Programming Language Comprehensive, January, 1996

INSTRUCTIONS. Do all 6 questions. You have 2 hours.

Question 1: The command sequence below is partially correct with respect to the pre- and post-conditions given. Explain why the command sequence terminates whenever the precondition holds. Give a loop invariant that, together with the assumption of termination, implies the postcondition.

$\{\mathrm{Y} \geq 0\}$
J := 1;
Z := X;
K := Y;
while K > 0 **do**
    if odd(K) **then** J := J * Z;
    K:=(K div 2);
    Z := Z * Z)
$\{\mathrm{J} = \mathrm{X}^{\mathrm{Y}}\}$

Question 2: Suppose we have a programming language **P** and we provide a denotational semantics for it. Suppose we then go on to provide a formal system of axioms and rules. Discuss the relationship between the denotational semantics and the formal system of axioms and rules. Give an example of the denotation of a construct from Tennent's mini-imperative language. What is the rule of inference corresponding to the **while**-statement.

Question 3: State Hoare's axiom for assignment statements, and explain why it is valid. Explain why a specification such as $\{\mathbf{false}\}\,\mathrm{C}\,\{0 = 1\}$ is valid.

Question 4: Give the semantic equation for the local-definition block

$$\mathbf{let}\ \iota\ \mathbf{be}\ P\ \mathbf{in}\ Q\,.$$

Explain how the informal expression

$$(\lambda\,\iota\,Q)P$$

explains the semantics of the local definition block.

Question 5: Give an example of a syntactical programming language construct which is NOT context-free, and briefly sketch an explanation for your example.

Question 6: What is the fundamental idea behind the semantics of

$$\mathbf{letrec}\ \iota : \Theta\ \mathbf{be}\ P\ \mathbf{in}\ Q$$

Explain the distinctions between

$$\mathbf{letrec}\ n : \mathbf{val[nat]}\ \mathbf{be}\ n + 1\ \mathbf{in}\ \mathbf{true}$$

and

$$\mathbf{let}\ n\ \mathbf{be}\ n + 1\ \mathbf{in}\ \mathbf{true}$$