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Regge Amplitudes in two-body processes

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abstract

A study is carried out of the Regge production amplitudes for $\pi_1(1405)$ in $\pi^- p \to \pi_1^0(1405)n$ and $\pi^- p \to \pi_1^-(1405)p$.

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1 Introduction

Consider a process

$$\pi^- p \rightarrow \pi_1^- p \tag{1}$$
$$\pi^- p \rightarrow \pi_1^0 n \tag{2}$$

A Regge-exchange diagram for these processes is illustrated in fig.1. Let $|J\mu\rangle$ be the helicity state describing π_1 and let μ_i and μ_f be the helicities for the initial and final nucleons. The purpose of this note is to exhibit Regge amplitudes appropriate for these processes and show the predicted form of the t-distribution given a particular set of values for μ, μ_i and μ_f.

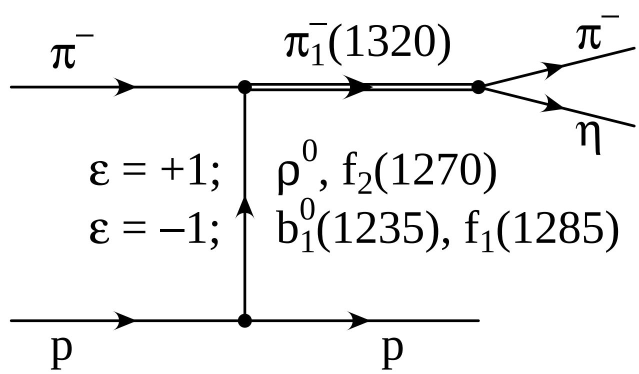

Figure 1: Resonance production via Reggeon exchange—an example.

2 Regge Amplitudes

Let $\alpha(t)$ be the Regge trajectory corresponding to an exchanged particle which may be ρ, $f_2(1270)$, $b_1(1235)$ or Pomeron. The Regge amplitude is, as $s \rightarrow \infty$,

$$A(s,t) \sim -\left(\frac{-t}{s_0'}\right)^{\frac{1}{2}m} \left[\frac{\mathrm{e}^{-i\pi\alpha(t)} + \mathcal{S}}{2\sin\pi\alpha(t)}\right] \gamma_\mu(t)\, \gamma_{\mu_i\mu_f}(t) \left(\frac{s}{s_0}\right)^{\alpha(t)} \tag{3}$$

where

$$m = |\mu| + |\mu_i - \mu_f| \tag{4}$$

$\mathcal{S} = \pm 1$ is the so-called signature of the Regge trajectory ($\mathcal{S} = (-1)^S$ where S is the spin of the particle responsible for the Regge trajectory); γ's are the residue functions free of kinematic sigularities; and s_0 is an arbitrary constant in the problem, commonly set to 1 GeV^2. This formula has been adapted from Eq. (6.4.9) of Collins [1].

The Regge trajectory is traditionally parameterized as follows:

$$\alpha(t) = \alpha_0 + \alpha_1 t \tag{5}$$

The residue functions are supposed to be damped exponentially in $-t$,

$$\gamma_\mu(t)\,\gamma_{\mu_i\mu_f}(t)\,\left(\frac{s}{s_0}\right)^{\alpha(t)} \sim\; e^{\frac{1}{2}bt} \tag{6}$$

independent of the helicities. These expressions show that, as $t \to 0$, one has

$$A(s,t) \to (-t)^{\frac{1}{2}m} \tag{7}$$

Let λ be the helicity of the exchanged particle. In the limit $t \to 0$, conservation of the z-component of spin at both the meson and nucleon vertices demands that

$$\lambda = \mu = \mu_f - \mu_i \tag{8}$$

so that one must conclude $m = 2|\lambda|$. This leads to the following $-t$ distribution:

$$\begin{aligned} \frac{d\sigma}{dt} &\sim (-t)^{2|\lambda|}\,e^{bt} \\ &\sim (-t)^{2|\mu|}\,e^{bt} \end{aligned} \tag{9}$$

It should be mentioned that the Regge amplitude (3) is valid only in the absence of cuts (initial- and final-state interactions). It is known, for instance, that the pion-exchange process involves a considerable amount of cuts—see Section 6.8 of Collins [1].

