The Phase Structure of Strings with Extrinsic Curvature

Mark Bowick*, Paul Coddington, Leping Han,
Geoffrey Harris and Enzo Marinari(†)

Dept. of Physics and NPAC,
Syracuse University,
Syracuse, NY 13244, USA

bowick@suhep.phy.syr.edu paulc@sccs.syr.edu han@sccs.syr.edu
gharris@sccs.syr.edu marinari@roma2.infn.it

(†): and Dipartimento di Fisica and INFN,
Università di Roma *Tor Vergata*
Viale della Ricerca Scientifica, 00173 Roma, Italy

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Abstract

We examine a model of non-self-avoiding, fluctuating surfaces as a candidate continuum string theory of surfaces in three dimensions. This model describes Dynamically Triangulated Random Surfaces embedded in three dimensions with an extrinsic curvature dependent action. We analyze, using Monte Carlo simulations, the dramatic crossover behaviour of several observables which characterize the geometry of these surfaces. We then critically discuss whether our observations are indicative of a phase transition.

1 Introduction

In this work, we investigate a theory of fluid, fluctuating random surfaces embedded in three dimensions. Various theories of fluctuating surfaces (string theories) arise in the description of many physical systems. Among these are the 3-dimensional Ising model, the strong interaction (QCD) in the infrared limit and fundamental unified theories of all interactions including gravity. Natural biological membranes, such as lipid bilayers, and artificial membranes, such as micelles and vesicles, also form a rich class of fluctuating surfaces together with interfaces, such as those in microemulsions, between two distinct three-dimensional bulk phases [1]. These latter systems are fluid because their component 'molecules' are loosely bound. Their constituents are arranged so that their net surface tension nearly vanishes; thus these membranes undergo large thermal fluctuations. These biological and chemical membranes exhibit self-avoidance, which we do not take into account in our simulations.

Just as field theories are described by sums over paths, string theories are formally characterized by a functional integral Z which can be written as a sum over surfaces weighted by $\exp(-S)$. Here S denotes the action associated with a particular surface. Note that we work in Euclidean space. To write down our action we introduce an explicit parametrization of a generic surface $\mathcal{M}$ in R^3 with coordinates (σ_1, σ_2) and the embedding $X^\mu(\sigma_i)$. μ runs from 1 to 3 (since we only study the case of a $3d$ embedding space). The induced metric (the pullback of the Euclidean R^3 metric via the embedding) is given by

$$h_{ij} = \partial_{\sigma_i} X^\mu \partial_{\sigma_j} X_\mu \ . \tag{1}$$

We will use Greek letters for the embedding space indices; Roman letters label the coordinate basis on each surface. A particularly natural choice for the action is the Nambu-Goto action, which is proportional to the area of the surface in the embedding space, and is given by

$$S = \mu \int \sqrt{\det(h)}. \tag{2}$$

This action is difficult to quantize, though, since it is non-polynomial. A further complication is that the measure in the path-integral constructed with this action is quite subtle, especially when the theory is discretized. By introducing additional degrees of freedom, in the form of an intrinsic metric g_{ij} on each surface, one can write down the Polyakov action [1].

$$S = \int \sqrt{|\det g|}(g^{ij}\partial_i X^\mu \partial_j X^\mu). \tag{3}$$

This action has proven to be much more tractable analytically. Note that it possesses a large degree of gauge freedom, associated with its invariance under reparametrizations of the metric and the intrinsic surface coordinates. Often, one gauge-fixes the metric to the form $g_{ij} = \exp(\rho)\delta_{ij}$; ρ is referred to as the Liouville mode. The Polyakov action is in fact independent of ρ [2], but this 'Weyl' symmetry is anomalous in general. The anomaly is expressed in terms of the central charge c. By carefully gauge-fixing the entire path-integral one can derive the anomaly condition

