BARYON CURRENT MATRIX ELEMENTS IN A RELATIVISTIC QUARK MODEL

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Current matrix elements and observables for electro- and photo-excitation of baryons from the nucleon are studied in a light-front framework. Relativistic effects are examined by comparison to a nonrelativistic model and can typically be of order 20-25%, but can be larger for certain matrix elements, such as radial transitions conventionally used to describe the Roper resonance. A systematic study shows that the violation of rotational covariance of the baryon transition matrix elements stemming from the use of one-body currents is generally small.

Much of what we know about excited baryon states has grown out of simple nonrelativistic quark models of their structure. These models were originally proposed to explain the systematics in the photocouplings of these states, which are extracted by partial-wave analysis of single-pion photoproduction experiments. Much more can be learned about these states from exclusive electroproduction experiments. Electroproduction experiments measure the Q^2 dependence of these form factors, and so simultaneously probe the spatial structure of the excited states and the initial nucleons. Both photoproduction and electroproduction experiments can be extended to examine final states other than $N\pi$, in order to find 'missing' states which are expected in symmetric quark models of baryons but which do not couple strongly to the $N\pi$ channel.[1,2] Such experiments are currently being carried out at lower energies at MIT/Bates and Mainz. Many experiments to examine these processes up to higher energies and Q^2 values will take place at TJNAF.

It is clear that, once the momentum transfer becomes greater than the mass of the constituent quarks, a relativistic treatment of the electromagnetic excitation is necessary. However, even at low momentum transfer, the ratio p_q/ω_q of the average quark momentum to its average energy is of order unity, which means that *relativistic effects will be significant in any model which describes valence quark degrees of freedom.*

The results reported here[9] make use of light-front Hamiltonian dynamics,[21] in which the constituents are treated as particles rather than fields. It shares with light-front approaches based upon field theories the property that certain combinations of boosts and rotations are independent of interactions which govern the quark dynamics, thus making it possible to perform relatively simple calculations of matrix elements in which composite baryons recoil with large momenta. In addition, we make use of a complete orthonormal set of basis states, composed of three constituent quarks, which satisfy rotational covariance. Such a basis is the natural starting point for dynamical models using the scheme of Bakamjian and Thomas.[22]

A consistent relativistic dynamical treatment of constituent quarks in baryons involves two main parts. First, the three-body relativistic bound-state problem is solved for the wavefunctions of baryons with the assumption of three interacting constituent quarks. Then these wavefunctions are used to calculate the matrix elements of one-, two- and three-body electromagnetic current operators. The conceptual and formal background for relativistic, directly interacting quarks is presented in detail in Ref. [21], and are outlined briefly below.

For quantum mechanical systems, relativistic invariance is equivalent to the requirement that there be a consistent set of generators of unitary transformations of inhomogeneous Lorentz transformations. For generators of spatial translations ($\mathbf{P}$), rotations ($\mathbf{J}$), boosts ($\mathbf{K}$) and time translations (H), that requirement is given by the commutation relations. Those relations common to Galilean-invariant systems are

$$[J^j, J^k] = i\epsilon_{jkl} J^l; \quad [P^\mu, P^\nu] = 0 \tag{1}$$

$$[J^j, P^k] = i\epsilon_{jkl} P^l; \quad [J^j, K^k] = i\epsilon_{jkl} K^l \tag{2}$$

$$[K^j, P^0] = -iP^j; \quad [J^j, P^0] = 0, \tag{3}$$

while those unique to Lorentz-invariant systems are

$$[K^j, K^k] = -i\epsilon_{jkl} J^l; \quad [K^j, P^k] = i\delta_{jk} P^0. \tag{4}$$

An inspection of Eq. (4) shows that the Hamiltonian is *necessarily* linked to at least some other generators, e.g., the boosts $\mathbf{K}$. In field theory, the generators are constructed via the energy-momentum stress tensor using the *exact* interacting fields. The approach taken here follows that of Bakamjian and Thomas,[22] with a direct interaction via a mass operator to construct consistent set of generators.

