3.2 Friction Stress

Friction is by nature a path-dependent dissipative phenomenon that requires the integration of the constitutive relation. In this study, a standard Coulomb friction model, with an additional limit on the allowable shear stress, is applied in an analogous way to the flow plasticity rule. This situation is analogical to the change of state from elastic to plastic in the theory of plasticity. The analogy to plasticity can be founded in Michalowski & Mroz's work (Michalowski, 1978). The basic formulations are summarized below (*Note:* A variable with ~ on top stands for a relative component between slave and master bodies, and *l, m=1,2; i,j, k=1,3* in this paper if without the special notation.).

Based on experimental observations, an increment decomposition is assumed

$$\Delta \tilde{u}_m = \Delta \tilde{u}_m^e + \Delta \tilde{u}_m^p , \tag{7}$$

where $D\tilde{u}_m^e$ and $D\tilde{u}_m^p$ represent the sticking (reversible) and the sliding (irreversible) part of $D\tilde{u}_m$, respectively. In addition, the slip is governed by the yield condition

$$F = \sqrt{f_m f_m} - \overline{F}\,, \tag{8}$$

where $\overline{F}$, the critical frictional stress, has three choices: $\overline{F} = mf_n$, $\overline{F} = F_{limit}$ and $\overline{F} = min(mf_n, F_{limit})$; f_m ($m=1,2$) is the frictional stress component along the tangential direction m; F_{limit} is an allowable value of shear stress; m is the friction coefficient, it may depend on the normal contact pressure f_n, the equivalent slip velocity $\dot{\tilde{u}}_{eq}^{sl}$ and the state variable j, *i.e.* $m = m(f_n, \dot{\tilde{u}}_{eq}^{sl}, j)$.

If $F<0$, contact is in the sticking state and treated as a linear elasticity, *i.e.*

$$f_m = E_t \tilde{u}_m^e = E_t \sum D\tilde{u}_m^e\,, \tag{9}$$

where E_t is a constant in the tangential direction.

When $F=0$, the friction changes its character from sticking to sliding. If slip occurs, according to the analogy to plasticity as mentioned above, $D\tilde{u}_m^p$ can be described from the 'flow rule' as

$$D\tilde{u}_m^p = D\overline{u}^p \frac{\partial F}{\partial f_m}\,, \tag{10}$$

where $D\overline{u}^p$ is the 'equivalent relative slip increment', and $D\overline{u}^p > 0$.

Combining with Eq. (8), the above equation can be rewritten as

$$D\tilde{u}_m^p = D\overline{u}^p f_m / \overline{F}\,. \tag{11}$$

From Eqs. (7) and (9),

$$f_m = E_t(\tilde{u}_m - \tilde{u}_m^p) = f_m^e - E_t D\tilde{u}_m^p\,, \tag{12}$$

where $f_m^e = E_t(\tilde{u}_m - \tilde{u}_m^p\big|_0)$, and $\tilde{u}_m^p\big|_0$ is the value of $\tilde{u}_m^p$ at the beginning of this step.

From the last two equations,

$$f_m = h_m \overline{F} \quad \text{and} \quad h_m = f_m^e \Big/ \sqrt{f_l^e f_l^e}\,. \tag{13}$$

The linearized form of the Eq. (13) can be rewritten as

$$df_l = \frac{\overline{F}E_t}{\sqrt{(f_1^e)^2 + (f_2^e)^2}}(d_{lm} - h_l h_m)d\tilde{u}_m + h_l m\left(df_n + \frac{\partial m}{\partial f_n}df_n\right) + h_l f_n\left(\frac{\partial m}{\partial \dot{\tilde{u}}_{eq}^{sl}}d\dot{\tilde{u}}_{eq}^{sl} + \frac{\partial m}{\partial j}dj\right), \quad (\text{if } \overline{F} = mf_n)$$

$$df_l = \frac{\overline{F}E_t}{\sqrt{(f_1^e)^2 + (f_2^e)^2}}(d_{lm} - h_l h_m)d\tilde{u}_m \quad (\text{if } \overline{F} = F_{limit})\,. \tag{14}$$

