1 The Cstat State Transition Model

Δ is a finite set of symbols:

$$\begin{aligned}\Delta \quad = \quad & \{\delta_{ij} \mid 3 \leq i < 6 \wedge 0 \leq j < 4\} \cup \\ & \{\delta_{ij} \mid 0 \leq i < 4 \wedge 3 \leq j < 6\} \cup \\ & \{\epsilon_{ij} \mid 0 \leq i < 6 \wedge 0 \leq j < 4\} \cup \\ & \{\epsilon_{ij} \mid 0 \leq i < 4 \wedge 0 \leq j < 6\}\end{aligned}$$

It's convenient to introduce a finite state transition model on Δ. This model has alphabet

$$\begin{aligned}\mathbf{A}^{\Delta} \quad = \quad \{ \; & c^O, c^I, c^I_-, q^O, q^I, i^O, i^I, \\ & \overline{c}^O, \overline{c}^I, \overline{c}^I_-, \overline{q}^O, \overline{q}^I, \overline{i}^O, \overline{i}^I \; \}\end{aligned}$$

The transition function $\mathbf{T}^{\Delta}$ is given in figure ??. In this diagram, all arcs are directed from left to right, or top to bottom.

The transition model describing the history of a channel is an "augmented" version of this finite state model. The state space *Cstat* is a subset of

$$\begin{aligned}\Delta \quad \times \quad & Ports_{\perp} \times Ports \times Ports_{\perp} \times Values_{\perp} \times \\ & Ports_{\perp} \times Ports \times Ports_{\perp} \times Values_{\perp},\end{aligned}$$

namely

$$\begin{aligned}Cstat \quad = \quad & \{(d, P_-, P, P_+, u, Q_-, Q, Q_+, v) \mid \\ & ((d = \delta_{ij} \vee d = \epsilon_{ij}) \Rightarrow \\ & \quad (\; (i < 3 \Rightarrow P_- \in Ports) \wedge \\ & \quad \;\; (4 \leq i \Rightarrow P_+ \in Ports) \wedge \\ & \quad \;\; ((i = 0) \vee (i = 2) \vee (i = 4)) \Rightarrow u \in Values) \wedge \\ & \quad \;\; (j < 3 \Rightarrow (Q_- \in Ports) \wedge \\ & \quad \;\; (4 \leq j \Rightarrow (Q_+ \in Ports) \wedge \\ & \quad \;\; ((j = 0) \vee (j = 2) \vee (j = 4)) \Rightarrow v \in Values) \;)) \wedge \\ & (d = \epsilon_{0j} \Rightarrow Q_- \in Ports) \wedge \\ & (d = \epsilon_{i0} \Rightarrow P_- \in Ports)\}.\end{aligned}$$

The restrictions ensure that various fields don't take the undefined value $\perp$ in various states—necessary in the definition of the transition function.

The alphabet of a *Cstat* system associated with $C \in Chans$ is

$$\begin{aligned}\mathbf{A}^{Cstat}_C \quad = \quad \{ \; & (\_,\_)^O.mes(E,\_), \\ & (\_,\_)^O.mes(\overline{E},\_), \\ & (\_,\_)^O.ini(E,\_,\_), \quad (\_,\_)^I.ini(E,\_,\_), \\ & (\_,\_)^O.ini(\overline{E},\_,\_), \quad (\_,\_)^I.ini(\overline{E},\_,\_), \\ & (\_,\_)^O.chg(E,\_,\_), \quad (\_,\_)^I.chg(E,\_,\_), \\ & (\_,\_)^O.chg(\overline{E},\_,\_), \quad (\_,\_)^I.chg(\overline{E},\_,\_), \\ & (\_,\_)^O.quo(E,\_), \quad (\_,\_)^I.quo(E,\_), \\ & (\_,\_)^O.quo(\overline{E},\_), \quad (\_,\_)^I.quo(\overline{E},\_) \; \}.\end{aligned}$$

where $E = out(C)$, $\overline{E} = in(C)$.