$$c_{\mathrm{matter}} + c_\rho - 26 = 0. \tag{4}$$

The first term depends on the space in which the surfaces are embedded, or, in other words, on the particular field theory that lives on the two-dimensional surface. For instance, the central charge associated with D bosons is D (1 per boson); this characterizes a string embedded in R^D [3] The -26 arises from the anomaly contribution from ghosts needed for gauge-fixing reparametrization invariance. Note that when $D = 26$, c_ρ, the Liouville central charge, vanishes and the Liouville mode effectively decouples from the theory. For $D < 26$ one can integrate the matter (X^μ) fields out and obtain an effective Liouville theory. The Liouville theory is highly non-linear; furthermore, its analysis is complicated by the fact that the two-dimensional metric on the surface in which the Liouville field lives depends on the field itself! Nevertheless, with a few general assumptions about the path integral measure, the spectrum

[1] An additional term, which is a topological invariant proportional to the Euler character of the surface is added to both the Polyakov and Nambu-Goto actions. The coefficient of this term is referred to as the 'string coupling'. It is the only dimensionless parameter in the theory; therefore it serves as the perturbative expansion parameter for string theory.

[2] It follows that substituting the solution to the equations of motion back into the Polyakov action yields the Nambu-Goto action. At the quantum level, though, it has only been shown that these actions are equivalent for $D = 26$[2]. In lower dimensions this equivalence has been questioned, for example, by Distler [3]. The work of Polchinski and Strominger also suggests that there are alternate quantizations [4].

[3] A fermion has a central charge of 1/2, so that by studying an Ising model on random fluctuating surfaces, we are effectively studying strings imbedded in 'fractional' dimensions.

of the theory at each order in perturbation theory can be computed when $c_{\rm matter} \leq 1$. Additional exponents, such as the Hausdorff dimension, which characterize both the intrinsic and extrinsic geometry of these surfaces, have also been computed from Liouville theory. These theories are often analyzed on surfaces of fixed area; in this case, the string susceptibility γ_h is determined by the dependence of the generating functional for surfaces of genus h on the area A

$$\ln(Z_h) = \exp(\mu_c A) A^{\gamma_h - 3}, \tag{5}$$

where μ_c is a non-universal constant. An analysis of the Liouville model yields

$$\gamma_h = 2 - \frac{(1-h)}{12}(25 - c_{\rm matter} + \sqrt{(1 - c_{\rm matter})(25 - c_{\rm matter})}). \tag{6}$$

These computations break down in the regime $1 < c_{\rm matter} < 25$, where exponents such as the string susceptibility given in (6) become imaginary.

Further progress in understanding string theory in a low number of dimensions has been made by mapping the sum over surfaces onto an integral over matrices. This mapping is realized by replacing continuum surfaces by their discrete cellular decompositions, such as their triangulations. In this construction, each triangle is equilateral with area 1 in the intrinsic metric; the coordination number at each vertex determines the intrinsic curvature of the surface. The coordinates i label the vertices of the triangulation. Then the discrete analogue of the intrinsic metric is the adjacency matrix C_{ij} whose elements equal 1 if i and j label neighbouring nodes of the triangulation, and vanish otherwise. Two-dimensional diffeomorphism invariance reduces, at this discrete level, to the permutation symmetry of the adjacency matrix. One of the keys, in fact, to the power of this construction is the preservation of this symmetry. The triangulation of a surface of genus h is then dual to a phi-cubed diagram of genus h. The large N ('t Hooft) expansion of an integral over $N \times N$ Hermitian matrices generates these Feynman graphs. In this case the area of the surfaces is not fixed, since all graphs of fixed genus are summed over. The action then acquires a contribution (from Legendre transformation) proportional to the product of the cosmological constant with surface area A. Orthogonal polynomial techniques can then be applied to analyze the matrix integral. Indeed the theory can be exactly solved in the 'double scaling limit' [5] in which the string coupling, cosmological constant and matter couplings (which are determined by the matrix 'potential') are tuned together. These solutions are exact, and thus include non-perturbative information about these string theories, albeit with ambiguities in certain cases. Unfortunately, the matrix models that represent theories with $c_{\mathrm{matter}} > 1$ are too difficult to solve exactly.