In summary, from Eqs. (6), (9) and (14), the contact stress acting on a slave node can be described as (denote $\dot{f}_3 = \dot{f}_n$)

$$\dot{f}_i = G_{ij}\dot{\tilde{u}}_j + \dot{f}_{j\,i}\ ,\qquad (15)$$

where $\boldsymbol{G}$ is the frictional contact matrix; $\dot{f}_{j\,i}$ is from the contribution of the terms related with j , when it is not a function of $\dot{\tilde{\boldsymbol{u}}}$; If $d\,j$ only is the function of the unknown variable $d\,\dot{\tilde{\boldsymbol{u}}}$, $\dot{f}_{j\,i} = 0$, *i.e.* all its contribution can be included in $\boldsymbol{G}$ at current state.

4. FINITE ELEMENT FORMULATION

4.1 Variational Principle

The updated Lagrangian rate formulation is employed to describe the nonlinear problem. The rate type equilibrium equation and the boundary at the current configuration are equivalently expressed by a principle of virtual velocity of the form (Xing, 1998a, 1998b)

$$\int_V \left\{ (s_{ij}^{J} - 2s_{ik}D_{kj})dD_{ij} + s_{jk}L_{ik}dL_{ij} \right\} dV = \int_{S_G} \dot{F}_i dv_i dS + \int_{S_C^1} \dot{f}_i^{1} dv_i^{1} dS + \int_{S_C^2} \dot{f}_i^{2} dv_2^{2} dS\,, \qquad (16)$$

where V and S denote respectively the domain occupied by the total body B and its boundary at time t; S_G is a part of the boundary of S on which the rate of traction $\dot{F}_i$ is prescribed; $d\boldsymbol{v}$ is the virtual velocity field which satisfies the boundary $d\boldsymbol{v} = \boldsymbol{0}$ on the velocity boundary; $\boldsymbol{L}$ is the velocity gradient tensor, $\boldsymbol{L} = \P\boldsymbol{v}/\P\boldsymbol{x}$; $\boldsymbol{D}$ and $\boldsymbol{W}$ are the symmetric and antisymmetric parts of $\boldsymbol{L}$, respectively.

The small strain linear elasticity and large strain rate-independent work-hardening plasticity are assumed, from which the elasto-plastic tangent constitutive tensor C_{ijkl}^{ep} is derived

$$s_{ij}^{J} = C_{ijkl}^{ep}D_{kl} = C_{ijkl}^{ep}L_{kl}\,. \qquad (17)$$

Substitution of Eq.(17) into Eq.(16) reads to the final form of the virtual velocity principle

$$\int_V \Sigma_{ijkl}\, L_{kl}dL_{ij}\,dV = \int_{S_G} \dot{F}_i dv_i dS + \int_{S_C} \dot{f}_i\, d\dot{\tilde{u}}_i dS\,, \qquad (18)$$

where $\mathring{a}_{ijkl} = C_{ijkl}^{ep} + (s_{jl}d_{ik} - s_{ik}d_{jl} - s_{il}d_{jk} - s_{jk}d_{il})/2$.

4.2 Contact Stress on A Slave Node

Assume that contact segment surfaces are described by $\boldsymbol{x} = \boldsymbol{x}(x_m)$, a slave node s has made contact with a master segment on point c (as shown in Fig. 2), and the contact stress acting on it in Eq.(18) can be described in the local contact coordinate system as follows

$$\dot{\boldsymbol{f}} = \dot{\hat{f}}_i\,\hat{\boldsymbol{e}}_i + \hat{f}_i\,\dot{\hat{\boldsymbol{e}}}_i\,. \qquad (19)$$

Here $\hat{\boldsymbol{e}}_i$, the base vector on the contact segment, is specified by

$$\hat{\boldsymbol{e}}_i = \hat{\boldsymbol{e}}_i(x,h) = \hat{\boldsymbol{e}}_i(x_m) \quad \text{and} \quad \dot{\hat{\boldsymbol{e}}}_i = \frac{\P\,\hat{\boldsymbol{e}}_i}{\P x_m}\dot{x}_m = E_{ijm}\,\hat{\boldsymbol{e}}_j\,\dot{x}_m\,, \qquad (20)$$

in which $E_{ijm} = \hat{\boldsymbol{e}}_{i,m}\cdot\hat{\boldsymbol{e}}_j$.