In light-front dynamics, the generators are reorganized into seven non-interacting operators $\{P^+; \mathbf{P}_\perp; J^3; K^3\}$, and three interacting generators $\{P^-; \mathbf{J}_\perp\}$. The Bakamjian construction consists of a mass operator

$$M_0 \to M = M_0 + U, \tag{5}$$

which determines the interacting generators:

$$P^- = \frac{M^2 + \mathbf{P}_\perp^2}{P^+}; \tag{6}$$

$$\mathbf{J}_\perp = \frac{1}{P^+}\Big\{\mathbf{e}_3 \times \Big[\tfrac{1}{2}(P^+ - P^-)\mathbf{E}_\perp - K^3\mathbf{P}_\perp\Big] + \mathbf{P}_\perp j^3 + M\mathbf{j}_\perp\Big\} \tag{7}$$

For a three-quark system, the non-interacting mass operator is

$$M_0 = \sum_{i=1}^{3} \sqrt{m_i^2 + \mathbf{k}_i^2}, \tag{8}$$

and interactions are added directly to the three-quark system:

$$M_0 \to M = M_0(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3) + U(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3). \tag{9}$$

The Capstick-Isgur interaction,[30] consisting of one-gluon exchange plus confining terms, satisfies all of the necessary formal requirements for the interaction U. This choice violates cluster separability, which is normally a problem for systems of particles which are individually observable, but not for systems of confined quarks.

The calculations reported here are described in detail in Ref. [9]. To calculate the current matrix element between initial and final baryon states, we expand in sets of free-particle states:

$$\begin{aligned}\langle M'j;\tilde{\mathbf{P}}'\mu'|I^+(0)|Mj;\tilde{\mathbf{P}}\mu\rangle \;=\; & (2\pi)^{-18}\int d\tilde{\mathbf{p}}_1'\int d\tilde{\mathbf{p}}_2'\int d\tilde{\mathbf{p}}_3'\int d\tilde{\mathbf{p}}_1\int d\tilde{\mathbf{p}}_2\int d\tilde{\mathbf{p}}_3\sum \\ & \langle M'j';\tilde{\mathbf{P}}'\mu'|\tilde{\mathbf{p}}_1'\mu_1'\tilde{\mathbf{p}}_2'\mu_2'\tilde{\mathbf{p}}_3'\mu_3'\rangle \\ & \times\langle\tilde{\mathbf{p}}_1'\mu_1'\tilde{\mathbf{p}}_2'\mu_2'\tilde{\mathbf{p}}_3'\mu_3'|I^+(0)|\tilde{\mathbf{p}}_1\mu_1\tilde{\mathbf{p}}_2\mu_2\tilde{\mathbf{p}}_3\mu_3\rangle \\ & \times\langle\tilde{\mathbf{p}}_1\mu_1\tilde{\mathbf{p}}_2\mu_2\tilde{\mathbf{p}}_3\mu_3|Mj;\tilde{\mathbf{P}}\mu\rangle. \end{aligned} \tag{10}$$

The electroweak current operator has a cluster expansion similar to that of its nonrelavistic counterpart:

$$I^\mu(x) = \sum_j I_j^\mu(x) + \sum_{j<k} I_{jk}^\mu(x) + \cdots. \tag{11}$$

Two-body currents I_{jk} are required for charge-changing (*e.g.*, π exchange) and/or non-local interactions, as they are in the nonrelativistic case, and they are also required for covariance of full current. In the front form, one- and two-body currents can be grouped separately. We compute only the contributions from one-body matrix elements, and assume that the struck quark carries the current of a free Dirac particle:

$$\langle\tilde{\mathbf{p}}'\mu'|I^+(0)|\tilde{\mathbf{p}}\mu\rangle = 1\delta_{\mu'\mu}. \tag{12}$$

The baryon state vectors are in turn related to wavefunctions as follows:

$$\begin{aligned}\langle\tilde{\mathbf{p}}_1\mu_1\tilde{\mathbf{p}}_2\mu_2\tilde{\mathbf{p}}_3\mu_3|Mj;\tilde{\mathbf{P}}\mu\rangle \;=\; & \left|\frac{\partial(\tilde{\mathbf{p}}_1,\tilde{\mathbf{p}}_2,\tilde{\mathbf{p}}_3)}{\partial(\tilde{\mathbf{P}},\mathbf{k}_1,\mathbf{k}_2)}\right|^{-\frac{1}{2}}(2\pi)^3\delta(\tilde{\mathbf{p}}_1+\tilde{\mathbf{p}}_2+\tilde{\mathbf{p}}_3-\tilde{\mathbf{P}}) \\ & \times\langle\tfrac{1}{2}\bar{\mu}_1\tfrac{1}{2}\bar{\mu}_2|s_{12}\mu_{12}\rangle\langle s_{12}\mu_{12}\tfrac{1}{2}\bar{\mu}_3|s\mu_s\rangle \\ & \times\langle l_\rho\mu_\rho l_\lambda\mu_\lambda|L\mu_L\rangle\langle L\mu_L s\mu_s|j\mu\rangle \\ & \times Y_{l_\rho\mu_\rho}(\hat{\mathbf{k}}_\rho)Y_{l_\lambda\mu_\lambda}(\hat{\mathbf{K}}_\lambda)\Phi(k_\rho,K_\lambda) \\ & \times D^{(\frac{1}{2})\dagger}_{\bar{\mu}_1\mu_1}[\underline{R}_{cf}(k_1)]D^{(\frac{1}{2})\dagger}_{\bar{\mu}_2\mu_2}[\underline{R}_{cf}(k_2)] \\ & \times D^{(\frac{1}{2})\dagger}_{\bar{\mu}_3\mu_3}[\underline{R}_{cf}(k_3)], \end{aligned} \tag{13}$$

The quantum numbers of the state vectors correspond to irreducible representations of the permutation group. The spins (s_{12}, s) can have the values $(0, \frac{1}{2})$, $(1, \frac{1}{2})$ and $(1, \frac{3}{2})$, corresponding to quark-spin wavefunctions with mixed symmetry (χ^ρ and χ^λ) and total symmetry (χ^S), respectively.[27] The momenta

$$\begin{aligned} \mathbf{k}_\rho &\equiv \frac{1}{\sqrt{2}}(\mathbf{k}_1 - \mathbf{k}_2); \\ \mathbf{K}_\lambda &\equiv \frac{1}{\sqrt{6}}(\mathbf{k}_1 + \mathbf{k}_2 - 2\mathbf{k}_3) \end{aligned} \tag{14}$$

preserve the appropriate symmetries under various exchanges of $\mathbf{k}_1$, $\mathbf{k}_2$ and $\mathbf{k}_3$. The set of state vectors formed using Eq. (13) and Gaussian functions of the momentum variables defined in Eq. (14) is complete and orthonormal. Since they are eigenfunctions of the overall spin, they satisfy the relevant rotational covariance properties. Any solution to a relativistic model with three constituent quarks can be written as a linear combination of these states. Thus, current matrix elements in any such model can be expressed in terms of the basis state coefficients and the matrix elements between basis state vectors. The use of this orthonormal basis allows us to examine the transition form factors for many different baryons simultaneously.

Rotational covariance represents a non-trivial constraint in light-front dynamics. It necessitates the existence of two-body current operators because of the interaction dependence of the four-vector current. One can test the extent to which rotational covariance is violated by constructing a quantity which should vanish under exact covariance, and comparing it to non-vanishing physical matrix elements. Helicity conservation yields the following constraint:

$$\sum_{\lambda'\lambda} D^{j\dagger}_{\mu'\lambda'}(\underline{R}'_{ch})\langle M'j';\tilde{\mathbf{P}}'\lambda'|I^+(0)|Mj;\tilde{\mathbf{P}}\lambda\rangle D^{j}_{\lambda\mu}(\underline{R}_{ch}) = 0, \quad |\mu' - \mu| \geq 2, \tag{15}$$

where

$$\underline{R}_{ch} = \underline{R}_{cf}(\tilde{\mathbf{P}}, M)\underline{R}_y(\frac{\pi}{2}), \quad \underline{R}'_{ch} = \underline{R}_{cf}(\tilde{\mathbf{P}}', M)\underline{R}_y(\frac{\pi}{2}), \tag{16}$$

and $\underline{R}_{cf}$ is a Melosh rotation. For elastic scattering from a nucleon, Eq. (15) is trivially satisfied. For a transition $\frac{1}{2} \to \frac{3}{2}$, there is a single non-trivial condition, while for $\frac{1}{2} \to \frac{5}{2}$, there are three.