$$
\begin{tikzcd}[row sep=small, column sep=small]
& q^I & i^O & i^I & c^O & c^I & q^O & \\
\overline{q}^I & & & & \delta_{30} \arrow[r, dash] \arrow[d, dash] & \delta_{40} \arrow[r, dash] \arrow[d, dash] & \delta_{50} \arrow[d, dash] & \\
\overline{i}^O & & & & \delta_{31} \arrow[r, dash] \arrow[d, dash] & \delta_{41} \arrow[r, dash] \arrow[d, dash] & \delta_{51} \arrow[d, dash] & \\
\overline{i}^I & & & & \delta_{32} \arrow[r, dash] \arrow[d, dash] & \delta_{42} \arrow[r, dash] \arrow[d, dash] & \delta_{52} \arrow[d, dash] & \\
\overline{c}^O & \delta_{03} \arrow[r, dash] \arrow[d, dash] & \delta_{13} \arrow[r, dash] \arrow[d, dash] & \delta_{23} \arrow[r, dash] \arrow[d, dash] & \delta_{33} \arrow[r, dash] \arrow[d, dash] & \delta_{43} \arrow[r, dash] \arrow[d] & \delta_{53} \arrow[r] & \delta_{03} \\
\overline{c}^I & \delta_{04} \arrow[r, dash] \arrow[d, dash] & \delta_{14} \arrow[r, dash] \arrow[d, dash] & \delta_{24} \arrow[r, dash] \arrow[d, dash] & \delta_{34} \arrow[r] \arrow[d, dash] & \epsilon_{00} & & \\
\overline{q}^O & \delta_{05} \arrow[r, dash] & \delta_{15} \arrow[r, dash] & \delta_{25} \arrow[r, dash] & \delta_{35} \arrow[d] & & & \\
& & & & \delta_{30} & & &
\end{tikzcd}
$$

$$
\begin{tikzcd}[row sep=small, column sep=small]
& \overline{c}^I_- & i^O & i^I & c^O & c^I & q^O & \\
c^I_- & \epsilon_{00} \arrow[r, dash] \arrow[d, dash] & \epsilon_{10} \arrow[r, dash] \arrow[d, dash] & \epsilon_{20} \arrow[r, dash] \arrow[d, dash] & \epsilon_{30} \arrow[r, dash] \arrow[d, dash] & \epsilon_{40} \arrow[r, dash] \arrow[d, dash] & \epsilon_{50} \arrow[d, dash] & \\
\overline{i}^O & \epsilon_{01} \arrow[r, dash] \arrow[d, dash] & \epsilon_{11} \arrow[r, dash] \arrow[d, dash] & \epsilon_{21} \arrow[r, dash] \arrow[d, dash] & \epsilon_{31} \arrow[r, dash] \arrow[d, dash] & \epsilon_{41} \arrow[r, dash] \arrow[d, dash] & \epsilon_{51} \arrow[d, dash] & \\
\overline{i}^I & \epsilon_{02} \arrow[r, dash] \arrow[d, dash] & \epsilon_{12} \arrow[r, dash] \arrow[d, dash] & \epsilon_{22} \arrow[r, dash] \arrow[d, dash] & \epsilon_{32} \arrow[r, dash] \arrow[d, dash] & \epsilon_{42} \arrow[r, dash] \arrow[d, dash] & \epsilon_{52} \arrow[d, dash] & \\
\overline{c}^O & \epsilon_{03} \arrow[r, dash] \arrow[d, dash] & \epsilon_{13} \arrow[r, dash] \arrow[d, dash] & \epsilon_{23} \arrow[r, dash] \arrow[d, dash] & \epsilon_{33} \arrow[r, dash] \arrow[d, dash] & \epsilon_{43} \arrow[r, dash] \arrow[d] & \epsilon_{53} \arrow[r] & \delta_{03} \\
\overline{c}^I & \epsilon_{04} \arrow[r, dash] \arrow[d, dash] & \epsilon_{14} \arrow[r, dash] \arrow[d, dash] & \epsilon_{24} \arrow[r, dash] \arrow[d, dash] & \epsilon_{34} \arrow[r] \arrow[d, dash] & \epsilon_{00} & & \\
\overline{q}^O & \epsilon_{05} \arrow[r, dash] & \epsilon_{15} \arrow[r, dash] & \epsilon_{25} \arrow[r, dash] & \epsilon_{35} \arrow[d] & & & \\
& & & & \delta_{30} & & &
\end{tikzcd}
$$