From Eqs. (19) and (20),

$$\dot{\boldsymbol{f}} = \dot{\hat{f}}_i\, \hat{\boldsymbol{e}}_i + \hat{f}_i E_{ijm}\, \hat{\boldsymbol{e}}_j\, \dot{x}_m . \tag{21}$$

Assuming that the tangential surface is spanned by the tangents to the parameter lines (as shown in Fig.2),

$$\hat{\boldsymbol{e}}_m = \frac{\partial \boldsymbol{x}}{\partial x_m}, \tag{22}$$

and the associated unit normal is

$$\hat{\boldsymbol{e}}_3 = \boldsymbol{n} = \hat{\boldsymbol{e}}_1 \times \hat{\boldsymbol{e}}_2 \big/ \left\| \hat{\boldsymbol{e}}_1 \times \hat{\boldsymbol{e}}_2 \right\| , \tag{23}$$

here x_m is the surface parameters; $\hat{\boldsymbol{e}}_i$ is the base vector of the local natural coordinate system on the master segment.

Considering the normal projection of the slave node onto the tangential plane, the coordinates of the contact point $\boldsymbol{x}_c$ should satisfy

$$\hat{\boldsymbol{e}}_m \cdot (\boldsymbol{x}_s - \boldsymbol{x}_c) = 0 . \tag{24}$$

Linearize the above equation with the unknowns, and note that $\boldsymbol{x}_c = \boldsymbol{x}_c(\boldsymbol{u}_c, x_m)$, we have

$$\dot{\hat{\boldsymbol{e}}}_m \cdot (\boldsymbol{x}_s - \boldsymbol{x}_c) + \hat{\boldsymbol{e}}_m \cdot (\dot{\boldsymbol{x}}_s - \dot{\boldsymbol{x}}_c) = 0 , \tag{25}$$

where $\dot{\boldsymbol{x}}_s = \dot{\boldsymbol{u}}_s$, $\dot{\boldsymbol{x}}_c = \dot{\boldsymbol{u}}_c + \hat{\boldsymbol{e}}_m \dot{x}_m$, $\dot{\hat{\boldsymbol{e}}}_m = \hat{\boldsymbol{e}}_{m,l}\, \dot{x}_l + \dot{\boldsymbol{u}}_{c,m}$.

Solving the above system yields the relationship between $\dot{x}_m$ and $\dot{\tilde{\boldsymbol{u}}}$ as

$$\dot{x}_m = \left\{ \left(\overline{C}_{ll}\, \dot{\boldsymbol{u}}_{c,m} - \overline{C}_{ml}\, \dot{\boldsymbol{u}}_{c,l} \right) \cdot \tilde{\boldsymbol{x}} + \left(\overline{C}_{ll}\, \hat{\boldsymbol{e}}_m - \overline{C}_{ml}\, \hat{\boldsymbol{e}}_l \right) \cdot \dot{\tilde{\boldsymbol{u}}} \right\} \big/ \wp \qquad (l \neq m \text{, no sum on } m \text{ and } l) , \tag{26}$$

where $\overline{C}_{ml} = C_{ml} - g_n\, \boldsymbol{n} \cdot \hat{\boldsymbol{e}}_{m,l}$, $C_{ml} = \hat{\boldsymbol{e}}_m \cdot \hat{\boldsymbol{e}}_l$, $\wp = \overline{C}_{11}\overline{C}_{22} - \overline{C}_{12}\overline{C}_{21}$, $\tilde{\boldsymbol{x}} = \boldsymbol{x}_s - \boldsymbol{x}_c$, $\dot{\tilde{\boldsymbol{u}}} = \dot{\boldsymbol{u}}_s - \dot{\boldsymbol{u}}_c$, while $\dot{\boldsymbol{u}}_s$ and $\dot{\boldsymbol{u}}_c$ are the velocity vectors at the slave node and the corresponding material position c of the master segment, respectively.

