Design and Analysis of Algorithms
Computer Science 45
Due: in class Friday, April 25, 1997

Neal Young

Problem Set 3

1. In class you figured out that the number of 5-digit decimal numbers with their digits summing to 27 was
$$[z^{27}]\left(\frac{1-z^{10}}{1-z}\right)^5.$$
(Recall that $[z^n]f(z)$ denotes the coefficient of z^n in $f(z)$.)
Figure out the actual number.[1]

2. Let S be the set of binary strings not containing "011" as a substring.
   (a) Give a deterministic finite automata accepting S.
   (b) Derive a generating function $\sum_n s_n z^n$ for S, where s_n is the number of strings in S of size n.[2]
   (c) Use your generating function to obtain the best estimates you can for s_n.

3. The goal of this problem is to derive a closed form for the generating function $\sum_{n=0}^{\infty}\binom{2n}{n}z^{2n}$.
   Let W be the set of binary strings with with equal numbers of 0's and 1's.
   (a) Why are there exactly $\binom{2n}{n}$ strings of length $2n$ in W?
   Let P be the subset of W such that every prefix of the string has as many 0's as 1's (essentially this is balanced parens).
   Let Q be the subset of P such that every *proper* prefix of the string has *more* 0's than 1's.
   (b) Argue that $P \equiv Q^*$.
   (c) Argue that $Q \equiv \{0\} \times P \times \{1\}$.
   (d) Argue that $W \equiv (\{\mathrm{up}, \mathrm{down}\} \times Q)^*$.
   (e) Use (b,c,d) to derive a generating function for W.

4. Exercise 33.7-2.

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[1] HINT: Use $(1-z^{10})^5 = 1 - 5z^{10} + 10z^{20} - 10z^{30} + 5z^{40} - z^{50}$.

[2] HINT: for each state q of your DFA, let S_q denote the set of strings taking the DFA from the start state to q. Set up an unambiguous context free grammar for the sets S_q, from this obtain a set of equations relating the generating functions for the S_q's. Solve the equations.