Figure 1: The Δ state transition function

The transition function is

$$
\begin{aligned}
&\mathbf{T}_C^{Cstat}(d, P^-, P, P^+, u, Q^-, Q, Q^+, v) = \\
&\quad \{((Q,P)^O.mes(E,\_) \mapsto (d, P^-, P, P^+, u, Q^-, Q, Q^+, v)) \mid \\
&\qquad\qquad (d = \delta_{3i} \vee d = \epsilon_{3i}) \wedge 0 \le i < 5\} \cup \\
&\quad \{((Q^+,P)^O.mes(\overline{E},\_) \mapsto (d, P^-, P, P^+, u, Q^-, Q, Q^+, v)) \mid \\
&\qquad\qquad d = \delta_{35} \vee d = \epsilon_{35}\} \cup \\
&\quad \{((Q,P)^O.chg(E, P^{+\prime}, u') \mapsto (d', P^-, P, P^{+\prime}, u', Q^-, Q, Q^+, v)) \mid \\
&\qquad\qquad (c^O \mapsto d') \in \mathbf{T}^\Delta(d) \wedge d' \ne \epsilon_{00}\} \cup \\
&\quad \{((Q,P)^O.chg(E, P^{+\prime}, u') \mapsto (d', P, P^{+\prime}, \bot, v, Q, Q^+, \bot, u')) \mid \\
&\qquad\qquad (c^O \mapsto d') \in \mathbf{T}^\Delta(d) \wedge d' = \epsilon_{00}\} \cup \\
&\quad \{((Q,P)^I.chg(E, P^+, u) \mapsto (d', P^-, P, P^+, \bot, Q^-, Q, Q^+, v)) \mid \\
&\qquad\qquad (c^I \mapsto d') \in \mathbf{T}^\Delta(d)\} \cup \\
&\quad \{((Q^-,P^-)^I.chg(E, P, v) \mapsto (d', P^-, P, P^+, u, Q^-, Q, Q^+, \bot)) \mid \\
&\qquad\qquad (c^I_- \mapsto d') \in \mathbf{T}^\Delta(d)\} \cup \\
&\quad \{((P,Q)^O.quo(E, u') \mapsto (d', P, P^+, \bot, u', Q^-, Q, Q^+, v)) \mid \\
&\qquad\qquad (q^O \mapsto d') \in \mathbf{T}^\Delta(d)\} \cup \\
&\quad \{((P^-,Q)^I.quo(E, u) \mapsto (d', P^-, P, P^+, \bot, Q^-, Q, Q^+, v)) \mid \\
&\qquad\qquad (q^I \mapsto d') \in \mathbf{T}^\Delta(d)\} \cup \\
&\quad \{((P,P^-)^O.ini(E, Q, u') \mapsto (d', P^-, P, P^+, u', Q^-, Q, Q^+, v)) \mid \\
&\qquad\qquad (i^O \mapsto d') \in \mathbf{T}^\Delta(d)\} \cup \\
&\quad \{((P,P^-)^I.ini(E, Q, u) \mapsto (d', P^-, P, P^+, \bot, Q^-, Q, Q^+, v)) \mid \\
&\qquad\qquad (i^I \mapsto d') \in \mathbf{T}^\Delta(d)\} \cup \\
&\quad \{((P,Q)^O.mes(\overline{E},\_) \mapsto (d, P^-, P, P^+, u, Q^-, Q, Q^+, v)) \mid \\
&\qquad\qquad (d = \delta_{j3} \vee d = \epsilon_{j3}) \wedge 0 \le j < 5\} \cup \\
&\quad \{((P^+,Q)^O.mes(\overline{E},\_) \mapsto (d, P^-, P, P^+, u, Q^-, Q, Q^+, v)) \mid \\
&\qquad\qquad d = \delta_{53} \vee d = \epsilon_{53}\} \cup \\
&\quad \{((P,Q)^O.chg(\overline{E}, Q^{+\prime}, v') \mapsto (d', P^-, P, P^+, u, Q^-, Q, Q^{+\prime}, v')) \mid \\
&\qquad\qquad (\overline{c}^O \mapsto d') \in \mathbf{T}^\Delta(d) \wedge d' \ne \epsilon_{00}\} \cup \\
&\quad \{((P,Q)^O.chg(\overline{E}, Q^{+\prime}, v') \mapsto (d', P, P^+, \bot, v', Q, Q^{+\prime}, \bot, u)) \mid \\
&\qquad\qquad (\overline{c}^O \mapsto d') \in \mathbf{T}^\Delta(d) \wedge d' = \epsilon_{00}\} \cup \\
&\quad \{((P,Q)^I.chg(\overline{E}, Q^+, v) \mapsto (d', P^-, P, P^+, u, Q^-, Q, Q^+, \bot)) \mid \\
&\qquad\qquad (\overline{c}^I \mapsto d') \in \mathbf{T}^\Delta(d)\} \cup \\
&\quad \{((P^-,Q^-)^I.chg(\overline{E}, Q, u) \mapsto (d', P^-, P, P^+, \bot, Q^-, Q, Q^+, v)) \mid \\
&\qquad\qquad (\overline{c}^I_- \mapsto d') \in \mathbf{T}^\Delta(d)\} \cup \\
&\quad \{((Q,P)^O.quo(\overline{E}, v') \mapsto (d', P^-, P, P^+, u, Q, Q^+, \bot, v')) \mid \\
&\qquad\qquad (\overline{q}^O \mapsto d') \in \mathbf{T}^\Delta(d)\} \cup \\
&\quad \{((Q^-,P)^I.quo(\overline{E}, v) \mapsto (d', P^-, P, P^+, u, Q^-, Q, Q^+, \bot)) \mid \\
&\qquad\qquad (\overline{q}^I \mapsto d') \in \mathbf{T}^\Delta(d)\} \cup \\
&\quad \{((Q,Q^-)^O.ini(\overline{E}, P, v') \mapsto (d', P^-, P, P^+, u, Q^-, Q, Q^+, v')) \mid \\
&\qquad\qquad (\overline{i}^O \mapsto d') \in \mathbf{T}^\Delta(d)\} \cup
\end{aligned}
$$