Thus, Eq. (21) can be rewritten as

$$\dot{\boldsymbol{f}} = \dot{\hat{f}}_i\, \hat{\boldsymbol{e}}_i + H_{jm}\, \hat{\boldsymbol{e}}_j \left\{ \left(\overline{C}_{ll}\, \dot{\boldsymbol{u}}_{c,m} - \overline{C}_{ml}\, \dot{\boldsymbol{u}}_{c,l} \right) \cdot \tilde{\boldsymbol{x}} + \left(\overline{C}_{ll}\, \hat{\boldsymbol{e}}_m - \overline{C}_{ml}\, \hat{\boldsymbol{e}}_l \right) \cdot \dot{\tilde{\boldsymbol{u}}} \right\} \quad (l \neq m \text{ and no sum on } l) , \tag{27}$$

where

$$H_{jm} = \hat{f}_i E_{ijm} \big/ \wp . \tag{28}$$

4.3 Evaluation of Contact Element Matrices

Now, we are concerned evaluation of matrices related with the node-to-point contact elements in the local Cartesian coordinate system as depicted in Fig.1. For an arbitrary case, the local Cartesian coordinate system on the contact interface (as shown in Fig. 1) is not the same as the above local natural coordinate system, it is defined as follows (see Fig. 2):

$$\boldsymbol{e}_3 = \boldsymbol{n} = \hat{\boldsymbol{e}}_3, \boldsymbol{e}_1 = \hat{\boldsymbol{e}}_1 \text{ and } \boldsymbol{e}_2 = \boldsymbol{e}_1 \times \boldsymbol{e}_3. \tag{29}$$

Assume a slave node s has contacted with point c on a surface element (master segment) E', and the surface element E' consists of g nodes, then ($p = 1, g$ in this paper)

$$\dot{\boldsymbol{u}}_c = N_p \dot{\boldsymbol{u}}_p, \quad \boldsymbol{x}_c = N_p \boldsymbol{x}_p, \tag{30}$$

here $\dot{\boldsymbol{u}}_p$ and $\boldsymbol{x}_p$ are the nodal velocity and position, respectively; N_p is the shape function value of the point c on the surface element E'. Thus the relative velocity and the relative position can be written as ($a = 1,(g+1)$ and $b = 1,(g+1)$ in this section)

$$\tilde{\dot{u}}_i = \dot{u}_{si} - \dot{u}_{ci} = \dot{u}_{scib} R_b, \qquad \tilde{x}_i = x_{si} - x_{ci} = x_{scib} R_b \tag{31}$$

in which

$$\dot{\boldsymbol{u}}_{sc} = \begin{bmatrix} \dot{u}_{s1} & \dot{u}_{11} & \dot{u}_{21} & \cdots & \dot{u}_{g1} \\ \dot{u}_{s2} & \dot{u}_{12} & \dot{u}_{22} & \cdots & \dot{u}_{g2} \\ \dot{u}_{s3} & \dot{u}_{13} & \dot{u}_{23} & \cdots & \dot{u}_{g3} \end{bmatrix} = \begin{bmatrix} \dot{\boldsymbol{u}}_s & \dot{\boldsymbol{u}}_1 & \dot{\boldsymbol{u}}_2 & \cdots & \dot{\boldsymbol{u}}_g \end{bmatrix},$$

$$\boldsymbol{x}_{sc} = \begin{bmatrix} x_{s1} & x_{11} & x_{21} & \cdots & x_{g1} \\ x_{s2} & x_{12} & x_{22} & \cdots & x_{g2} \\ x_{s3} & x_{13} & x_{23} & \cdots & x_{g3} \end{bmatrix} = \begin{bmatrix} \boldsymbol{x}_s & \boldsymbol{x}_1 & \boldsymbol{x}_2 & \cdots & \boldsymbol{x}_g \end{bmatrix},$$

$$\boldsymbol{R} = \begin{bmatrix} 1 & -N_1 & -N_2 & \cdots & -N_g \end{bmatrix}^T. \tag{32}$$

Thus

$$\dot{\boldsymbol{u}}_{c,m} = N_{p,m} \dot{\boldsymbol{u}}_p = \dot{\boldsymbol{u}}_{sc\,a} R_{a,m} \qquad (p = 1, g). \tag{33}$$