It has generally been suspected that these analytic techniques fail for $c_{\mathrm{matter}} > 1$ because the string theory becomes pathological. In this regime, a negative mass-squared particle, which is sometimes referred to as the 'tachyon', comes on shell [4]. It is thought that its presence might engender the proliferation of long tubes with thickness of the scale of the ultraviolet cutoff on surfaces that dominate the string functional integral. Cates, by analyzing the Liouville action, has put forth another perhaps related explanation of this pathology [7]. He points out that in the presence of vortex configurations of the form $\rho \sim -\ln(r)$, reducing the world-sheet cutoff to zero no longer causes the spacetime cutoff to vanish. He then shows that these vortices have negative free-energy when $c_{\mathrm{matter}} > 1$ and proliferate to the extent that the Liouville partition function becomes ill-defined with a finite spacetime cutoff and zero cosmological constant. For positive cosmological constant, it then appears plausible that these vortex configurations would still appear, though perhaps the partition function will be finite. The predominant surfaces in the functional integral would thus be subject to large fluctuations of their internal geometry. Recent simulations of multiple Potts models with c_{matter} less than, equal to, and greater than 1 coupled to gravity have been performed [8]. These simulations, however, do not show any dramatic changes in the behaviour of the internal geometry of the dominant surfaces as c_{matter} is increased beyond 1. More analytic and numerical work clearly needs to be done to determine what exactly happens as c_{matter} becomes greater than 1.

Monte Carlo simulations of strings embedded in flat space for $D > 1$ do indicate that these theories fail to describe the fluctuations of two dimensional *smoothly embedded* surfaces in the continuum limit. The normals to the surfaces dominating the simulations are uncorrelated over the distance of a few lattice spacings. These surfaces also appear to have a large (greater than 8), or perhaps infinite, extrinsic Hausdorff dimension; they resemble

[4] More precisely, one encounters these instabilities in Liouville theory when the quantity $c - 24\Delta > 1$, where c denotes the central charge of the matter theory which describes the embedding of the surfaces and Δ is the conformal weight of the lowest weight state in this theory [6]. Since here we are considering flat space, $c = D$ and $\Delta = 0$.

branched polymers [5]. Such configurations should not describe, for instance, the domains of the 3d Ising model or QCD strings. We would thus like to find a modified string theory that is dominated by smoother surfaces.

The tachyon, and apparently these related instabilities, can be eliminated in particular cases by introducing fermionic coordinates and supersymmetry on the worldsheet, and implementing an appropriate projection of states. The fermions, presumably, effectively smooth out the surfaces. This would be consistent with what has been observed for one-dimensional geometries; the random walk of a spin one-half particle has Hausdorff dimension one and thus appears to be smooth [9]. Many authors have proposed an alternative modification of the string action [10, 11, 12, 13] via the addition of a term that directly suppresses extrinsic curvature[6]. We shall examine this class of theories in this talk.

To characterize the geometry of our surfaces further, we associate with each point in our generic surface $\mathcal{M}$ tangent vectors ($t^\mu_i \in T\mathcal{M}$) and a normal vector $n^\mu \in T\mathcal{M}^\perp$. The extrinsic curvature matrix K_{ij} (the second fundamental form) can be defined by

$$\partial_i n^\mu = -K_{ij} t^{\mu j} \ . \tag{7}$$

The eigenvalues of this matrix are the inverses of the radii of curvature of $\mathcal{M}$. One usually describes the geometry of these surfaces in terms of the mean curvature [15, 16]

$$H = \frac{1}{2} h^{ij} K_{ij} \ , \tag{8}$$

and the Gaussian curvature

$$K = \epsilon^{ik} \epsilon^{jl} K_{ij} K_{kl} \ . \tag{9}$$

One can show that the Gaussian curvature can be computed solely from the metric h_{ij}, while the mean curvature depends explicitly on the embedding X^μ.