d	$\widetilde{p}^s(P^-)$	$\widetilde{p}^s(P)$	$\widetilde{p}^s(P^+)$
ϵ_{0j}	$\pi_2(E, Q^-, P)$	π_0	
δ_{0j}	$\pi_2(E, Q, P)$	π_0	
η_{1j}	$\pi_3(E, Q, P)$	π_0	
η_{2j}		π_0	
η_{3j}		$\pi_1(E, Q)$	
η_{4j}		$\pi_2(E, Q, P^+)$	π_0
η_{5j}		$\pi_2(E, Q, P^+)$	π_0
ϵ_{05}	$\pi_2(E, Q^-, P)$	$\pi'_0(Q^+, Q)$	
δ_{05}	$\pi_2(E, Q, P)$	$\pi'_0(Q^+, Q)$	
η_{15}	$\pi_3(E, Q, P)$	$\pi'_0(Q^+, Q)$	
η_{25}		$\pi'_0(Q^+, Q)$	
η_{35}		$\pi'_1(E, Q^+, Q)$	

d	$\widetilde{p}^s(Q^-)$	$\widetilde{p}^s(Q)$	$\widetilde{p}^s(Q^+)$
ϵ_{i0}	$\pi_2(\overline{E}, P^-, Q)$	π_0	
δ_{i0}	$\pi_2(\overline{E}, P, Q)$	π_0	
η_{i1}	$\pi_3(\overline{E}, P, Q)$	π_0	
η_{i2}		π_0	
η_{i3}		$\pi_1(\overline{E}, P)$	
η_{i4}		$\pi_2(\overline{E}, P, Q^+)$	π_0
η_{i5}		$\pi_2(\overline{E}, P, Q^+)$	π_0
ϵ_{50}	$\pi_2(\overline{E}, P^-, Q)$	$\pi'_0(P^+, P)$	
δ_{50}	$\pi_2(\overline{E}, P, Q)$	$\pi'_0(P^+, P)$	
η_{51}	$\pi_3(\overline{E}, P, Q)$	$\pi'_0(P^+, P)$	
η_{52}		$\pi'_0(P^+, P)$	
η_{53}		$\pi'_1(\overline{E}, P^+, P)$	

Figure 2: The partial function $\widetilde{p}^s$ for $s = (d, P^-, P, P^+, u, Q^-, Q, Q^+, v)$

$$\{((Q, Q^-)^I.ini(\overline{E}, P, v) \mapsto (d', P^-, P, P^+, u, Q^-, Q, Q^+, \perp)) \mid (\overline{i}^I \mapsto d') \in \mathbf{T}^\Delta(d)\}.$$

The elements of the bisimulation between the *Cstat* system and the *Pstat*, *Mstat* system are the partial functions $\widetilde{p}^s$, $\widetilde{m}^s$. these functions are defined in the tables of figures **??** and **??**. Here, η means δ *or* ϵ. In figure **??**, i and j run from 0 to 4; in figure **??** they run from 0 to 5.

d	$\widetilde{m}^s(P^-, Q^-)$	$\widetilde{m}^s(P^-, Q)$	$\widetilde{m}^s(P, P^-)$	$\widetilde{m}^s(Q, P)$
ϵ_{0j}	$\langle chg(\overline{E}, Q, u)\rangle$			
δ_{0j}		$\langle quo(E, u)\rangle$		
η_{1j}			$\langle\rangle$	
η_{2j}			$\langle ini(E, Q, u)\rangle$	
η_{3j}				$\langle\rangle$
η_{4j}				$\langle chg(E, P^+, u)\rangle$
η_{5j}				

d	$\widetilde{m}^s(Q^-, P^-)$	$\widetilde{m}^s(Q^-, P)$	$\widetilde{m}^s(Q, Q^-)$	$\widetilde{m}^s(P, Q)$
ϵ_{i0}	$\langle chg(E, P, v)\rangle$			
δ_{i0}		$\langle quo(\overline{E}, v)\rangle$		
η_{i1}			$\langle\rangle$	
η_{i2}			$\langle ini(\overline{E}, P, v)\rangle$	
η_{i3}				$\langle\rangle$
η_{i4}				$\langle chg(\overline{E}, Q^+, v)\rangle$
η_{i5}				

Figure 3: The partial function $\widetilde{m}^s$ for $s = (d, P^-, P, P^+, u, Q^-, Q, Q^+, v)$