Combining with Eqs. (15) and (31)-(33), Eq. (27) can be rewritten as

$$\dot{\boldsymbol{f}} = \left\{ G_{hk} R_a \boldsymbol{e}_h + \left(H_{jm} \hat{\boldsymbol{e}}_j \left(\left(\overline{C}_{ll} R_{a,m} - \overline{C}_{ml} R_{a,l} \right) \boldsymbol{e}_k \cdot \tilde{\boldsymbol{x}} + R_a \left(\overline{C}_{ll} \hat{\boldsymbol{e}}_m - \overline{C}_{ml} \hat{\boldsymbol{e}}_l \right) \cdot \boldsymbol{e}_k \right) \right) \right\} \dot{u}_{scka} + \dot{f}_{jh} \boldsymbol{e}_h \tag{34}$$

$$(h = 1, 3, \; l \neq m \text{ and no sum on } l).$$

Thus the term related with contact in Eq. (18) can be described as

$$\dot{f}_i (d\dot{u}_{si} - d\dot{u}_{ci}) = d\dot{u}_{scib} \left(\left[\overline{K}_{fik} \right]_{ba} \dot{u}_{scka} + R_b \dot{f}_{ji} \right), \tag{35}$$

where

$$\left[\overline{K}_{fik} \right]_{ba} = R_b \, \boldsymbol{e}_i \cdot \left\{ G_{hk} R_a \boldsymbol{e}_h + \left(H_{jm} \hat{\boldsymbol{e}}_j \left(\left(\overline{C}_{ll} R_{a,m} - \overline{C}_{ml} R_{a,l} \right) \boldsymbol{e}_k \cdot \tilde{\boldsymbol{x}} + R_a \left(\overline{C}_{ll} \hat{\boldsymbol{e}}_m - \overline{C}_{ml} \hat{\boldsymbol{e}}_l \right) \cdot \boldsymbol{e}_k \right) \right) \right\}. \tag{36}$$

$$(h = 1, 3, \; l \neq m \text{ and no sum on } l)$$

4.4 Time Integration Algorithm

The time integration method is one of key issues to formulate a nonlinear finite element method. It is well known that the fully implicit method is often subjected to bad convergence problems, mostly due to changes of contact and friction states. In order to avoid this, we employ an explicit time integration procedure as follows. It is assumed that under a sufficiently small time increment all rates in Eq. (18) can be considered constant within the increment from t to $t + Dt$ as long as no drastic change of states (for example, elastic to plastic at an integration point, contact to discontact or discontact to contact on the contact interface, stick to slip or slip to stick in friction on the contact interface) takes place. The *R*-minimum method (Yamada, 1968) is extended and used here to limit the step size in order to avoid such drastic changes in state within an incremental step.

Thus all the rate quantities used to derive Eq. (18) are simply replaced by incremental quantities as

$$D\boldsymbol{u} = \boldsymbol{v}Dt\,,\quad Ds = s^{J}Dt \quad \text{and} \quad D\boldsymbol{L} = \boldsymbol{L}Dt\,. \tag{37}$$

Finally, in combination with Eqs. (35)-(37), Eq.(18) can be rewritten as

$$(\boldsymbol{K}+\boldsymbol{K}_f)D\boldsymbol{u} = D\boldsymbol{F} + D\boldsymbol{F}_f\ . \tag{38}$$

Here $\boldsymbol{K}$ is the standard stiffness matrix corresponding to body B; $D\boldsymbol{u}$ is the nodal displacement increment; $D\boldsymbol{F}$ is the external force increment subjected to body B on S_G; $\boldsymbol{K}_f$ and $D\boldsymbol{F}_f$ are the stiffness matrices and the force increments of all the node-to-point contact elements. From Eqs. (18), (35) and (36), for one node-to-point contact element E, they can be described as

$$\left[K^{E}_{f\,ik}\right]_{ba} = -\int_{S_c^E}\left[\overline{K}_{f\,ik}\right]_{ba} dS\,, \tag{39}$$

$$\left[DF^{E}_{f\,i}\right]_{b} = \int_{S_c^E} R_b\, \dot{f}_{j\,i}\, dS\,. \tag{40}$$

Note $\boldsymbol{K}_f$ is unsymmetrical due to the nonlinear friction and the geometry curvature, thus the total stiffness matrix $(\boldsymbol{K}+\boldsymbol{K}_f)$ is also unsymmetrical.