The result of combining Eqs. (10)–(13) is a six-dimensional integral over two relative three momenta. These integrations are performed numerically, as the angular integrations cannot be performed analytically. The integration algorithm is a multidimensional quadrature technique recently generalized and extended to higher degree and generalized.[31] Uncertainties are typically on the order of a few percent for the largest matrix elements. The light-quark mass is taken[7,30] to be $m_u = m_d = 220$ MeV.

Using the techniques outlined above we can form the light-front current matrix elements for nucleon elastic scattering $\langle M_N\,\frac{1}{2};\tilde{\mathbf{P}}'\mu'|I^+(0)|M_N\,\frac{1}{2};\tilde{\mathbf{P}}\mu\rangle$, from Eq.(10).

We have evaluated Eq.(13) using a simple ground-state harmonic oscillator basis state, $\Phi_{0,0}(k_\rho, K_\lambda) = \exp\left\{-[k_\rho^2 + K_\lambda^2]/2\alpha_{HO}^2\right\}/(\pi^{\frac{3}{2}}\alpha_{HO}^3)$, where the oscillator size parameter $\alpha_{\rm HO}$ is taken[27,1] to be 0.41 GeV, and using the (CI) wavefunctions which result from the full solution of the relativized model mass operator,[30] expanded up to the $N = 6$ oscillator shell. Eq. (12) applies equally well to quark spinor and nucleon spinor current matrix elements, so we can extract $F_1(Q^2)$ and $F_2(Q^2)$ for the nucleons directly from the above light-front matrix elements.

Figure 1 compares the proton and neutron G_E and G_M calculated with these two wavefunctions, and the modified-dipole fit to the data. Our choice of quark mass for the relativistic calculation, while motivated by previous work,[7,30] gives a reasonable fit to the nucleon magnetic moments. The single-oscillator relativistic calculation yields proton charge and magnetic radii close to those found from the slope near Q^2=0 of the dipole fit to the data. The relativistic calculation using the relativized model wavefunctions falls off too slowly with Q^2, which is due to the larger probability of higher-momentum components in these wavefunctions. This confirms the results of previous work[32] using these wavefunctions, where the nucleon form factors were fit by the adoption of relatively soft form factors for the quarks.

Figure 2 compares the axial-vector form factor $G_A(Q^2)$ and $G_E(Q^2)$ for the proton calculated with the CI wavefunctions. Relativistic effects are known[7] to reduce the axial coupling constant g_A from the static nonrelativistic quark model value of 5/3 to more like the physical value of 1.25 using simple single-Gaussian wavefunctions; using the CI wavefunctions g_A is reduced further[15] due to the higher momenta of the quarks in these wavefunctions. As expected from the data for $G_A(Q^2)$, the axial form factor falls with Q^2 more slowly than G_E.

Figure 3 shows our relativistic results for the $A_{\frac{1}{2}}$, $A_{\frac{3}{2}}$, and $C_{\frac{1}{2}}$ helicity amplitudes for electroexcitation of the $\Delta\frac{3}{2}^+(1232)$ from nucleon targets using $N = 6$ CI wavefunctions for the initial and final momentum-space wavefunctions, compared to relativistic results using the single oscillator-basis state above. The parameters α_{HO} and $m_{u,d}$ are the same as above. The relativistic calculation does not solve the problem of the long-standing discrepancy between the measured and predicted photocouplings (which are essentially transition magnetic moments as the transition is almost purely M1), although the behavior of the single-oscillator relativistic calculation is similar to the faster-than-dipole fall off found in the data. Like the nucleon magnetic and axial moments, the photocouplings are reduced further by the adoption of the CI wavefunctions, and the form factors drop more slowly with Q^2. A reasonable fit to the Q^2 dependence of the data is achieved by Cardarelli *et al.*[33] using the CI wavefunctions and soft quark form factors which fit the nucleon form factors.

We have also plotted the numerical value of the rotational covariance condition (multiplied by the normalization factor $\zeta\sqrt{4\pi\alpha/2K_W}$ for ease of comparison to the physical amplitudes), given by the left-hand side of Eq. (15), for $|\mu' - \mu| = 2$. At lower values of Q^2 the rotational covariance condition expectation value is a small fraction of the transverse helicity amplitudes, but approximately the same size as $C_{\frac{1}{2}}$ and larger than the value of $E2/M1$ implied by our $A_{\frac{1}{2}}$ and $A_{\frac{3}{2}}$.

