Final Exam., CIS 675

Design and Analysis of Algorithms

May 7, 1996

Note: Answer all questions. Points assigned to each problem are indicated before the problem. Print your name and social security number **clearly**. If you take the Hartmann's Fifth on any question, indicate it clearly on the cover sheet.

Name: ____

SSN: ____

1. (10 points) Can the 0-1 integer knapsack problem be solved in polynomial time if the capacity of the knapsack is $W = n^2$, where n is the number of objects and ith object has non-negative weight w_i and profits p_i for $i = 1, \ldots, n$? If yes, then why is the 0-1 integer knapsack problem NP-complete?

2. (5 points) *TRUE* or *FALSE* (give reasons): if P = NP then NP = co-NP?

3. (10 points) You are given n numbers $a_1, a_2, \ldots, a_n$. Give a $\mathrm{O}(n)$ algorithm to find the median of $\{a_{ij} : i, j = 1, \ldots, n\}$ where,

$$a_{ij} = \min_{i,j}\{a_i, a_j\}; \;\; i, j = 1, \ldots, n$$

4. (5 points) Suppose we use a random hash function h to hash n distinct keys into an array of length m. What is the expected number of collisions? More precisely, what is the expected cardinality of the set $\{(x, y) : h(x) = h(y)\}$.

5. (10 points) Use Ford–Fulkerson method to find the maximum flow for the following directed weighted graph from the source node s to the destination node t.