Programming Languages Comprehensive Exam

August 25, 1996

The proctor has a copy of *Semantics of Programming Languages,* by R.D. Tennent. You may ask the proctor at most twice to let you see Tennent's book to look up a definition if you need to, but you will find that the attached pages copied from Tennent's book will suffice for all five of the questions.

Question 1) Give a rigorous proof, using our axioms and rules of inference, of the validity of the specification

$\{A \geq 0\}$
X := A;
Y := 0;
while X > Y **do**
    (X := X - 1; Y := Y + 1);
$\{X = A \text{ div } 2\}$

If you can't get your proof to work out completely, then give as much of the proof as you can and discuss what is obstructing your proof, for some partial credit.

Question 2) There is a positive integer, V, called Vanogradoff's number. The definition of V is NOT relevant to this problem, but what IS relevant is that nobody *knows* what V is (although for reasons also not relevant to this problem it is strongly *believed* that V = 1).

Suppose we define a recursive function h as shown below using **letrec**. The semantics of the simple applicative language, together with the semantics of **letrec**, gives a method for computing the values of **letrec** expressions. Computing the value of the letrec expression given below involves the solution of the equation

$$p = [\![P]\!](u \mid h \mapsto p)\,.$$

Consider

$[\![$ **letrec** h : **val**[**nat**] $\rightarrow$ **val**[**nat**] **be**
    λ n:**val**[**nat**].**if** n = 0 **then** 0 **else** 1 + h(n **div** 2)
in h(n) $]\!]$ (u | n $\mapsto$ V)

Show the evaluation of this expression through two recursive calls. Show all steps that are relevant to showing how recursive evaluation works. Deciding what is relevant is part of giving a good answer to this question.

Question 3) Calculate and simplify as much as you can:

$$[\![\textbf{let x be let x be y in x in let x be x in x}]\!] \, u$$

Show all steps.

Question 4) Let D and E be domains. Prove that a continuous function $f : |D| \longrightarrow |E|$ must be monotone.

Question 5) Suppose that D is a domain and $f : |D| \longrightarrow |D|$. ($f$ might not be continuous.) Suppose that for every chain d in D^ω,

$$f(\bigsqcup_{i \in \omega} d_i) \leq \bigsqcup_{i \in \omega} f(d_i) \, .$$

This property is called "finitary". (If the property holds for all directed sets and not just chains it is called compactness.) Give an example of a domain D and a finitary function f which is not monotone. Show that if f is finitary and monotone, then it is continuous.