Given the controversy surrounding the nature of the baryon states assigned to radial excitations of the nucleon in the nonrelativistic model[35], in Figure 4 we compare nonrelativistic and relativistic calculations, for both proton and neutron targets, using for the final wavefunction a simple radially excited basis state which can be used to represent the P_{11} resonance $N(1440)\frac{1}{2}^+$. For the initial state we have used the single oscillator-basis ground state wavefunction above.

There are large relativistic effects, with differences between the relativistic and nonrelativistic calculations of factors of three or four. Interestingly, the transverse amplitudes also change sign at low Q^2 values approaching the photon point. The large amplitudes at moderate Q^2 predicted by the nonrelativistic model (which are disfavored by analyses of the available single-pion electroproduction data[36]) appear to be an artifact of the nonrelativistic approximation. This disagreement, and that of the nonrelativistic photocouplings with those extracted from the data for this state,[6] have been taken as evidence that the Roper resonance may not be a simple radial excitation of the quark degrees of freedom but may contain excited glue.[37,38] The strong sensitivity to relativistic effects demonstrated here suggests that this discrepancy for the Roper resonance amplitudes has a number of possible sources, including relativistic effects.

We also find in the case of proton targets that there is a sizeable $C^p_{\frac{1}{2}}$, reaching a value of about $40 - 50 \times 10^{-3}$ $\text{GeV}^{\frac{1}{2}}$ at Q^2 values between 0.25 and 0.50 GeV^2, and increasing at lower Q^2 values. Correspondingly, there will be a sizeable longitudinal excitation amplitude.

We have also calculated helicity amplitudes for the final state $N\frac{1}{2}^-(1535)$, for both proton and neutron targets. Here we use the same single oscillator-basis state initial momentum-space wavefunction as above, and final state wavefunctions which are made up from momentum-space wavefunctions with one or the other oscillator orbitally-excited. In this case configuration mixing due to the tensor part of the hyperfine interaction is included in the final-state wavefunction. Since the two types of orbitally excited states are degenerate in mass before the application of tensor spin-spin interactions, they are substantially mixed by them. The results for the helicity amplitudes for $N\frac{1}{2}^-(1535)$ excitation are compared to the corresponding nonrelativistic results in Figure 5.

In contrast to the results shown above, here there is reduced sensitivity to relativistic effects in the results for the transverse amplitudes $A_{1/2}$, with the main effect being a hardening of the Q^2 behavior of the transition form factor; this is not the case for the $C_{1/2}$ amplitudes. For both targets the substantial nonrelativistic $C_{1/2}$

amplitudes are reduced to essentially zero in the relativistic calculation.

1 Discussion and Summary

The results outlined above establish that there can be considerable relativistic effects at all values of Q^2 in the electroexcitation amplitudes of baryon resonances, even at $Q^2 = 0$. In particular, our results show that the Q^2 dependence of the nonrelativistic amplitudes is generally modified into one resembling a dipole falloff behavior, as has been shown in the case of the nucleon form factors. However, we consider it remarkable that relativistic effects account for a large part of discrepancy between the nonrelativistic model's predictions and the physical situation.

Electroexcitation amplitudes of the P_{11} Roper resonance $N(1440)\frac{1}{2}^+$ and $N(1710)\frac{1}{2}^+$ states, as well as those of the $\Delta(1600)\frac{3}{2}^+$, are substantially modified in a relativistic calculation. Given the controversial nature of these states,[37,38] we consider this an important result. Our results show that relativistic effects tend to reduce the predicted size of the amplitudes for such states at intermediate and high Q^2 values, in keeping with the limited experimental observations for the best known of these states, $N(1440)\frac{1}{2}^+$.