Since natural-parity exchanges forbid the $\mu = 0$ amplitude, it follows that the $-t$ distribution should dip for $t = 0$ in this case. One notes further that the amplitude with $\mu = 0$, if allowed, dominates in general over that with $\mu = \pm 1$. For unnatural-parity processes, therefore, one expects in general a spike at $t = 0$, unless the $\mu = 0$ amplitude happens to be small for some dynamical reason. The GAMS data exhibits a strong dip at $t = 0$ in the $a_2(1320)$ region. Nevertheless, the waves P_0 and D_0 (unnatural-parity exchanges) are found to be dominant in the region $-t < 0.15$ GeV^2—this is a mystery. On the other hand, both the KEK and VES data show a dip at $t = 0$ accompanied by strong D_+ and P_+ waves; this is consistent with the Regge picture outlined here.

It is an established folklore that the natural-parity states are produced with a forward dip, as exemplified by the $a_2(1320)$ production, while the unnatural-parity states are produced with a forward spike at $t = 0$. One may therefore speculate that the $\pi_1(1405)$ resonance is likewise produced with a forward dip. As stated in the previous paragraph, natural-parity exchanges necessarily generate a forward dip. It may very well be that the natural-parity states, produced via natural-parity exchanges, are always accompanied by a dip at $t = 0$ in the absence of cuts. One should note in addition that the helicity states are used exclusively in this appendix—the z-components of spin in the Jackson frame correspond to the helicities only in the limit $t = 0$.

A more restrictive form of the Regge amplitude (3) was given by Irving and Worden [2] [see their equation (AA.10)]:

$$A(s,t) \sim -\left(\frac{t'}{s'_0}\right)^{\frac{1}{2}n} \left[\frac{\mathrm{e}^{-i\pi\alpha(t)} + \mathcal{S}}{2\sin\pi\alpha(t)}\right] \beta_\mu\, \beta_{\mu_i\mu_f} \left(\frac{s}{s_0}\right)^{\alpha(t)} \tag{10}$$

where

$$n = |\mu - (\mu_f - \mu_i)| \tag{11}$$

The key ideas here are that the exponent m of (3) has been replaced by a more general one—resulting only from conservation of the angular momentum for $t' \simeq 0$ in reactions (1) and (2)—and that the residue functions β are now constant and independent of $-t$. Note also that $t' = (-t) - (-t)_{\mathrm{min}}$ is introduced above. Parity conservation at each vertex implies

$$\begin{aligned} \beta_\mu &= -\epsilon\, \nu_J (-)^\mu\, \beta_{-\mu} \\ \beta_{\mu_i\mu_f} &= \epsilon\, (-)^{\mu_i - \mu_f} \beta_{-\mu_i - \mu_f} \end{aligned} \tag{12}$$

where ϵ is the naturality of the Reggeon and J and ν_J are the spin and naturality of $\pi_1(1405)$.

The meson coupling constant can be cast into that in reflectivity basis

$${}^\epsilon\beta_\mu = \theta(\mu)\, [\beta_\mu - \epsilon\, \nu_J (-)^\mu\, \beta_{-\mu}] \tag{13}$$

Because of parity conservation, this turns into

$$\begin{aligned} {}^\epsilon\beta_\mu &= 2\theta(\mu)\beta_\mu \\ &= 0, \qquad \text{if} \quad \mu < 0, \\ &= \beta_\mu, \qquad \text{if} \quad \mu = 0, \\ &= \sqrt{2}\,\beta_\mu, \qquad \text{if} \quad \mu > 0 \end{aligned} \tag{14}$$

One sees, therefore, that the production amplitude that is fit in parial-wave analyses is in fact given by (10), i.e.

$${}^\epsilon V_{J\mu\, \mu_i\mu_f} = 2\theta(\mu)\, A(s,t) \tag{15}$$

The reader is referred to a previous note by the author[3] for a detailed exposition of the techniques of the partial-wave analyses. One may note that, in the previous note, $|J\mu\rangle$ (helicity state) was substituted by $|\ell m\rangle$ (Jackson frame) and $\{\mu_i\mu_f\}$ by an index k.

