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1. M.L. Stein & P.R. Stein, "New experimental results on the Goldbach conjecture", *Math. Mag.*, v. 29, 1965, pp. 72-80.

2. G.H. Hardy & J.E. Littlewood, "Some problems of 'partitio numerorum'; III: On the expression of a number as a sum of primes", *Acta Math.*, v. 44, 1923, pp. 1-70.

Table 1

Lower bounds $N_2^{min}(n)$

$N_2^{min}(n)$

(3) $$N_2(n) > \frac{n}{2(\log n)^2} \quad \text{for all } n > 4.$$

Now let us look at the Goldbach curve and the Littlewood-Hardy formula in more detail. Fig. 2 shows a portion of the Goldbach curve at large $n$
As a final check on the Littlewood-Hardy conjecture, the difference $D(n) = |\rho(n) - N_2(n)|$

**3. The conjectures of Stein and Stein.** The two conjectures made by Stein and Stein have also been checked and found to to hold for $n \leq 2 \times 10^7$
We can also look at the number of solutions of $N_2(n) = k$

| $S^{min}(k)$ | $k$ | $S^{min}(k)$ | $k$ |
|---|---|---|---|
| 10 | 2 | 60 | 1202 |
| 20 | 12 | 70 | 2486 |
| 30 | 88 | 80 | 7520 |
| 40 | 252 | 90 | 21347 |
| 50 | 382 | 100 | 24041 |

Table 2

Lower bounds $S^{min}(k)$

The truth of conjecture II is not so clear. Stein and Stein argued that this conjecture was plausible since the Goldbach curve "almost always" has a local maximum where $n \equiv 0$
To see how these exceptions can arise, let us look at the smallest one beyond the range of the Stein and Stein calculation, which is $N_2(190188) = 2069$
We have seen, however, that the equation $N_2(n) = k$

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# A Computer Study of the Goldbach Curve

**By Paul D. Coddington**

**Abstract.** The Goldbach curve $N_2(n)$

**1. Introduction.** The Goldbach conjecture states that every even number greater than 2 can be represented as the sum of two primes. Although it is undoubtedly true, it has remained unproven since its proposal by Goldbach in a letter to Euler in 1742. Stein and Stein [1] investigated Goldbach's conjecture by computing the Goldbach curve $N_2(n)$

CONJECTURE I. *For every integer* $k > 0$

CONJECTURE II. *For every integer* $k > 4$

Conjecture II is quoted by Stein and Stein as being true only for $k > 8$

Stein and Stein also devised a formula which approximates the Goldbach curve, apparently unaware that Littlewood and Hardy had previously proposed a much simpler formula for the asymptotic behavior of $N_2(n)$

$$N_2(n) \sim \rho(n) = C_2 \frac{n}{(\log n)^2 - 2\log n} \prod_{p|n} \frac{p-1}{p-2}, \tag{1}$$

where $p$

$$C_2 = \prod_p \left(1 - \frac{1}{(p-1)^2}\right) \approx 0.6601618. \tag{2}$$

The $2\log n$

In this work the calculation of the Goldbach curve has been extended to values of $n$

**2. The Goldbach curve and the Littlewood-Hardy conjecture.** A notable feature of the Goldbach curve is that it is not "smooth" – two neighboring values of $n$

It is instructive to look at the upper and lower bounds (i.e. the envelope) of the Goldbach curve. Let us define the minimum extent of the curve, $N_2^{min}(n)$

| $N_2^{min}(n)$ | $n$ | $N_2^{min}(n)$ | $n$ |
|---|---|---|---|
| 10 | 632 | 1000 | 195368 |
| 20 | 1448 | 1500 | 317972 |
| 50 | 4688 | 2000 | 445268 |
| 100 | 11672 | 5000 | 1315298 |
| 150 | 19246 | 10000 | 2947918 |
| 200 | 27908 | 15000 | 4719452 |
| 500 | 85616 | 20000 | 6594752 |

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