We have also found that the rotational covariance violation is a small fraction of the larger amplitudes for the Q^2 values considered here. In cases where the dynamics causes an amplitude to be intrinsically small, the uncertainty in our results for these amplitudes becomes larger. In particular, the calculated ratios $E2/M1$ and $C2/M1$ for the electroexcitation of the $\Delta(1232)\frac{3}{2}^+$ in the *absence* of configuration mixing of D-wave components into the initial and final state wavefunctions[18] are probably 100% uncertain, and are thus consistent with zero at all Q^2.[40] This may not be the case in the presence of such configuration mixing, and we intend to investigate this possibility, since $\Delta(1232)$ electroproduction is the subject of current experiments at MIT/Bates and several proposed experiments at CEBAF.[41]

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40. Although it may not be obvious from Fig. 3, our results for the transverse amplitudes at $Q^2 = 0$ yield E2/M1 and C2/M1 which are *exactly* zero.
41. V. Burkert *et al.*, CEBAF proposal PR-89-037, ; P. Stoler *et al.*, CEBAF proposal PR-91-002; S. Frullani *et al.*, CEBAF proposal PR-91-011; J. Napolitano, P. Stoler *et al.*, CEBAF Experiment E94-14.

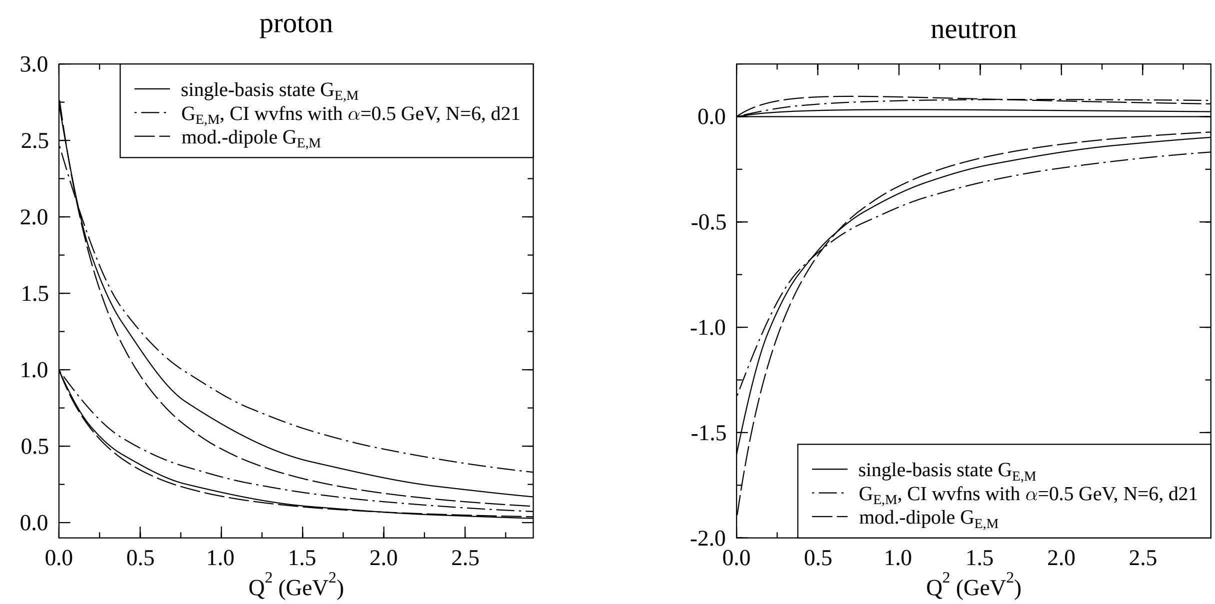

Figure 1: Proton and neutron elastic form factors G_E and G_M.

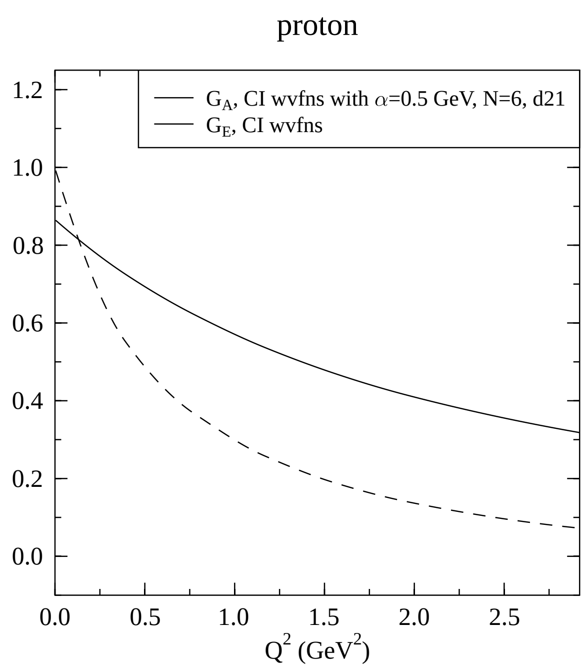

Figure 2: Proton axial form factor G_A and G_E calculated with the CI wavefunctions.