[5] As above, singular configurations also dominate the Gaussian theory, which is essentially a theory of free random walks, rather than surfaces.

[6] In fact, integrating the fermions out of the Green-Schwarz superstring yields an action similar to the one we consider, but with the addition of a complex Wess-Zumino type term [14].

Our lattice model is constructed by triangulating each surface, as we discussed above in the context of matrix models. Each node of the triangulation is embedded in R^3 by the functions X_i^μ; i labels the ith node and μ runs from 1 to 3. We also associate a normal vector $(n^\mu)_{\hat{k}}$ with each triangle (indices with hats label the triangles). We shall study the theory defined by the action

$$S = S_G + \lambda S_E = \sum_{i,j,\mu} C_{ij}(X_i^\mu - X_j^\mu)^2 + \lambda \sum_{\hat{k},\hat{l},\mu} C^{\hat{k}\hat{l}}(1 - n_{\hat{k}}^\mu \cdot n_{\hat{l}}^\mu). \qquad (10)$$

This model has been examined in references [17, 18, 19, 20, 21, 22, 23]. For $\lambda = 0$ this is simply a discretization of the Polyakov string action. The final term, which depends on the discretized extrinsic curvature, introduces a ferromagnetic interaction between surface normals, which one might hope would cause smoother surfaces to dominate the partition function. We would like to know if there is a smooth phase and a phase transition (at some finite λ_c) between this phase and the crumpled phase observed at $\lambda = 0$. If this were so, an interesting continuum limit of this lattice model could perhaps be constructed at this phase transition point, yielding a new continuum string theory.

The action we simulate is in fact a particularly natural discretization of

$$S = \int \sqrt{|\det g|}(g^{ij}\partial_i X^\mu \partial_j X^\mu + \frac{\lambda}{2} g^{ij} h^{kl} K_{ik} K_{jl}) \ . \qquad (11)$$

Note that the second term in the action is manifestly positive, Weyl and reparametrization invariant, and that λ is a dimensionless coupling. So, it is not clear whether or not this term is relevant. Mean field (large D) and perturbative RG calculations have been performed using similar actions, such as

$$S = \int d^2\sigma \ (\mu_0 \sqrt{\det h} + \frac{1}{\alpha}\sqrt{\det h}(h^{ij}K_{ij})^2) \ , \qquad (12)$$

in the regime in which the string tension μ_0 is small (unlike the usual particle physics limit of string theory, which is characterized by large μ_0). After integrating out fluctuations of the embedding X^μ between momentum scales Λ and $\tilde{\Lambda}$, it is found that the renormalization of the extrinsic curvature coupling is given to one-loop order by

$$\beta(\alpha) \equiv \Lambda \frac{d\alpha}{d\Lambda} = -\frac{3}{4\pi}\alpha^2 \ , \qquad (13)$$

so that α is driven to infinity in the infra-red. This theory thus exhibits asymptotic freedom. Surfaces are smooth (the normals are correlated) below a persistence length [24]

$$\xi_p \sim \exp(\frac{4\pi}{3\alpha_{bare}}) \tag{14}$$

and are disordered above this scale. Some intuition into this result can be gained by observing that this theory is similar to the $O(3)$ sigma model, which is asymptotically free [9]. The normals to M are the analogues of $O(3)$ vectors, though in this case they are constrained to be normal to a surface governed by the action (11).

The analytic results, therefore, do not indicate that we should anticipate a finite λ phase transition. Note, though, that the RG calculations are based on a Nambu-Goto type action although we simulate an extension of the Polyakov action. Since the two actions are not clearly equivalent, *particularly* when extrinsic curvature dependent terms are added, we cannot simply assume that these analytic and our numerical results should agree.