5. CONTACT SEARCHING

In cases that two or more bodies come in contact with each other, the search algorithms are normally split into a global and a local search. For the global search, several methods have been proposed, such as the regular cell algorithm (e.g. Santos, 1993), the Hierarchy-Territory (HITA) algorithm (Zhong, 1993), the position code algorithm (Oldenburg, 1994), the bucket sorting algorithm (Benson, 1990 and Belyschko, 1987), the spherical sorting algorithm (Papadopoulos, 1993), etc. The last three methods are mainly subjected to the finite-element-type mesh description of the contact surface, and the HITA and the position code algorithms are recommended in terms of the computational efficiency. In this study, the position code algorithm is employed for the global contact search between deformable bodies. For the local search, several algorithms have also been proposed, such as the pinball algorithm (Belyschko, 1991), the node-to-segment algorithm (Benson, 1990) etc. Here, according to basic characters of the node-to-point contact element, we take the normal vector $\boldsymbol{n}_s$ (in Fig. 3) of the slave node s for contact searching to avoid the 'deadzone' problem, while use the normal vector of the contact point c on the master segment to define the precise contact position of a slave node on the segment, which can be obtained from the normal vectors at the contact segment nodes. Note the normal vector at a node used in our code is determined as a weighted average of the normal vectors of the surfaces surrounding this node, where the weighting factors are proportional to the area of the corresponding surface segment (Xing, 1999). If the 8-node hexahedron solid element is used to discretize the body, for a local search, let (in Fig. 3)

$$\begin{aligned} &\boldsymbol{V}_i = \overrightarrow{ik} \times \overrightarrow{is} \quad (i = 1,4) \\ &k = i + 1 \; if \; i = 1,3, \; otherwise \; k = 1 \end{aligned} . \tag{41}$$

If all the $\boldsymbol{V}_i$ (in Fig.3) keep the same or the reverse direction as $\boldsymbol{n}_s$, point c will locate on this segment. Then the distance between the slave node s and point c is calculated and compared with a prescribed accuracy sector. If within the prescribed zone, the slave node s is in contact with this master segment on point c, and the exact location of point c and the penetration of the slave node s will be obtained and saved for further computation.

The following measures are also taken for contact search:

1). Contact candidates. The candidates of contact segments and slave nodes are marked during the pre-processing, then only these marked elements are considered during the contact searching and the calculation to save the computation cost.

2). Automatic extensions of master surfaces. To meet the requirement of the contact territory, the master surfaces can be extended automatically along the surface perimeter after one or some increment steps.

6. PARALLEL SOLVER

The 'Earth Simulator' (Earth simulator, 1999), a high performance massively parallel processing computer being developed in Japan, will have the following architecture: MIMD-type distributed memory parallel system consisting of computing nodes with shared memory vector type multi-processors. And it will have the following performance:

The peak performance: 40 TFLOPS
Total number of processor nodes: 640
Number of PE's for each node: 8
Peak performance of each PE: 8 GFLOPS
Peak performance of each node: 64 GFLOPS
The total main memory: 10 TB
Shared memory/node: 16 GB

In the analysis of the practical engineering and the active faults in the crust, a large-scale complex geometry has to be taken into account, thus parallel computing is necessary on the 'Earth Simulator'. Several related researches are being carried out. Based on the domain decomposition method, the parallelization of the nonlinear finite element system has been conducted in RIKEN for several years (e.g. Nikishikov, 1996). With choosing candidate contact surfaces as subdomain boundaries, a preliminary parallel version for a contact problem using the direct solvers was developed on IBM SP2 multiple processor computer for some special cases (Xing, 1998). Because the node-to-point contact element strategy with an explicit integration algorithm was proposed and applied as above, there exists no convergence problem, but it is not so efficient for a very large-scale computing. Thus, for a huge scale computing, a parallel version using iterative solvers is being developed (Miyamura, 2001). Meanwhile, GeoFEM group (GeoFEM, 2001) employed the so-called augmented Lagrangian multiplier (ALM) method to treat the contact problems and iterative solvers with localized preconditioning method for a large-scale computing, in which three kinds of iterations are necessary to obtain a suitable penalty parameter using ALM method and get to acceptable convergence points for both the Newton-Raphson solution and the iterative solvers. This may cause a convergence problem due to the existing iterations (e.g. Zavarise, 1998; Weiss, 1999), especially for the nonlinear friction contact problem. Thus the parallel sparse solver is investigated here.