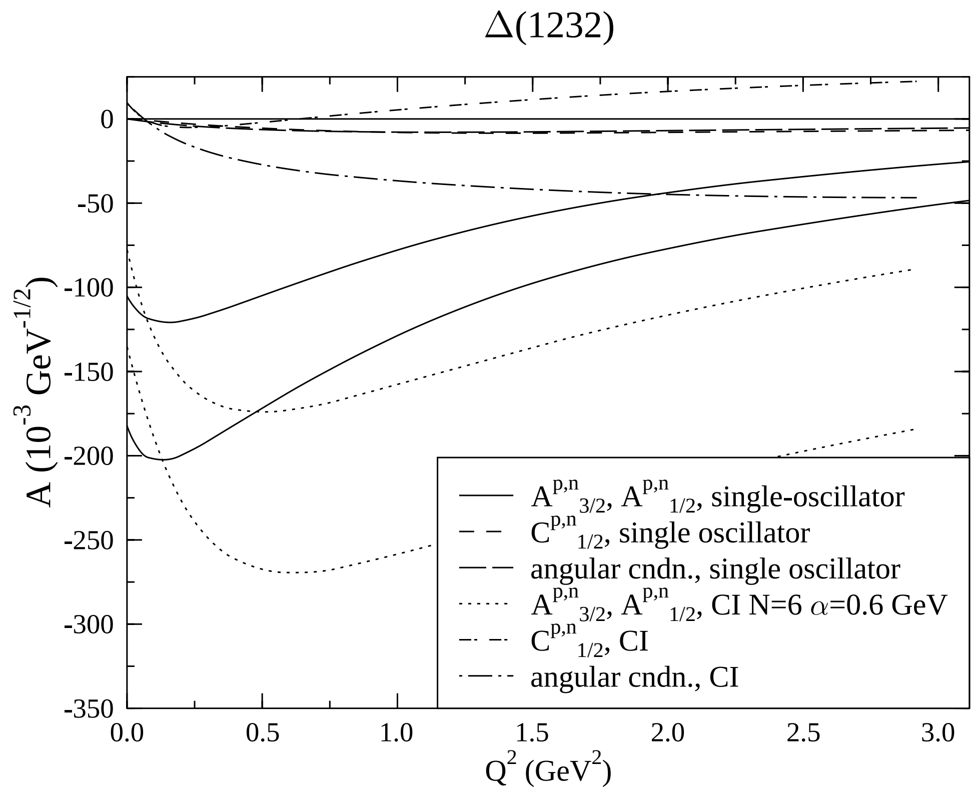

Figure 3: Single-basis state and CI wavefunction relativistic $\Delta(1232)$ electroexcitation helicity amplitudes and rotational covariance condition.