2 The Simulation

We have considered triangulations with the topology of the torus, to minimize finite size effects. The above action was used, with the BRST invariant measure utilized also by Baillie, Johnston and Williams [18], so that

$$Z = \sum_{G \in T(1)} \int \prod_{\mu,i} dX_i^\mu \prod_i q_i^{\frac{D}{2}} \exp(-S_G - \lambda S_E) \ , \tag{15}$$

where $D = 3$, q_i is the connectivity of the ith vertex and $T(1)$ refers to the set of triangulations of genus 1. We used the standard Metropolis algorithm to update our configurations. To sweep through the space of triangulations we performed flips on randomly chosen links. Flips were automatically rejected if they yielded a degenerate triangulation. After a set of $3M$ flips was performed, $3M$ randomly selected embedding coordinates were updated via random shifts from a flat distribution. Most of the Monte Carlo simulations were performed on HP-9000 (720 and 750 series) workstations; we also collected some data by simulating lattices on each of the 32 nodes of a CM-5.

N	$C(\lambda)^{\max}$	λ_c
36	3.484(8)	1.425(35)
72	4.571(15)	1.410(15)
144	5.37(14)	1.395(30)
288	5.55(7)	1.410(25)
576	5.81(17)	1.425(30)

Table 1: The maximum of the specific heat and its position, with errors, for different lattice sizes.

We ran on lattices ranging in size from $N = 36$ to 576 (N signifies the number of vertices) with 4 to 7 different values of λ for each N. Most of the data was this data was taken in the region $\lambda \in (1.325, 1.475)$. For small N, the runs consisted of 3×10^6 sweeps, while we performed longer runs (of up to 27×10^6 sweeps for $N = 576$) for larger lattices, because the autocorrelation times for our simulations were very large. (The correlation time for the radius of gyration was greater than 10^6 sweeps for $N = 576$!) To determine our observables as a function of λ we used a histogram reconstruction procedure. We patched different histograms by weighting them with the associated statistical indetermination (which was estimated by a jack-knife binned procedure). Various consistency checks indicate that this procedure is very reliable.

3 Observables

We measured the edge action S_E and the associated specific heat $C(\lambda) \equiv \frac{\lambda^2}{N}(< S_E^2 > - < S_E >^2)$. In Fig. 1 we plot the specific heat curve (constructed via the histogram procedure) and we tabulate its maximum and peak position for various lattice sizes in Table 1.

We see that the specific heat peak grows vigorously with N for small lattices, but that this growth quickly levels off for larger N. These observations agree fairly well with previous work [20, 21, 22]. For the larger lattices it appears that the peak position shifts very slowly towards higher values of λ, though this increase is not statistically significant. The shape of the peak does not change dramatically with N; it narrows perhaps a bit between $N = 144$ and $N = 576$.

To determine how the mean size of the dominant surfaces depend on λ, we measured the squared radius of gyration R_G;

$$R_G \equiv \frac{1}{N} \sum_{i,\mu} (X_i^\mu - X_{\text{com}}^\mu)^2 \ , \tag{16}$$

where the com subscript refers to the center of mass of the surface. By measuring the N dependence of the gyration radius, one can extract a value for the extrinsic Hausdorff dimension, which is given by

$$R_G \sim N^\nu \sim N^{\frac{2}{d_{\text{extr}}}} \ . \tag{17}$$

We plot R_G in Fig. 2(a), clearly the size of the mean surface size increases dramatically for λ near 1.4. In Fig. 2(b), we plot the effective Hausdorff dimension, given by

$$\nu(N) \equiv \frac{\log \frac{R_G(N)}{R_G(\frac{N}{2})}}{\log(2)} \ . \tag{18}$$

In the large λ limit $\nu \to 1$ and $d_{extr} \to 2$, as expected for flat surfaces. In the low λ limit d_{extr} becomes very large. In the pseudo-critical region ν is a linear function of λ. Curiously enough, the latter curve yields a Hausdorff dimension of 4, a value characteristic of branched polymers, near the location of the specific heat peak. This value is not particularly reliable though because of finite-size effects and also because it changes rapidly in this region. In ref. [22] a value compatible with ours ($D_H(\lambda_c) > 3.4$) is quoted for the critical theory. We stress however (and also here we are in complete agreement with [22]) that the Hausdorff dimension in the pseudo-critical region depends heavily and quite unusually on N.