In the limit $-t \to 0$, one sees that, from (8), the helicity λ of the exchanged Reggeon is equal to $\mu = \mu_f - \mu_i$ and so $n = 0$ [see (11)]. Away from the forward region, one can have $n = 1$ if $\mu = 1$ and helicity non-flip at the nucleon vertex or $\mu = 0$ and helicity flip at the nucleon vertex. On the other hand, one could have $n = 2$ if $\mu = 1$ and $\{\mu_f, \mu_i\} = \{-,+\}$ (note that $n = 0$ if $\mu = 1$ and $\{\mu_f, \mu_i\} = \{+,-\}$). The t' distribution is given by

$$\frac{\mathrm{d}\sigma}{\mathrm{d}t'} \sim (t')^n\, \mathrm{e}^{-b\,t'} \tag{16}$$

where $n = 0$, 1 or 2 and

$$\left(\frac{s}{s_0}\right)^{\alpha(t)} \sim \ \mathrm{e}^{-\frac{1}{2}b\,t'} \tag{17}$$

It may be instructive to study the meaning of the term in the square bracket in (3). Let it be represented by

$$f(t) = -\frac{\mathrm{e}^{-i\pi\alpha(t)} + \mathcal{S}}{2\sin\pi\alpha(t)} \tag{18}$$

The numerator is called the signature factor. If Im α is small, it is equal to $+2$ for a Regge pole of even signature when Re α passes through an even integer, and it is zero when Re α is at odd integer values. For odd signature it is equal to -2 when Re α is odd, while it is equal to zero when Re α is even. Note that for wrong-signature integer values of Re α, the function $f(t)$ is simply equal to $+i$.

The denominator of (18) generates the propagator corresponding to the Regge pole. Let M be the mass and S be the spin of a state on the Regge trajectory. Then, one has, for t sufficiently close to M^2,

$$\alpha(t) \simeq S + \alpha'_{_\mathrm{R}}\,(t - M^2) + i\,\alpha_{_\mathrm{I}} \tag{19}$$

where the subscripts stand for real and imaginary parts. From this one finds, for $t \simeq M^2$ and small Im α,

$$f(t) \simeq \left(\frac{1}{\pi\alpha_{_\mathrm{I}}}\right) \frac{\alpha_{_\mathrm{I}}/\alpha'_{_\mathrm{R}}}{M^2 - t - i\,\alpha_{_\mathrm{I}}/\alpha'_{_\mathrm{R}}} \tag{20}$$

This is the propagator corresponding to the Regge pole. Note that the second factor above is the conventional Breit-Wigner formula and it is equal to $+i$ at $t = M^2$. One may then identify

$$M\Gamma = \frac{\alpha_{_\mathrm{I}}}{\alpha'_{_\mathrm{R}}} \tag{21}$$

where Γ is the experimental width of the resonance corresponding to the Regge pole. The $-t$ distribution could be modified, using this formula,

$$\frac{\mathrm{d}\,\sigma}{\mathrm{d}\,t} \sim \frac{(-t)^{2|\lambda|}\,\mathrm{e}^{bt}}{(-t + M^2)^2 + (M\Gamma)^2} \tag{22}$$

If M^2 is large compared to $-t$, then the denominator above is nearly constant. For example, if a process involves exchange of the $f_2(1270)$, then the denominator can be set to 1 for $-t \ll 1\mathrm{GeV}^2$.

References

[1] 'An Introduction to Regge Theory and High Energy Physics,' P.D. B. Collins, Cambridge University Press (1977).

[2] A. C. Irving and R. P. Worden, Physics Reports **34**, 117(1977).

[3] 'Amplitude Analysis of two-pseudoscalar systems—Version VII,' S. U. Chung, BNL-QGS-95-41 (1996).