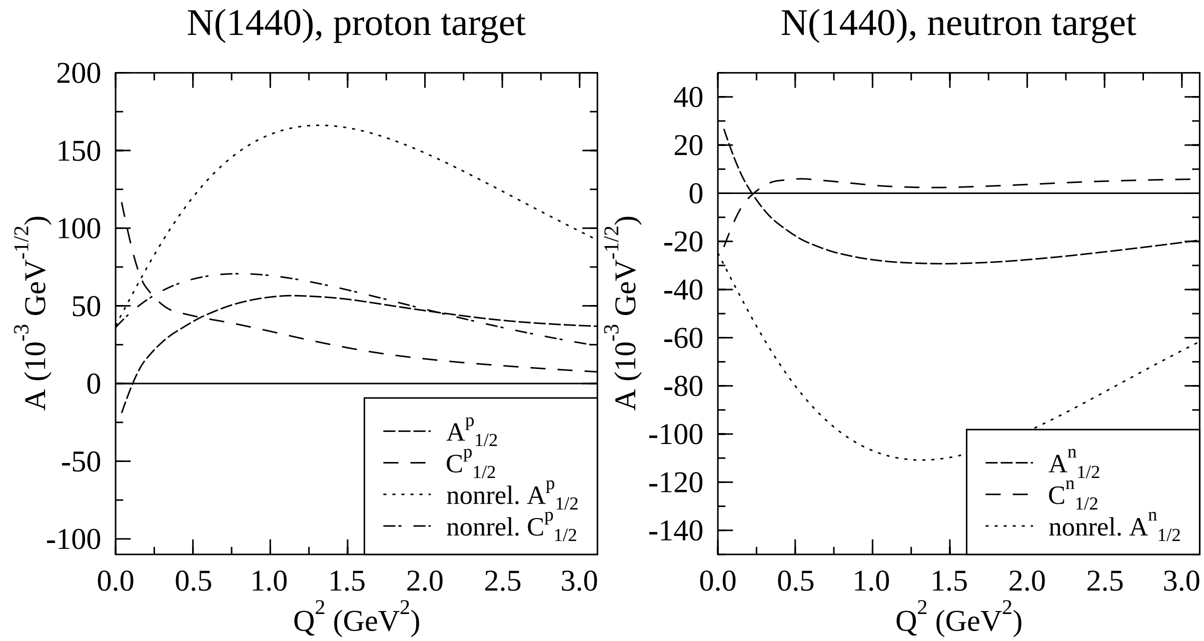

Figure 4: Single-basis state relativistic $N(1440)$ electroexcitation helicity amplitudes, and corresponding nonrelativistic Breit-frame helicity amplitudes ($C^n_{1/2}$ is zero in the nonrelativistic model).

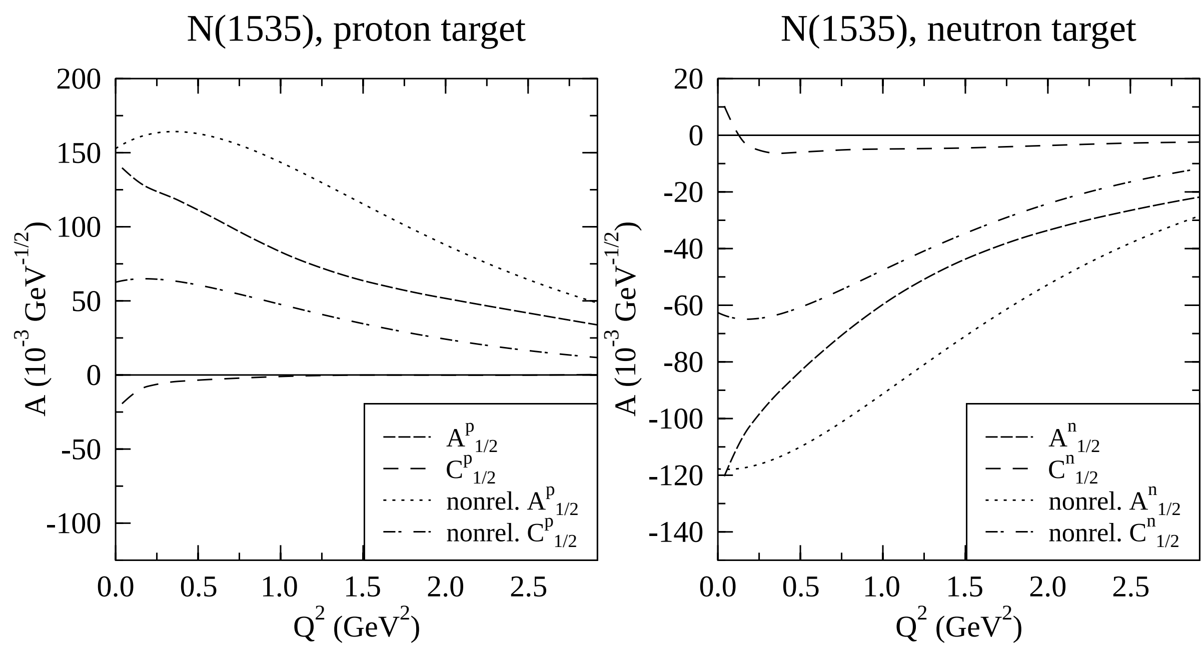

Figure 5: Single-basis state relativistic $N(1535)$ electroexcitation helicity amplitudes, and corresponding nonrelativistic Breit-frame transverse helicity amplitudes.