In both the high and low λ regions finite size effects are quite small (compatible with zero to one standard deviation). In the pseudo-critical region, on the contrary, finite size effects are large. This effect cannot be explained by the shift in λ which one gets from the shift of the peak of the specific heat, which is far too small.

We also measured the magnitude of the extrinsic Gaussian curvature, $\int \mid K \mid \sqrt{\mid h \mid}$, given by

$$\mid \mathcal{K} \mid = \frac{1}{N} \sum_i \mid 2\pi - \sum_{\hat{j}} \phi_i^{\hat{j}} \mid \ . \tag{19}$$

Here $\phi_i^{\hat{j}}$ denotes the angle subtended by the $\hat{j}$th triangle at the ith vertex. This quantity, plotted in Fig. 3, measures the magnitude of the deficit angle in the embedding space averaged over all vertices. Note that the mean Gaussian curvature decreases rapidly in the neighborhood of $\lambda = 1.4$, indicating that a sharp crossover is occuring in this system. From this plot we can see that finite size effects increase with λ. They do not appear to peak in the region about $\lambda = 1.4$ as one might expect for a typical phase transition.

The magnitude of the intrinsic Gaussian curvature, $\mid \mathcal{R} \mid$, given by

$$\mid \mathcal{R} \mid = \frac{\pi}{3N} \sum_i \mid 6 - q_i \mid \ , \tag{20}$$

is shown in Fig. 4. When the intrinsic and extrinsic metrics are equal, the intrinsic and extrinsic deficit angles are identical and $K = R/2$. Both extrinsic and extrinsic curvatures behave in a qualitatively similar manner; $\mid \mathcal{R} \mid$ drops off rapidly, just as $\mid \mathcal{K} \mid$ does. Through the peak region, though, $\mid \mathcal{K} \mid$ decreases by about a factor of 5 while $\mid \mathcal{R} \mid$ diminishes to only about .6 of its value on the left-hand side of the peak. Since the action explicitly suppresses mean curvature, and the mean and extrinsic Gaussian curvature are closely related (for instance, $H^2 > \frac{K}{2}$), we would expect that for large λ extrinsic fluctuations would be suppressed much more than fluctuations of intrinsic geometry.

The question of the equivalence of the Nambu-Goto and Polyakov actions motivated us to study the correlations between intrinsic geometry (which is not introduced independently in the Nambu-Goto formulation) and extrinsic geometry. We measured the quantity which we refer to as $\mathcal{K} * \mathcal{R}$

$$\mathcal{K} * \mathcal{R} \equiv \frac{\int KR}{\sqrt{\int K^2 \int R^2}} = \frac{\sum_i (2\pi - \sum_{\hat{j}} \phi_i^{\hat{j}})(6 - q_i)}{\sqrt{\sum_i (2\pi - \sum_{\hat{j}} \phi_i^{\hat{j}})^2 \sum_i (6 - q_i)^2}} \ . \tag{21}$$

This quantity is 1 when the metrics are equal, 0 if they are un- correlated, and negative when these curvatures are anti-correlated. The plot of $\mathcal{K} * \mathcal{R}$ in Fig. 5 indicates that intrinsic and extrinsic geometry are strongly correlated for small λ, but as one passes through the peak region they become decorrelated. This is not particularly surprising, given that the action directly suppresses only extrinsic fluctuations. Note that RG calculations based on the Nambu-Goto action plus an extrinsic curvature term perturb about a background that is both intrinsically and extrinsically flat. Given the observed decorrelation between intrinsic and extrinsic geometry, we would not anticipate that this background appears in the low- temperature limit of the model which we simulate.