Recently, the parallel sparse solver is widely used in the complex engineering analysis due to its stability and easy implementation into an existing serial code. For one node of the Earth Simulator, it has 16 GB memory, 8 processors and 64 GFLOPS peak performance as mentioned above and can work independently as a shared memory supercomputer (such as SGI Onyx2). It may have the ability to solve some typical practical engineering problems, the active faults in the localized region and some virtual friction experiments et al. If this parallel sparse solver used together with the node-to-point contact element strategy described as above, there will be no any iterations in the code, thus no convergence problem exists here at all. As for the efficiency of the parallel sparse solver for nonlinear frictional contact analysis, no result was reported, thus it will be investigated here using the parallel unsymmetrical sparse solver PSLDU on the SGI Onyx2 computer, which has 6 processors. In which multiple processors may be used to solve the linear equations, but only one of them is used also for other work, such as contact search and stiffness matrix assembling. And the average time cost of this processor per step is used here to investigate the efficiency of the solver if without any special notation.

Firstly, the parallel speedup for models with different degrees of freedom (DOF) are measured and compared (as shown in Fig.4), in which the contact node numbers are 140, 795, 285 and 399 respectively corresponding to the models from the smaller to the larger as shown in Fig. 4. And the speedup is calculated as

$$Speedup = \frac{Real\ clock\ time\ (1\ processor)}{Real\ clock\ time\ (multiple\ processors)}. \tag{42}$$

Fig.4 shows that all parallel speedups are increased with the increase of processor numbers, but it depends on the model size very much. For the above models, the larger the model size is, the better the speedup is. This is due to the relatively more time cost required to solve the linear equations for the larger model.

Secondly, the influence of contact node numbers on the computing cost is also investigated here. The assembling process of 37 tubes to a tubesheet using hydraulic expansion is taken as an example (as shown in Fig. 5). A tube and tubesheet assembling is one of the most important processes for a heat exchanger. All the tubes are fixed to the tubesheet with welding at the bottom; this is modeled with 'tied' (or stick) algorithm in our code (as shown in Fig. 5). Due to symmetry, only one-twelfth of the structure has been considered, being discretized into 30024 nodes (90072 DOF) and 21247 elements. The tubesheet is only supported along the central line direction of the tubes at the outside nodes of the bottom edge (see Fig.5). The case that all the tubes are hydraulically bulged at the same time and to be gradually contacted with the tubesheet is calculated. Fig. 6 shows the relationship between the average computing time per step and the numbers of contact nodes when 4 processors are used to calculate the assembling process. From this, the parallel sparse solver is powerful for such a scale calculation, but it is very sensitive to contact node numbers, and the time cost rises with the increase of contact node numbers. This may need further related research in the future.

From the above, the parallel sparse solver is a good choice for at least a medium scale calculation, because its stability, efficiency and easier implementation (nearly no additional modification is needed to implement it into an existing sequential code). Thus, the Earth Simulator may be used to calculate at least 640 medium-scale problems using the parallel sparse solver at the same time.

