CIS 600 Computational Geometry	Spring 1995
Lecture 10: Thursday 2/14/95	
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Point Location Problem in Planar Subdivision using Triangulation Refinement Method (Kirkpatrick)

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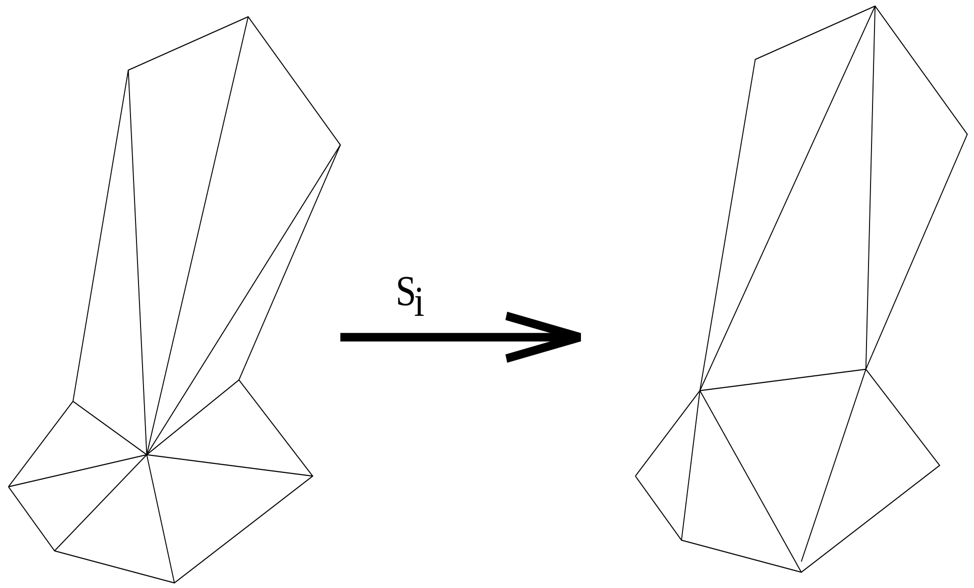
Procedure POINT-LOCATION
    begin if(z not belong to TRIANGLE(root) then print "z belongs to unbounded region"
              else begin v:= root
                 while( v has descendents) do
                  for each u belonging to T(v) do if(z belongs to TRAINGLE(u))
                                                  then v :=u;
           print v;
     end
end.
```

Figure 1: Point Location Algorithm

For this algorithm to be efficient it should satisfy the following properties:
(a) Longest path from root to any leaf should be as small as possible.
(b) Number of descendants should be less.

We must show that there is a creterion for selecting the sets of vertices to be removed. The criterion is(here K is an integer, to be carefully chosen);" Remove a set of nonadjacent vertices of degree less than K." The order in which the vertices are inspected and possibly



removed are irrevalent: one starts arbitrarily with one, and marks its neighbors(they cannot be reomoved); then one continues unitl no more are unmarked.

The criterion does the trick. Since removal of a vertex of a degree less than K gives rise to polygon with less than K edges, each of the replaced triangles intersects at most K - 2 = H new triangles.

Goal: We will like to remove vertices with small degree i.e.
degree(v) $\leq$ H

When going from S_{i-1} to S_i, we want to remove atleast fraction 0 $\leq$ 1- $\alpha \leq$ 1 of vertices in S_{i-1}.
For property (a) to hold good, choose vertices of small degree
For property (b) to hold good, remove a fraction (1-α) of the points in S_{i-1} each time. ($0 < \alpha < 1$).
Thus K =$\lceil log_{1/\alpha} n \rceil$
We know that S = a planar subdivision.
Let m = number of edges, n = number of vertices
1) From Euler's formula on planar graphs we know that m $\leq$ 3n -6
2) Unless there are no internal vertices (in which case the problem is trival), each of the

three vertices on the boundary has degree at least three. Since there are 3n-6 edges, and each edge contributes 2 to the total degree, the total vertex degree is $< 6N$. This immediately yield that there are atleast n/2 vertices of degree less thab 12. Therefore let k = 12. Let v be the number of selected vertices: since each selected vertex may eliminate at most k-1 = 11 adjacent vertices and the three boundary vertices are nonselectable, we have

$v \geq \lfloor \frac{1}{12}(\frac{n}{2} - 3) \rfloor$

Therefore each time number of vertices removed is $\approx \frac{1}{24}$th of the previous. Therefore H $\leq$ 11

$\Rightarrow$ k = O(log n).

For any graph $\sum$ deg (v) = 2m

$\sum deg(v) \leq 6n - 12 < 6n$

$\Rightarrow$ There are atleast n/2 vertices of degree less than 12.

Storage requirment:

Storage is required for:

(1) Number of nodes in D.

(2) Number of edges in D.

From Euler's formula on planar graphs we know that n - m + f = 2

where n = number of nodes, m = number of edges, f = number of faces

$\Rightarrow$ f = 2 + m - n

Also m $\leq 3n - 6$

$\Rightarrow f \leq$ 2 + 3n -6 -n ¡ 2n

$\Rightarrow f < 2n$

Suppose S_i has n_i vertices

S_i has atmost $2n_i$ faces(triangles).

$n_i \leq \alpha\ n_{i-1}$

Therefore total number of triangles is atmost

$2(n_1 + n_2 + n_3 + ... + n_k) \leq 2n_1(1 + \alpha + \alpha^2 + ... + \alpha^k)$

$< \frac{2n_1}{1-\alpha}$.

$\Rightarrow$ Total number of triangles = O(n_1).

Thus space is linear .

(Because edges ≤ 12 * number of nodes).

Building D:

Triangulation can be done in O(n)

Going from S_{i-1} to S_i requires two steps.

(1) Find an independent set and remove vertices

(2) Retriangulate new faces.

Step (1) can be performed in O(n_{i-1}).

Step (2) can be performed in O(n_{i-1}).