We also measured various other observables which characterize both the intrinsic and extrinsic geometry of these surfaces. These measurements are discussed in another write-up of this work [23]. They all exhibit sharp crossover behaviour in the region near $\lambda = 1.4$. We found that the auto-correlation times of these observables grew rapidly as λ increased, but we did not note any maximum in these times in the region about $\lambda = 1.4$.

The crossover behaviour became also quite apparent when we examined typical snapshots of our simulated surfaces for various values of λ. In Figs. 6 (a)-(d) we present pictures of typical surfaces for $\lambda = .8, 1.3, 1.5$, and 2.0. Note that the surfaces rapidly become smoother and the normal-normal correlation length increases significantly as one passes from the second to the third of these pictures, which correspond to only slightly different values of λ.

4 Interpretation

This model of crumpled surfaces appears to exhibit sharp crossover behaviour in the region around $\lambda = 1.4$. The sharp change in the magnitude of the Gaussian curvature, the radius of gyration and other observables indicates that the normals acquire long-range correlations, up to the size of the systems we examine. The zero string tension measurement of [22] also shows that the disordered regime differs from the regime in which the surfaces are ordered (up to scale of the lattices that are simulated) by only a small shift in λ. This evidence might indicate the presence of a phase transition at this point. Since the peak growth rapidly diminishes for large N, such a phase transition would likely be higher than second order. Still, the apparent absence of diverging correlation times and, in some cases, increasing finite size effects in the peak region leads us to question whether we are actually observing a typical phase transition.

There are indeed other possible interpretations of our data. Note that the surfaces which we simulate are quite small. For instance, if the surfaces in our simulations had an intrinsic dimension of 2.87 (characteristic of $D =$

0 gravity), they would have roughly a linear size of fewer than 10 lattice spacings [7].

Perhaps the simplest alternative explanation for the presence of this peak is suggested by the arguments of Kroll and Gompper [25]. They argue that the peak occurs when the persistence length of the system approaches the size of the lattice ($\xi_p \sim N^{\frac{1}{d}}$); d denotes the intrinsic Hausdorff dimension. Fluctuations on a larger scale become more important. When this scale is greater than the lattice size these fluctuations are suppressed. Thus one might surmise that the specific heat will drop for large λ. Typically, the persistence length grows as $\xi_p \sim \exp(C\lambda)$; C is inversely proportional to the leading coefficient of the beta function. We would expect that the peak position should shift to the right with increasing N in this scenario as

$$\lambda_{peak}(N') - \lambda_{peak}(N) = \frac{\ln(\frac{N'}{N})}{dC}. \tag{22}$$

Quite a large value of C is needed to explain the rapid crossover; roughly values of $C \sim 10, d_{intr} \sim 3$ are more or less consistent with the magnitude of the peak shift and crossover width. The RG calculations using different forms of the action yield $C = \frac{4\pi}{3}$ (see equation 14), but this may not apply to the action we simulate. This reasoning also indicates that the peak should widen as the lattice size increases; we do not observe this at all. It seems plausible though that these arguments, based only on the leading term of the high λ expansion, are too naive.

An alternative scenario, which builds on the ideas in the above paragraph, is suggested by the tantalizing similarities between the results of our fluid surface simulations and what has been observed for the $d = 4$ $SU(2)$ Lattice Gauge Theory [26] and for the $d = 2$ $O(3)$ model.