7. APPLICATION TO ACTIVE FAULTS

Japan locates on the boundaries of Eurasia, North America and Philippine Sea Planes. Pacific Plate subducts beneath Eurasia Plate from southeast at a speed of 9 cm per year, and Philippine Sea Plate subducts from south at a speed of 5 cm or less per year. Many large earthquakes occurred repeatedly on the plate boundaries. For modeling such earthquakes, several researches were published using the analytic and the finite element method (e. g. Stuart, 1988; Kato, 1997; Hirohara, 1999). They all assumed that the plate is a dip with a constant angle and applied in a two dimensional model. However, from the measured data, the practical plate is a little more complicated, as shown in Fig.7 for the Pacific plate around Japan (Kanai, 2000). Here, a part of Northeast fault model with the real-shaped subducting Pacific plate around Japan (Kanai, 2000) (as shown in 8) is taken as an application example. Fig. 9 shows the 3-dimensional meshes (with a scale of 1:100,000, unit: mm) of the local region and the boundary conditions. The displacement constraints used are also shown in Fig. 9 except that the plate is fixed along the *y* direction at the positions C and D (see Fig. 8). Here, all the materials are taken the same parameters as: density $r = 2.60\, g/cm^3$, Young modulus $E=44.8\ GPa$, and Poisson ratio $g = 0.12$. As for the loading conditions, the combined action of the self-gravity and the hydraulic pressure of water is investigated. And the widely accepted rate- and state-dependent friction law proposed by Dieterich (1978,1979) and Ruina (1983) is applied here to describe the complex phenomena along the interface between the active faults. That is

$$m = t / f_n = m_0 + j + a\, ln\left(V/V_{ref}\right), \quad dj/dt = -\left[(V/L)\left(j + b\, ln\left(V/V_{ref}\right)\right)\right], \tag{43}$$

and for a steady state,

$$m^{ss} = t^{ss}/f_n = m_0 + (a-b)ln\left(V/V_{ref}\right), \tag{44}$$

where

$$a = V(\partial t/\partial V)_j \,/\, f_n = (\partial t/\partial lnV)_j \,/\, f_n \text{ and } a - b = \left(dt^{ss}/dlnV\right)/f_n, \tag{45}$$

here, *a* and *b* are empirically determined parameters; *a* represents the instantaneous rate sensitivity, while *a-b* characterizes the long-term rate sensitivity. Depending on whether *a-b* is positive or negative, the frictional response is either velocity strengthening or velocity weakening, respectively. *L* is the critical slip distance; V_{ref} and *V* are an arbitrary reference velocity and a sliding velocity, respectively; j is the state variable; f_n is the effective normal contact stress; m_0 is the steady friction coefficient at reference velocity V_{ref}. The above special friction form is substituted into Eq. (11) in three dimension by replacing *V* with the relative velocity $\dot{\tilde{u}}_{eq}^{sl}\ (=\sqrt{\dot{\tilde{u}}_m \dot{\tilde{u}}_m})$ and implemented into our code. The calculated results with different friction coefficients are compared and shown in Fig. 10, which demonstrates that the friction coefficient along the active fault interface has obvious influence on their relative movement. The bigger the friction coefficient is, the less the relative movement along the interface is. Also this is affected by the distribution of different friction coefficients due to the relative slip velocity along the interface. As for the detail analysis of the plate movement, it will be published in the journal on the geophysics.

8. CONCLUSIONS

A static-explicit FEM code has been developed to simulate the static or the quasi-static 3-dimension friction contact between multi-elasto-plastic bodies and extended to simulate the active faults in the crust with a more general nonlinear friction law. An arbitrarily shaped contact element strategy, named as node-to-point contact element strategy, is proposed and applied according to the static-explicit characters, which overcomes the main convergence problems existing in the implicit treatment of contact. Meanwhile, for the multi-deformation-body contact problems, an efficient contact-searching algorithm suitable for the node-to-point contact element strategy has been proposed and implemented in our code. Moreover, combining with the above contact strategy, the parallel sparse solver is very stable (no convergence problem) and powerful for the nonlinear friction contact problems, but its efficiency also depends much on the contact node numbers, this may need further research. Finally, a model for the plate movement in the northeast zone of Japan under gravitation is taken as an example to be calculated with different friction behaviors. The preliminary results demonstrate the stability, efficiency and usefulness of this algorithm for the nonlinear friction multibody contact problems on a shared memory supercomputer (such as SGI Onyx2) or a node of Earth Simulator.

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Figures

Fig.1 Bodies in contact with each other
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Fig. 4 Speedup for different cases
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Fig. 10 Displacement distribution at different friction conditions (a). $m = 0.5$; (b). $m = 0.3$; (c). $m = 0.3\,(U_z \le 1500 \text{ or } U_z \ge 2780)$, otherwise $m = 0.5 - 0.025\,\ln(\tilde{u}_{eq}^{sl} / 0.01)$