The $O(3)$ model, which is thought to be asymptotically free, exhibits a specific heat peak near $\beta = 1.4$ (first measured via Monte Carlo simulations by Colot [27]). The origin of this peak is understood [28]; it is due to the fluctuations of the sigma particle, a low-mass bound state of the massless $O(3)$ pions. The sigma induces short-range order and contributes to the specific heat as a degree of freedom only at high temperatures (when the correlation length in the system becomes smaller than its inverse mass). The peak thus

[7] Of course, our lattices are too small, by one or two orders of magnitude, to really exhibit a convincing fractal structure.

occurs at the beginning of the crossover regime, when the correlation length is several lattice spacings.

According to the low temperature expansion, the correlation length grows as $\xi \sim \exp(2\pi\beta)/\beta$. Thus one would expect a fairly rapid crossover in the $O(3)$ model; the correlation length should increase by roughly a factor of 9 when β is shifted by about 0.35. Such a crossover is indeed observed, though it is not so apparent that it is as dramatic as the crossover behaviour observed for fluid surfaces. [8]

Recent simulations of the $O(3)$ model [29] indicate that the specific heat peak grows significantly when the lattice size L is increased from 5 to 15, and that virtually no growth in peak height is evident as L is increased further up to 100. Furthermore, the peak position shifts to the right as L grows and then appears to stabilize for large L. This is more or less what we observe in our simulations of fluid surfaces, on lattices of small size. We point out these similarities largely to emphasize that there does exist an asymptotically free theory (with low mass excitations) which exhibits crossover behaviour qualitatively similar to that observed in our simulations. The analogy is perhaps deeper, though, since the fluid surface action (with extrinsic curvature) in certain guises looks like a sigma model action. It would not therefore be so surprising from this point of view to find a sigma particle in these theories, perhaps associated with $(\hat{n}^2-1)$, in which $\hat{n}$ denotes the unit normal to our surfaces.

Another additional possibility is that fluctuations of the intrinsic geometry (the Liouville mode) are responsible for short-range order and contribute to the specific heat peak.

5 Conclusion

We have introduced a model of fluid random surfaces with an extrinsic curvature dependent action and explored its phase diagram. Unfortunately, we have been unable to determine if our model undergoes a phase (crumpling) transition at finite coupling. We have observed dramatic crossover behavior for particular observables in our Monte Carlo simulations, but on the other

[8] To compare quantitatively the width of the crossover regimes for these two models it would be necessary to measure a correlation length (perhaps extracted from the normal-normal correlation function) in these random surface simulations.

hand, the correlation times and certain finite-size effects do not behave as one would expect in the presence of a phase transition. The behavior of other lattice models also indicates that it is possible that we are observing the effects of finite-mass excitations on small lattices, rather than a phase transition. We hope that future work will clarify this murky state of affairs, to determine if there indeed exists a crumpling transition for fluid surfaces.

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7 Figure Captions

Fig. 1 The specific heat $C(\lambda)$ as a function of λ. As in all other pictures, filled circles and a dotted line correspond to $N = 144$, crosses and a dashed line indicate $N = 288$, and empty squares and a solid line represent $N = 576$.

Fig. 2a The gyration radius R_G defined in (16), plotted as in Fig. 1.

Fig. 2b The effective inverse Hausdorff dimension ν as a function of λ, as defined in (17). The filled dots and the dashed curve are from a fit to the $N = 288$ and $N = 144$ data, while the empty dots and solid curve represent the fit to $N = 576$ and $N = 288$.

Fig. 3 The extrinsic Gaussian curvature $\mid \mathcal{K} \mid$ defined in (19), plotted as in Fig. 1.

Fig. 4 The intrinsic curvature $\mid \mathcal{R} \mid$ defined in (20), plotted as in Fig.1.

Fig. 5 The intrinsic extrinsic curvature correlation, as defined in (21), plotted as in Fig.1.

Fig. 6a A snapshot of a 576 node surface of toroidal topology for $\lambda = 0.8$.

Fig. 6b As in Fig. 6a for $\lambda = 1.3$.

Fig. 6c As in Fig. 6a for $\lambda = 1.5$.

Fig. 6d As in Fig. 6a for $\lambda = 2.0$.